Chapter 10: TEPC Response Functions

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TEPC Response Function

Preface
The tissue equivalent proportional counter had the purpose of providing the energy absorbed from a radiation field and an estimate of the corresponding linear energy transfer (LET) for evaluation of radiation quality to convert to dose equivalent. It was the recognition of the limitations in estimating LET which lead to a new approach to dosimetry, microdosimetry, and the corresponding emphasis on energy deposit in a small tissue volume as the driver of biological response with the defined quantity of lineal energy. In many circumstances, the average of the lineal energy and LET are closely related and has provided a basis for estimating dose equivalent. Still in many cases the lineal is poorly related to LET and brings into question the usefulness as a general purpose device. These relationships are examined in this paper.

Introduction
The energy deposited within a gas from ionizing radiation consists of discrete events with energy partitioning among excitation and ionization processes. As a result, the gas proportional counter allows a measure of the energy deposited in the gas by these discrete ionization events and an accounting of the number of such events. These instruments were first utilized in galactic cosmic ray studies of the bimodal attenuation in the atmosphere of these radiations. This bimodal attenuation consists of, as interpreted by McClure and Pomerantz (1950), a fast attenuating high charge and energy component discovered only a few years earlier (Frier et al. 1948) and a more slowly attenuating component associated with light ion induced nuclear disintegration events. It was only after the introduction of Linear Energy Transfer (LET) as an explanation of radiation quality (Lea 1946, Zirkle et al. 1952) that the use of gas proportional counters were developed as a means of measuring LET for dosimetric purposes (Rossi and Rosenzweig 1955). It was in the difficulties of interpreting the LET results so measured that the randomness of the deposition events became understood and the realization that LET itself was possibly less meaningful than the uncorrected data of the energy deposit spectrum (Rossi 1959, Kellerer 1996). It has been suggested that the usual LET dependent quality factor, Q, be replaced by a microdosimetric (lineal energy) dependent quality factor (ICRU 1986) for use in protection practice.

One of the recognized difficulties of implementing a lineal energy dependent quality factor is limitation in estimating the lineal energy spectra in computational shield design practice (ICRU 1986). In spite of great advances in understanding the physical processes associated with microdosimetry (ICRU 1983), a simplified system of estimation using a triangular lineal energy response model with its many limitations was suggested (ICRU 1986). It was the decision of the ICRP (1991) not to implement such a system although a re-evaluation of the LET dependent quality factor was given. Still, the microdosimeter has been very useful as a dosimetric tool as an indicator of spectral distribution of radiation components of different LET quality. However, the use of such an
instrument to evaluate the degree to which the environment is understood is limited by lack of knowledge of the relation of the radiation fields to the microdosimeter response except under restricted field conditions (Kellerer and Chmelevsky 1975). It is noted by Kellerer and Chmelevsky (1975) that LET is only one of the many factors which determines energy deposition in microscopic sensitive site. Other factors are ion range, energy loss straggling, and energy dissipation by delta-arys. Various regions influenced by these different factors have been plotted (Kellerere and Chmelevsky, 1975) as a function of site diameters and particle energies and there is a narrow region for which the energy deposit can be approximately equated to the value of LET.

Tissue equivalent proportional counters (TEPC) are widely used in both ground and flight measurements of radiation quality. For example, Badhwar et al. (1994) estimated dose equivalent and the derived quality factors at various locations inside the Space Shuttle using onboard TEPCs which are accurate over a wide range of LET. The results reveal that the Q value is higher due to GCR than to Earth’s trapped radiation for the Shuttle orbits. The derived quality factors using ICRP-26 (ICRP, 1977) and ICRP-60 (ICRP, 1991) were obtained assuming the measured TEPC lineal energy (y) spectrum to be the LET spectrum (Badhwar et al. 1994). This assumption was necessary since it is difficult to convert a lineal energy spectrum to an LET spectrum, especially for radiation sources as complex as GCR. This is also true for the complex radiation fields existing at the proposed HSCT altitudes. On the other hand, the lineal energy spectra can be obtained when the energy and charge of incident particles are given, by using Monte Carlo method (Zaider et al. 1983, Wilson and Paretzke 1981) which is often time-consuming or, alternatively, by using analytic methods (Wilson and Paretzke 1994, Olko and Booz 1990) for limited ion species and energies. Recently, a generalized analytic model was developed to calculate energy deposition of direct ion events in a micron-size detector for incident ions of arbitrary species and energy (Xapsos et al. 1996). This model is used for the TEPC response calculations made in this study. In the following, a brief description of the analytical model (Xapsos et al.,1996) and its validation are given. The uncertainties related to the use of a TEPC in measuring radiation quality is examined by applying the analytical model.

**TEPC Response Model**

As an ion traverses a detector volume of micron size, the amount of ionization in the volume will depend on the actual path length of the ion in the volume, the energy transported by delta rays out of the volume, and the energy partitioning between ionization and inelastic excitations. Ions which do not traverse the volume but pass by close enough may also deposit some of their energy by injecting delta rays into the volume. The latter case is usually referred to as indirect events (Kellerer, 1971) or “touchers” (ICRU-36, 1983) and the former as direct events (Kellerer, 1971) or “crossers” (ICRU-36, 1983). The analytical approach developed by Xapsos et al. (1996; 1994) to obtain a solution for the ionization spectrum produced in a small volume by the passage of monoenergetic ions is currently limited to the direct events. Herein we will briefly describe the approach for use in the estimate of instrument response and resulting Q values.
As the detector (or target) size becomes smaller, the randomness of energy deposition processes become increasingly important. The relative variance for the single event, V derived by Kellerer (1968) is

\[ V = V_L + V_s + V_L \cdot V_s + V_{\text{str}} + V_F \]  

(1)

where \( V_L \) is the relative variance of the LET distribution of incident particles, \( V_s \) is the relative variance of the particle’s path length distribution through the volume, \( V_{\text{str}} \) is the relative variance of energy-loss straggling and \( V_F \) is the relative variance of Fano fluctuations related to the energy partitioning. For monoenergetic particles, \( V_L \) is zero and equation (1) reduces to

\[ V = V_s + V_{\text{str}} + V_F \]  

(2)

Equation (2) indicates that the probability distribution function for ionization produced by the random traversal of an ion through the volume requires the knowledge of the path length distribution and the energy-loss straggling including the Fano fluctuation. The probability distribution function for the ion’s path length can be easily obtained from the chord length distribution of the detector (target) volume assuming that the ion is energetic enough to travel in straight lines through the volume. The energy-loss straggling approximates a lognormal distribution (Xapsos et al.1996). This results because with each collision, the ion loses some random fraction of its energy that is proportional to its energy before the collision. This observation is consistent with the application of the lognormal distribution to the related problem of energy deposition distributions (Wilson and Paretzke, 1994; Condon and Breit, 1935; Lepson, 1976; Burke, 1975). Given \( p_s(x, E_j) \) as the probability density distribution function for the lognormal process to produce \( x \) number of ionizations related to a path length \( s \) of an incident ion \( j \) with energy \( E_j \), the overall probability density distribution is then

\[ f(x, E_j) = \int_s p_s(x, E_j) \cdot c(s) \, ds \]  

(3)

where \( c(s) \) is the chord length density distribution function. If the representation of the lognormal process is given as

\[ p_s(x, E_j) = \frac{1}{\sqrt{2\pi\sigma_s x}} \cdot e^{-\frac{1}{2} \left( \frac{\ln x - \mu_s}{\sigma_s} \right)^2} \]  

(4)

then the parameters of the lognormal distribution are related to the mean and relative variance of the number of ionizations as follows (Aitchison and Brown, 1957)
\[ \mu_s = \ln (\bar{x}) - 0.5\sigma_s^2 \]  \hspace{1cm} (5)

and

\[ \sigma_s^2 = \ln (1 + V_x) \]  \hspace{1cm} (6)

where \( V_x \) is the sum of \( V_{\text{str}} \) and \( V_F \) for \( x \) number of ionizations.

Unlike the earlier work of Wilson and Paretzke (1994) and Olko and Booz (1990), the evaluation of parameters \( \mu_s \) and \( \sigma_s \) needed in calculating energy-loss straggling, equation (4), does not rely on curve fitting from existing Monte Carlo results. The parameters are strongly dependent on the detector medium and size, particle types and energy values. The approach here is to obtain an analytical expression for the relative variance \( V_x \) so that equation (3) is readily soluble for any given size of detector exposed to an arbitrary ion field.

The relative variance of energy-loss straggling is

\[ V_{\text{str}} = \frac{\delta_2}{\bar{\varepsilon}} \]  \hspace{1cm} (7)

where \( \delta_2 \) is the energy-weighted mean of the energy deposited per ion-electron collision in the site and \( \bar{\varepsilon} \) is the average energy deposited in the site by a single ion track (see equation 4 in Xapsos et al. 1996). For a micron size volume of tissue traversed by an ion with energy greater than 3 MeV/A or so, the track width will be large enough to allow some of its deposited energy carried away from the volume by the delta rays. The fraction of such energy loss is treated analytically (Xapsos 1992) and is included in the evaluation of \( \bar{\varepsilon} \). An approximate form for \( \delta_2 \) is also available from the work of Vail and Burke (1984) for use in evaluation of \( V_{\text{str}} \). It is also easy to evaluate the relative variance of Fano fluctuation which is given as (Kellerer 1968)

\[ V_F = F \frac{W}{\bar{\varepsilon}} \]  \hspace{1cm} (8)

where \( F \) is the Fano factor (Fano 1947) and \( W \) is the average energy required to produce an ion pair by the incident radiation. The values for \( W \) in various media are widely available from the literature.

The ionization spectrum detected by a TEPC due to random passage of ion \( j \) with differential energy flux \( \phi_j(E_j) \) is then
\[ \Phi_j(x) = \int \phi_j(E_j) f(x, E_j) \, dE_j \]  \hspace{1cm} (9)

\( \Phi_j(x) \) can be easily converted to a lineal energy differential spectrum, \( \psi_j(y) \), through the relation \( y = x \frac{W}{c} \), where \( c \) is the average chord length and \( y \) is lineal energy. The derived quality factor is then given by

\[ Q_j(y)_{\text{TEPC}} = \frac{\int y Q(y) \psi_j(y) \, dy}{\int y \psi_j(y) \, dy} \]  \hspace{1cm} (10)

where \( Q(y) \) is assumed (Badhwar et al. 1994) to be the ICRP quality factor, \( Q(\text{LET}) \).

**Model Validation**

**Monte Carlo:** The validity of Xapsos model was demonstrated previously by Xapsos et al. (1996) showing good agreement with Monte Carlo results of Olko and Booz (1990). The comparison was for the probability density distribution as a function of ionization produced by a 1 MeV proton randomly incident on a 100 nm diameter water sphere. Also presented (Xapsos et al., 1996) were analytical results for a 1 MeV proton randomly incident on 1 micron and 10 nm diameter spheres of silicon where it is illustrated that the distribution for 1 micron diameter is less affected by energy straggling than that for a nm diameter sphere. As the diameter increases the distribution approaches the microscopic limit resembling more closely the chord length distribution of the sphere which is a right triangle. This is in agreement with the trend presented in the graphs by Kellerer and Chmelevsky (1975) delineating regions of site diameters and energies influenced by various factors other than LET. Here we further present results to illustrate the differences in the distribution due to the varying ion energy. Figure 1 shows the probability distribution as a function of lineal energy \( y \) for a one-micron diameter water sphere irradiated randomly by a single proton of various energies (from 0.3 to 5 MeV). The results for all the energies are generally in good agreement with the Monte Carlo histograms (Olko and Booz, 1990) showing the same general trend in straggling effect that is less important for the lower proton energies as evidenced by the changing shape of distributions. Although there is a noticeable discrepancy for the lowest energy (0.3 MeV), it has been verified (Shinn et al., 1999) that there is no error in the results calculated using the current model.

**Shuttle Flight Experiments:** Space qualified instruments are usually small and of light weight with no exception for the TEPCs used in monitoring radiation health of astronauts in Shuttle flights. The detector head used in the Shuttle simulates a 2-micron tissue site which is of cylindrical shape with equal diameter and height (Badhwar et al., 1994). The associated chord-length distribution contains a sharp peak occurring at the chord length equal to the diameter (also the height) with a low flat background. This right circular cylindrical shape allows a better resolution in the measured LET spectral components since many particles traverse near the diameter or through the end surfaces (Badhwar et al., 1992). However, the overall measured lineal energy distribution of GCR fluences for the Shuttle flights show very little resolution due to the overriding effect of energy loss straggling as seen in figure 2 for STS-56. The calculated LET spectrum is obtained using a new version of the HZETRN code (Shinn and Wilson, 1992;
Wilson et al., 1995) to account for the particle-field change as the GCR particles penetrate and interact with the Shuttle structural materials. The figure indicates existing spikes in calculated LET spectrum disappear as the HZETRN results were post-processed with the TEPC response function, in agreement with the measured results. The lower prediction by HZETRN at lineal energy below 2 keV/micron is likely due to the neglect of pions, kaons, and electromagnetic cascade in HZETRN (Shinn et al. 1998) although the response model needs further improvement such as including indirect delta-ray events and wall effects (Rademacher et al., 1998).

**Laboratory Experiments:** The response model used in the HZETRN post processing has been validated with Monte Carlo comparisons for relatively low energy proton beams. Further validation of the model for high energy particles relevant to space radiations as related to HZETRN code are accomplished by comparing with existing ground-based experimental results. Figure 3 shows dose distributions obtained for the 3.9 GeV nitrogen beam at the Princeton Particle Accelerator (Rodgers et al., 1973). The experimental results are for a 2-micron diameter wall-less counter of spherical shape situated behind a 2-cm thick water column. The present calculation is seen to agree well with the experiment except for the low y (below 10 keV/micron) region for which contributions from indirect delta ray events are not yet considered in the current model. For the same reason, the calculated peak tends to be slightly higher because of normalization. Preliminary results of an extended model (Xapsos, 1999) to include indirect delta ray effect is shown (figure 4) to yield a very good comparison with the data. Other example of validating the model with high energy laboratory data obtained by using 160 MeV proton beam on a 1-micron wall-less counter at the plateau location of water column (Kliauga et al., 1978) is given in figure 5. The agreement between present calculation and experimental results is reasonably good except for the region above 2 keV/micron. This discrepancy is due to the fact that the present calculation assumes only slowing down of the proton beam through water column without considering nuclear fragments which are of high LET.

**Implications**

In the comparison with Monte Carlo calculations and laboratory experiments, it has been illustrated how energy-loss staggling influences lineal energy distribution measured by a spherically shaped detector. Since cylindrically shaped TEPCs are advantageous in spectral resolution and often used in the modern days as in the case of Shuttle experiments, it is of importance to examine the effect of some of the above mentioned factors (Kellerer and Chmelevsky, 1975) on these detectors using the existing response model (direct delta rays only). Figure 6 shows results for the Shuttle TEPC randomly irradiated by protons of various (low) energies. As the energy increases the peak becomes less sharp due to the increase in energy-loss straggling. Moreover, the location of the peaks do not correspond well with the LET values [see inserted table in figure 6] of the monoenergetic protons incident on the gas volume and tends to be located at higher values for all the cases except the lowest energy (0.075 MeV) protons whose range is shorter than some of the detector chord lengths. Very low energy protons as well as short range heavy ions are important contributors to the risk estimate for GCR exposure due to their high LET values although they are less abundant (Cucinotta et al. 1996). It is not clear from the results given in Fig. 6 that a consistent estimate of radiation quality will result from the TEPC measurement.
The effect of the microdosimetric distribution on the estimates of radiation quality can be best seen from the ratio of derived quality factor (see equation 10) over the nominal value as defined by the ICRP for each GCR ion of varying LET and ion charge. For all the results given in the following the LET values correspond to the ion energies above the Bragg peak. Figs. 7 and 8 show such ratio for ICRP-26 and ICRP-60, respectively, with the instrument-derived quality factor being predicted by the analytic model. In general, the deviation from unity is surprisingly large and appears to be greater for ICRP-60 than ICRP-26. This can be explained by the fact the quality factor which is used in equation (10) as a multiplier to the microdosimetric distribution varies less smoothly over the entire LET range for ICRP-60. For example, the peak of predicted lineal energy spectrum for monoenergetic alpha particles of 50 keV/µm in LET is near 72 keV/µm, as shown in Fig. 9. The maximum of the ICRP-60 quality factor is closer to this peak value than that of ICRP-26. As a result, the ratio for ion charge Z = 2 at 50 keV/µm is higher for ICRP-60 (Fig. 8) than for ICRP-26 (Fig. 7). Also, no systematic trend is seen in the predicted ratio across various ion charges nor across varying LET values. For a constant LET of 100 keV/µm, the ratio for ICRP-60 stays at a nearly constant value of 0.81 for all the ions but the ratio for ICRP-26 decreases from 1.1 for light ions to 0.95 for heavy ions. This decrease can be understood by comparing (see Fig. 10) the TEPC lineal energy spectra for alpha particle and titanium, both having an LET of 100 keV/µm. The distribution for titanium tends to weigh more in the region below 100 KeV/µm where the quality factor for ICRP-26 decreases sharply. Note that the difference in the location of these distribution peaks is due to the fact that the track structure for the high-energy titanium ion is wider than the detector site and a substantial fraction of delta rays carrying energy lost by the ion in fact escapes the site.

As mentioned earlier, the model currently used in estimating microdosimetric distribution does not include indirect delta-ray events (except for figure 4) and wall effects; nonetheless, the characteristics of these results probably would not change significantly once these effects were added. The indirect delta-ray events will add very small energy events that result in insignificant dose equivalent contribution and will have minimal effect on the calculated ratio. On the other hand, the wall effect will tend to contribute at the high lineal energy tails of spectra that may slightly increase the ratio depending on the LET of the ion.

**Concluding Remarks**

Although microdosimeters are useful dosimetric tools to indicate spectral distribution of radiation components of different quality, relating the measured spectrum to knowledge of the environment requires understanding of the detector response functions. The dose equivalent resulting from the measurement may be inaccurate by as much as a factor of two depending on the ion field. For this reason, “measured” dose equivalent may be of limited usefulness in environmental characterization in some applications.

**References**


Xapsos, M. A. private communication; 1999.


Figure 1. Probability density distributions of lineal energy for a 1-µm diameter water sphere irradiated by a single randomly incident proton of various energies.
Figure 2. GCR differential LET or lineal energy spectra, in comparison with TEPC data for a 57 degree Shuttle flight.
Figure 3. Dose distributions measured by a 2-µm diameter spherical, wall-less TEPC 2-µm depth in a water column irradiated by 3.9-GeV nitrogen ion beam.
Figure 4. Dose distribution measured by a 2-micron diameter spherical, wall-less TEPC 2-micron depth in a water column irradiated by 3.9-GeV nitrogen ion beam. Calculation includes indirect delta ray effect.

(Rogers et al., 1973)
Figure 5. Microdosimetric spectra (dose times lineal energy, y) as a function of y, at 1.9 cm depth (plateau location) in water column irradiated by 160 MeV proton beam, as measured by a wall-less, 1-µm diameter spherical TEPC.
Fig. 6. Probability density distributions of lineal energy for a cylindrical TEPC irradiated by a single randomly incident proton of various energies. The TEPC has equal dimension of a diameter and height which is 1.8 $\mu$m in water.
Fig. 7(a). LET < 100 keV/µm

Figure 7(b) LET > 100 keV/µm.

Fig. 7. Ratio of TEPC-derived quality factor over the nominal value for various charged ions of varying LET. ICRP-26 quality factor is used in the calculation.
Fig. 8(a). LET < 100 keV/µm.

Figure 8(b) LET ≥ 100 keV/µm.

Fig. 8. Ratio of TEPC-derived quality factor over the nominal value for various charged ions of varying LET. ICRP-60 quality factor is used in the calculation.
Fig. 9. Probability density distribution of lineal energy for a cylindrical TEPC irradiated by a single randomly incident alpha-particle of 50 keV/μm in LET. The TEPC has the same dimensions as in Fig. 6.

Fig. 10. Probability density distribution of lineal energy for a cylindrical TEPC irradiated by a single randomly incident alpha-particle or titanium ion of 100 keV/μm in LET. The TEPC has the same dimensions as in Fig. 6.