Improved Multi-Axial, Temperature and Time Dependent (MATT) Failure Model

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Introduction

An extensive effort has recently been completed by the Space Shuttle's Reusable Solid Rocket Motor (RSRM) nozzle program to completely characterize the effects of multi-axial loading, temperature and time on the failure characteristics of three filled epoxy adhesives (TIGA 321, EA913NA, EA946). As part of this effort, a single general failure criterion was developed that accounted for these effects simultaneously. This model was named the “Multi-Axial, Temperature, and Time Dependent” or MATT failure criterion.

Due to the intricate nature of the failure criterion, some parameters were required to be calculated using complex equations or numerical methods. This paper documents some simple but accurate modifications to the failure criterion to allow for calculations of failure conditions without complex equations or numerical techniques.

Theoretical

The following Multi-Axial Temperature and Time (MATT) dependent failure model was originally proposed:

\[ AP^2 J_2 + B P I_1 = 1 \]  

Here \( J_2 \) is the second deviatoric stress invariant, and \( I_1 \) is the first stress invariant. \( A \) and \( B \) are shape parameters that define the ellipsoidal nature of the failure envelope. These parameters have been shown to be independent of temperature or time.

For a constant \( P \) value, this failure criterion is equivalent to the Tsai-Wu failure model and equivalent to a modified Drucker-Prager failure model.

\[ \sigma_f = N_o \left[ \frac{t_f}{1+\beta} \right]^{1+\beta} \]  

Here \( N_o \) and \( \beta \) are experimentally determined failure parameters, \( \sigma_f \) is an applied stress as a function of time with a failure time \( t_f \). The linear cumulative damage equation can be simplified for the following basic loading conditions.

For constant loading rate conditions:

\[ t_f = \left( 1+\beta \right) \left[ \frac{N_o}{\sigma_f} \right]^\beta \]  

For constant stress (or creep) loading conditions:

\[ t_f = \left( \frac{N_o}{\sigma_f} \right)^\beta \]  

Using these simple relationships and normalizing the \( A \) and \( B \) parameters, the MATT failure criterion can be modified to the following form for constant loading rate evaluations:

\[ A_o B_o \left[ \frac{t_f}{1+\beta} \right]^{\frac{2}{1+\beta}} J_2 + B_o \left[ \frac{t_f}{1+\beta} \right]^{\frac{1}{1+\beta}} I_1 = 1 \]  

For constant load studies:

\[ A_o B_o \left[ t_f^2 \right]^{\frac{1}{1+\beta}} J_2 + B_o t_f^2 I_1 = 1 \]  

Here the \( B_o \) parameter takes on additional meaning because it becomes a combined MATT failure parameter. \( B_o \) contributes to definition of both shape and size of the failure ellipse (compare with equation 1). The \( A_o \) parameter noted in this equation can be different than that used in equation 1. Here, \( A_o \) is a shape parameter that is specifically normalized to the linear cumulative damage term seen in equations 5 and 6 (\( A \) in equation 1 can be normalized to anything).
For the materials evaluated in this paper, it was determined that the $B_0$ and the $\beta$ parameters are a linear function of temperature:

$$\beta = m_\beta T + b_\beta$$

(7)

$$B_0 = m_{N_0} T + b_N$$

(8)

Here the $m_\beta$, $b_\beta$, $m_{N_0}$, and $b_N$ terms are the traditional slope and intercept parameters, and $T$ is the temperature.

Calculations of failure can be obtained by substituting the results of equations 7 and 8 into equations 5 and 6. As will be shown in subsequent sections, failure for a wide range of multi-axial, temperature, and time conditions can be defined by the MATT equation using only five coefficients ($A_0$, $m_\beta$, $b_\beta$, $m_{N_0}$, $b_N$).

Experimental

Extensive test data were used to characterize failure of TIGA 321 and EA946, and limited test data were used to characterize failure of EA913NA. As will be seen, even with limited characterization data, accurate failure models are developed. Testing was conducted below the glass transition temperature for TIGA 321 and EA913NA. For EA946, testing was conducted in the glass transition regime. The results of this characterization are of particular interest due to this low glass-transition temperature.

For this study, the time and temperature dependent nature of the adhesives was characterized using tensile adhesion test specimens. These tests were used to determine the $m_\beta$, $b_\beta$, $m_{N_0}$, and $b_N$ terms. Shear adhesion tests were used to characterize the effects of multi-axial loading on failure. From these tests, the $A_0$ parameter was obtained. Tests were conducted under temperature conditions that ranged from (20 °C to 45 °C), with failure times that ranged from several minutes to several hours, and with pure tension or pure shear.

Verification of the accuracy of the failure model was evaluated using napkin ring test specimens and creep loading of tensile adhesion buttons. The materials were tested under temperature conditions that ranged from (20 °C to 45 °C), with failure times that ranged from several minutes to several months, and with a wide range of multi-axial loading (tension/compression combined with shear).

Results and Discussion

The results of tensile and shear adhesion tests can be seen in the Figures 1-3 for the adhesives TIGA 321 and EA946. These figures include both the raw test data and the MATT predictions of failure. Each data point is an average of several tests (the number varies from 6 to 16 depending on the condition, the adhesive, and the test). The coefficients of variation for these tests are in Table 1.
Figures 5-8 show the multi-axial and creep test data and predictions that were used for verification of the material model for TIGA 321 and EA946. Each data point represents the average of 2 to 13 data points. Table 2 contains the coefficients of variation for this testing.

Table 2. Coefficients of Variation for the Multi-Axial and Creep Tests

<table>
<thead>
<tr>
<th>Adhesive</th>
<th>Multi-Axial</th>
<th>Creep</th>
</tr>
</thead>
<tbody>
<tr>
<td>TIGA 321</td>
<td>13%</td>
<td>15%</td>
</tr>
<tr>
<td>EA913NA</td>
<td>11%</td>
<td>21%</td>
</tr>
<tr>
<td>EA946</td>
<td>23%</td>
<td>20%</td>
</tr>
</tbody>
</table>

Conclusions

The improved MATT failure criterion has been shown to be accurate for a wide range of conditions. Coefficients of variation for all the data combined are seen in Table 3.

Table 3. Coefficients of Variation for All Tests

<table>
<thead>
<tr>
<th>TIGA 321</th>
<th>EA913NA</th>
<th>EA946</th>
</tr>
</thead>
<tbody>
<tr>
<td>11%</td>
<td>9%</td>
<td>17%</td>
</tr>
</tbody>
</table>

Of particular interest is the accuracy of the model for the adhesive EA946. Failure for this adhesive is characterized and the model is verified for conditions that pass through the glass transition of the adhesive. The test data indicate the generality of the failure criterion for a wide range of materials (even through the glass transition).

References

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