MATHEMATICAL MODELING OF FOOD SUPPLY FOR LONG TERM SPACE MISSIONS USING ADVANCED LIFE SUPPORT

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ABSTRACT

A habitat for long duration missions which utilizes Advanced Life Support (ALS), the Bioregenerative Planetary Life Support Systems Test Complex (BIO-Plex), is currently being built at JSC. In this system all consumables will be recycled and reused. In support of this effort, a menu is being planned utilizing ALS crops that will meet nutritional and psychological requirements. The need exists in the food system to identify specific physical quantities that define life support systems from an analysis and modeling perspective. Once these quantities are defined, they need to be fed into a mathematical model that takes into consideration other systems in the BIO-Plex. This model, if successful, will be used to understand the impacts of changes in the food system on the other systems and vice versa.

The Equivalent System Mass (ESM) metric has been used to describe systems and subsystems, including the food system options, in terms of the single parameter, mass. There is concern that this approach might not adequately address the important issues of food quality and psychological impact on crew morale of a supply of fresh food items. In fact, the mass of food can also depend on the quality of the food.

This summer faculty fellow project will involve creating an appropriate mathematical model for the food plan developed by the Food Processing System for BIO-Plex. The desired outcome of this work will be a quantitative model that can be applied to the various options of supplying food on long-term space missions.
INTRODUCTION

When humans conduct long range space missions such as the establishment of permanent bases on the Lunar surface or travel to Mars, they will continue to need food, water and air. For long term missions it will not be feasible to resupply these life support elements from Earth. Systems will need to be developed to produce food, purify the water supply and regenerate oxygen. Of significant importance is the development of agricultural systems to produce food, convert carbon dioxide to oxygen through photosynthesis, provide potable water through evapo-transpiration and recycle organic wastes.

The Bioregenerative Planetary Life Support Systems Test Complex (BIO-Plex) is a habitat for long duration missions which utilizes Advanced Life Support (ALS). In this system all consumables will be recycled and reused. In support of this effort, a menu is being planned utilizing ALS crops that will meet nutritional and psychological requirements. The baseline ALS crops are wheat, soybeans, peanuts, rice, potato, sweet potato and various salad plants.

The need exists in the food system to identify specific physical quantities that define life support systems from an analysis and modeling perspective. This model, if successful, will be used to understand the impacts of changes in the food system on the other systems and vice versa.

Within ALS the Equivalent System Mass (ESM) metric has been used to describe systems and subsystems, including the food system options, in terms of the single parameter, mass. The technique is described in the Baseline Values and Assumptions Document (BVAD) (Drysdale, et. al., June, 1999). It is possible this approach does not adequately address the important issues of food quality and psychological impact on crew morale of a supply of fresh food items.

The objective of the proposed project will be the development, within the ESM framework, of a quantitative modeling perspective relative to the food processing subsystem for long-term space missions. More precisely, work will proceed on two primary issues. Firstly, a modified computation of the ESM for wheat will be studied which adequately credits the air and water regeneration capabilities of the wheat crop. Such a modified ESM computation will be particularly useful in comparing different cultivars of wheat as potential ALS crops. Secondly, a food metric will be proposed which includes both food quality and the ESM metric as essential factors.

This report will conclude with a discussion of several topics for further study concerning the ALS food plan for long-term space missions.
MODIFIED ESM COMPUTATION FOR WHEAT

The standard ESM computation for food does not generally consider the air and water regeneration capabilities of crops being grown as part of the food system for a long duration space mission. A modification of the ESM computation will be introduced that hopefully gives adequate mass credits for air and water regeneration to crops. Data exists concerning the performance of wheat for air revitalization and food production during the Phase III test of the Lunar-Mars Life Support Test Project (Barta & Henderson, 1998). This data is a suitable basis for the study of wheat production during the BIO-Plex experiments, and, more generally, for the study of wheat production on the surface of the moon or Mars.

The Mars scenarios described in the Advance Life Support Systems Modeling and Analysis Reference Missions Document (JSC-39502) all include an approximate stay on the surface of 600 days. Consequently, the examples considered in this report will focus on a 600-day period of crop growth of wheat. The modified ESM for wheat will be denoted by \( ESM_{\text{wheat}} \) and the details of its computation follow.

Modified ESM for Wheat:

\[
ESM_{\text{wheat}} = M_E + \text{Pow}_P + V_P + \text{Biomass} - \text{CO}_2 \text{ Credit} - \text{H}_2\text{O Credit},
\]

where

- \( M_E \) = mass of equipment used for growing/harvesting the wheat crop
- \( \text{Biomass} \) = total mass of the wheat crop over the course of the mission
- \( \text{CO}_2 \text{ Credit} \) = mass credit for \( \text{CO}_2 \) uptake by the wheat crop computed using the Air data in the ALS Reference Missions document
- \( \text{H}_2\text{O Credit} \) = mass credit for \( \text{H}_2\text{O} \) transpiration by the wheat crop computed using the Water data in the ALS Reference Missions document

It is appropriate to comment on the means by which these quantities will be computed. Some of the quantities will almost certainly be the same no matter which cultivar of wheat is grown. In particular, the same equipment will be used no matter which cultivar is used. Similarly, the power requirements of the wheat crop will likely be independent of the particular cultivar being grown. Consequently, \( M_E \) and \( \text{Pow}_P \) will be considered as constants that are independent of cultivar. \( M_E \) could at best be approximated at this stage.
since the particular equipment to be used in growing, maintaining, and harvesting wheat grown on the Mars surface has yet to be completely determined. In fact, perhaps a fractional amount, based on the portion of the growing area devoted to wheat, of the mass of the equipment might be more appropriate since it is reasonable to assume that crop-growing/harvesting equipment will be designed for more than one type of crop. Biomass for wheat can be easily computed by known characteristics of the particular cultivar of wheat being used. Much data is available concerning the Apogee wheat and a recent seminar held at Johnson Space Center by Dr. Bruce Bugbee (Bugbee, 2000) contained additional information about the Perigee wheat as a possible replacement for the Apogee wheat. $V_p$ can readily be computed using the ESM volume mass penalty (9.08 kg/m$^3$) developed in the ALS Reference Missions document by simply computing the volume of the space taken up by the growing plants.

The CO$_2$ Credit is computed using the rate at which the wheat assimilates carbon dioxide per unit area per day (0.098 kg/m$^2$ per day for Apogee wheat (Barta and Henderson)) and the appropriate proportional amount of the ESM from Table 3.1.1 of Drysdale, Maxwell, et.al. for Air. A similar computation using the Water data from the same Table 3.1.1 together with the rate at which wheat transpires water (600 grams of water for each gram of seed yield (Bugbee)) is done to compute the value of the H$_2$O Credit. More precisely,

$$\text{CO}_2 \text{ Credit} = (0.098 \text{ kg/m}^2 \text{ /day}) \times (\text{growing area}) \times (\text{total number of days}) \times (10.5/5760)$$

and

$$\text{H}_2\text{O Credit} = 600 \times (\text{grain yield per day per m}^2) \times (\text{growing area}) \times (\text{total number of days}) \times (3.8/64512)$$

It should be noted that a careful study of Table 3.1.1 of Drysdale, Maxwell, et.al. shows that 5760 kg of revitalized CO$_2$ (1 kg/person/day for a crew of 6 for 960 days) corresponds to an ESM for Air of 10.5 metric tons. In a similar way, 64512 kg of water (11.2 kg/person/day (based on Lange and Lin (1998)) for 6 people for 960 days) corresponds to an ESM value of 3.8 metric tons for Water. The units of CO$_2$ Credit and H$_2$O Credit will be metric tons. The data in Table 1 below will allow for a comparison of the ESM$_{\text{wheat}}$ values for Apogee and Perigee wheat for a 600-day mission. In Table 1 the crop growing areas have been adjusted to produce the same total amount of grain yield for the two crops. The H$_2$O data in Table 1 comes form Dr. Bugbee’s seminar, while the other Apogee data in Table 1 comes from the Phase III report of Barta and Henderson. It appears that the Perigee wheat has not yet been grown under the strict conditions under which the Apogee wheat was grown during the Phase III test, but Dr. Bugbee’s seminar and a subsequent phone conversation with him lead to the data listed for Perigee. In particular, Perigee’s grain yield and CO$_2$ uptake capability are assumed to be 90% of the corresponding figures for Apogee wheat.
Table 1. Summary of Apogee and Perigee Wheat Characteristics

<table>
<thead>
<tr>
<th>characteristic</th>
<th>Apogee Wheat</th>
<th>Perigee Wheat</th>
</tr>
</thead>
<tbody>
<tr>
<td>height</td>
<td>50 cm</td>
<td>45 cm</td>
</tr>
<tr>
<td>harvest index</td>
<td>35%</td>
<td>35%</td>
</tr>
<tr>
<td>grain yield</td>
<td>1.74 kg/m²</td>
<td>1.57 kg/m²</td>
</tr>
<tr>
<td>growing cycle</td>
<td>80 days</td>
<td>80 days</td>
</tr>
<tr>
<td>CO₂ uptake</td>
<td>0.098 kg/m²/day</td>
<td>0.090 kg/m²/day</td>
</tr>
<tr>
<td>H₂O transpiration</td>
<td>600*(1.74/80 kg/m²/day)</td>
<td>600*(1.57/80 kg/m²/day)</td>
</tr>
<tr>
<td>growing area</td>
<td>20 m²</td>
<td>22.2 m²</td>
</tr>
<tr>
<td>total harvest (600 d)</td>
<td>261 kg</td>
<td>261 kg</td>
</tr>
</tbody>
</table>

The information in Table 1 leads to the following computations:

**Apogee wheat:**

\[
\text{ESM}_{\text{wheat}} = M_E + \text{Pow}_p + (0.00908 \text{ MT/m}^3) \times (20 \text{ m}^2) \times (0.5 \text{ m}) + \\
(600/80) \times (1.74 \text{ kg/m}^2)(1/.35) \times (20 \text{ m}^2) - \\
(0.098 \text{ kg/m}^2/\text{day}) \times (20 \text{ m}^2) \times (600 \text{ days}) \times (10.5 \text{ MT/5760 kg}) - \\
600 \times (1.74/80 \text{ kg/m}^2/\text{day}) \times (20 \text{ m}^2) \times (600 \text{ days}) \times (3.8 \text{ MT/64512 kg})
\]

\[
= M_E + \text{Pow}_p + 0.0908 + 0.746 - 2.14 - 9.22 \text{ (metric tons)}
\]

\[
= M_E + \text{Pow}_p - 10.52 \text{ (metric tons)}
\]

**Perigee wheat:**

\[
\text{ESM}_{\text{wheat}} = M_E + \text{Pow}_p + (0.00908 \text{ MT/m}^3) \times (22.2 \text{ m}^2) \times (0.45 \text{ m}) + \\
(600/80) \times (1.57 \text{ kg/m}^2)(1/.35) \times (22.2 \text{ m}^2) - \\
(0.090 \text{ kg/m}^2/\text{day}) \times (22.2 \text{ m}^2) \times (600 \text{ days}) \times (10.5 \text{ MT/5760 kg}) - \\
600 \times (1.57/80 \text{ kg/m}^2/\text{day}) \times (22.2 \text{ m}^2) \times (600 \text{ days}) \times (3.8 \text{ MT/64512 kg})
\]

\[
= M_E + \text{Pow}_p + 0.0907 + 0.747 - 2.19 - 9.24 \text{ (metric tons)}
\]

\[
= M_E + \text{Pow}_p - 10.59 \text{ (metric tons)}
\]

Assuming Apogee and Perigee wheat have very similar nutritional values, and they are grown and harvested using the same types of equipment, the numbers above indicate that Perigee wheat would be a slightly better choice. In any event, the credits for air and water regeneration were significant factors.

**DEVELOPMENT OF A FOOD METRIC**

There are three primary variables to include in the design of a food metric which can be applied to any potential food plan for long duration space travel: nutritional value, palatability (sometimes called food quality), and variation in the diet. A Food Quality Index (FQI) rating will be introduced which includes these variables. Subsequent to that, a food metric will be proposed which includes the Food Quality Index and the ESM metric as essential factors.
Food Quality Index

The following letters represent the three key variables mentioned above.

- $n$ denotes the nutritional value (based on RDA) of the food plan, where $0 < n \leq 10$.
- $p$ denotes the palatability (based on a Hedonic scale) of the food plan, where $0 < p \leq 10$.
- $v$ denotes the cycle length in days of the diet, where $0 < v \leq 20$.

We let $FQI$ denote the Food Quality Index. It is defined as the function of the three variables $n, p, v$ given by

$$FQI(n, p, v) = n \cdot \log(p^{sv})$$

Prior to an analytic study of $FQI$, it is appropriate to consider the qualitative properties of the above function. This function has value zero when $n = 0$ or $p = 1$, and has maximum value of 100 when $n = 10, p = 10, v = 20$. The $FQI$ function places heavy emphasis on nutritional value $n$ and diet cycle length $v$, but it is jointly dependent on all three variables. If any one of the three variables $n, p, v$ are very low in value then the value of $FQI$ is low. Algebraically, the $\log(p)$ factor indicates that palatability values in the upper range of the scale are not considered significantly different from one another.

Table 2 below contains a sampling of values of $FQI$ for a variety of values of $n, p, v$. The values in Table 2 are sorted by increasing $FQI$ values and are used to indicate the general properties of the $FQI$ rating. Some of the listed combinations of $n, p, v$ values (for example, $n = 2, p = 5, v = 8$) would not match an actual space mission food plan because of unacceptably low $n$ or $p$ values. It is important to demonstrate, however, that the proposed $FQI$ gives low ratings to such combinations.

**Table 2. Sample $FQI$ Values for Various $n, p, v$ Values**

<table>
<thead>
<tr>
<th>$n$</th>
<th>$p$</th>
<th>$v$</th>
<th>$FQI(n, p, v)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1</td>
<td>3</td>
<td>0.00</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>20</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>8</td>
<td>8.84</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>1</td>
<td>12.18</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>1</td>
<td>17.89</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>20</td>
<td>21.07</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>7</td>
<td>21.37</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>10</td>
<td>24.71</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>3</td>
<td>30.98</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>5</td>
<td>31.61</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>5</td>
<td>40.00</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>6</td>
<td>43.82</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>8</td>
<td>50.60</td>
</tr>
<tr>
<td>10</td>
<td>7</td>
<td>10</td>
<td>59.76</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>20</td>
<td>67.61</td>
</tr>
<tr>
<td>10</td>
<td>7</td>
<td>18</td>
<td>80.17</td>
</tr>
<tr>
<td>10</td>
<td>7</td>
<td>20</td>
<td>84.51</td>
</tr>
<tr>
<td>10</td>
<td>9</td>
<td>20</td>
<td>95.42</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>20</td>
<td>100.00</td>
</tr>
</tbody>
</table>
The values in this table show, for example, that a food plan with \( n = 8, p = 7, v = 20 \) rates better than a plan with \( n = 8, p = 10, v = 8 \). The \( FQI \) rating is particularly valuable in rating one food plan against another. Since the \( FQI \) function is nonlinear it is not necessarily the case that a food plan with an \( FQI \) rating of, say 80, is twice as good as a food plan with an \( FQI \) rating of 40. However, the food plan with the higher rating would definitely be preferable to the one with the lower rating.

The \( FQI \) rating can be applied to a food system with prepackaged food as well as to a food system consisting of a mixture of fresh ALS crops and some amount of resupply items. In fact, the \( FQI \) rating can be applied to any food system as long as the appropriate values for \( n, p, \) and \( v \) are obtained from the food science group.

**Details of a Proposed Food Metric**

A food metric based on the Equivalent System Mass (ESM) metric and the Food Quality Index (\( FQI \)) rating is proposed. It is reasonable to assume that as mass goes down the value of \( FQI \) does not increase. Consequently, the ratio of \( FQI \) to ESM will give a metric which rates highly food plans which simultaneously have low mass and high food quality index values. More precisely, denoting the food metric value by \( FMV \), for a food plan \( A \) with ESM value \( E_A \) and \( FQI \) rating \( FQI_A \) the food metric computation for plan \( A \) is given by

\[
FMV \text{ for plan } A = \frac{FQI_A}{E_A}
\]

In this computation the quantity \( E_A \) can be computed using the traditional ESM approach for prepackaged food systems or using the modified ESM method proposed in this document if there is a component of crop growth in the food plan. The units of ESM (usually, metric tons) and \( FQI \) are not the same but the magnitudes of their values are comparable. So, the value of \( FMV \) should be thought of as a numerical value to be used for comparison purposes when comparing one food plan against another. Higher \( FMV \) values are better in the sense that between two plans with equal mass factors the better plan is the one with higher \( FQI \), and between two plans with equal \( FQI \) factors the better plan is the one with the lower mass.

**CONCLUSIONS**

The models proposed here are significant in that they contain the important factors of food quality and nutritional value as important components. The work described in this paper should be thought of as early work in the development of an appropriate mathematical model describing the food system for long duration space missions. A natural process in the formulation of a mathematical model is the testing and refinement of the proposed model. The models described here are ready to be studied carefully by
means of the testing/refinement process. The traditional ESM metric does not sufficiently take the factors of food nutrition and quality into account. Consequently, a food metric that considers food quality and nutrition along with mass could be very valuable in the overall rating of different food plans, especially if that metric can be applied to food systems with a mixture of packaged items and fresh crops. The food metric put forth in this paper has precisely these attributes.

TOPICS FOR FURTHER STUDY

The work described herein leads to several natural areas of further work. Foremost would be the extension of the modified ESM computation for wheat to the other ALS crops. This project would involve the gathering of data similar to that listed in Table 1 for the other ALS crops. Once this work is completed the modified ESM numbers could be used to study the problem of which crop scenario would be most beneficial for the BIO-Plex experiments and various ones of the Mars mission scenarios. It seems certain the food plans developed for these various scenarios will be crop driven rather than menu driven in the sense that what meals are prepared depends on the crops available, more so than having the menu items determine the cropping scenario. This is somewhat contrary to the approach used by Hunter, et. al. (Hunter, et. al., 1998).

Much work can be done in the area of the testing and refining of the food metric proposed in this paper. It is very important to consider food quality and nutrition in any metric applied to food systems. Of course, in the case of space missions, the mass of the various systems is also a crucial factor. To date the model does not take into account the psychological benefits of a supply of fresh food items for long duration space missions. There seems to be little firm data concerning this possibly significant component of the overall food plan. Consequently, a suitable metric could eventually be quite different from the food metric put forth in this paper, but the food metric proposed here is a functioning model upon which to base further study.
REFERENCES

Bugbee, Bruce, Frantz, Jonathan, (July 13, 2000), Seminar, "Environmental Factors Affecting Plant Growth in BIO-Plex", Johnson Space Center, Houston, Texas


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