AERODYNAMIC HEAT-POWER ENGINE OPERATING ON
A CLOSED CYCLE

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SUMMARY

Hot-air engines with dynamic compressors and turbines offer new prospects of success through utilization of units of high efficiencies and through the employment of modern materials of great strength at high temperature. Particular consideration is given to an aerodynamic prime mover operating on a closed circuit and heated externally. Increase of the pressure level of the circulating air permits a great increase of limit load of the unit. This also affords a possibility of regulation for which the internal efficiency of the unit changes but slightly. The effect of pressure and temperature losses is investigated. A general discussion is given of the experimental installation operating at the Escher Wyss plant in Zurich for a considerable time at high temperatures.

INTRODUCTION

Considerable progress has been made during recent years in the further development of the gas turbine; in particular, the safety of operation has been increased, so that industrial installations operating on the constant pressure process could be taken into operation (reference 1). These installations, however, are for special purposes such as stand-by capacity of power stations and their arrangement: compressor - combustion chamber - turbine. An open circuit with oil as fuel would not satisfy the requirements of high efficiency, nor were they able to compete with steam plants using coal as fuel. There is a

question, therefore, as to whether and how it is possible to obtain economies equal to those of best steam installations without sacrificing safety of operation. The following survey seems to show that this object can be attained with means available today. The present paper discusses basic relations, and certain simplifications have been made in the calculations presented therein.

The Escher Wyss Company, Zurich, following the suggestions of the authors (reference 2), has undertaken the development and construction of an experimental installation of considerable size. This plant is at present in successful operation.

**MERITS OF HOT-AIR TURBINES**

**Carnot Cycle**

If a maximum of mechanical energy is to be produced from a flow of heat from a source of high temperature $T_h$ to a lower ambient temperature $T_t$, the process is indicated by the Carnot cycle (fig. 1). It consists of two isotherms $AB$ and $CD$, along which the quantities of heat $Q_{zu}$ and $Q_{ab}$ are added and removed, respectively, and of two adiabatic curves for compression $DA$ and expansion $BC$. Its theoretical efficiency

$$\eta_c = \frac{(T_h - T_t)}{T_h}$$

depends upon the temperatures only. The surrounding temperature $T_t$ can be assumed at $300^\circ$ absolute even; the maximum temperature depends on whether the cycle takes place periodically as in internal-combustion engines and constant-volume turbines, or as a steady process as in steam turbines and in constant-pressure gas and hot-air turbines. We shall consider cycles with steadily changing conditions; therefore, the temperature $T_h$ must be chosen low enough so that the construction materials will stand up under loads from pressure, centrifugal forces, etc., and give adequate safety of operation.

Let us assume as maximum temperature for this case, $1000^\circ C$ absolute (equal to $1268^\circ F$). This leads to $\eta_c = 70$ percent. It follows that a very good efficiency
can be obtained with temperatures as they occur - for instance, in furnace construction - provided it is possible to keep the deviations of the actual cycles from the Carnot cycle low.

The obstacles to realizing the Carnot cycle in practical process are well known; this would require a temperature increase from 300° to 1000° C absolute through adiabatic compression; for diatomic gases this corresponds to compression ratio of 67.5. Inasmuch as the cycle requires an additional isothermal compression - that is, with a compression ratio of 4 - the cycle would require an over-all compression ratio of 270. It is evident that a dynamic compressor for this pressure ratio and the high final temperatures would cause great difficulties in construction.

The steam-turbine cycle, in a long development, has gradually approached the Carnot cycle. Heat is added essentially during isothermal evaporation of the working fluid and removed during isothermal condensation. Furthermore, it has been possible to effect a compensation for the nonadiabatic heating of the water in the boiler through feed water preheating with bleeder or exhaust steam in such a way that no impairment of efficiency occurs. Unfortunately, the vapor pressures of the water at high temperatures are so high that the high temperature $T_h$ of the Carnot cycle must be held rather low. It is well known that superheating does not help much and that the efficiency can be improved only by plants with several different working fluids (reference 3) (mercury, diphenylxyloxyd (reference 4), etc.).

The steam cycle, which has the great advantage of non-mechanical compression, has largely arrived at its natural limits. For gases, in particular for air, no such obstacle exists. High temperatures are attainable without requiring high pressures. Hence the temperature limits of stresses of the structural materials may be approached without necessarily approaching the pressure load limits. This means, however, that the cycle must be different from the Carnot cycle. While it is possible for a given pressure ratio $P_A/P_G$ to choose the initial pressure $P_G$ without affecting the efficiency so low that the final pressure $P_A$ remains acceptable (this requires a closed circuit), this results in such a decrease of energy transfer per pound of working fluid that the size of the installation becomes excessive compared with its capacity.
Double Isothermal Cycle

It is of interest to note that for hot-air plants there is a cycle fully equivalent to the Carnot cycle which operates with moderate pressure ratios and with moderate absolute pressure. This cycle requires a process of heat exchange which is closely related to the process of feed-water preheating, by which the steam cycle is approximated to the Carnot cycle as discussed above. Figure 2 shows the temperature entropy diagram of this process.

The isothermal compression takes place along CD. Heat is added at constant pressure along DA. The addition of heat takes place in a heat exchanger in such a way that the heat given off from B' to C' in the optimum case is exactly equal to the heat required for increasing the temperature of the same weight of air from D' to A' by the same number of degrees. The heat of the fuel is then to be transmitted to the machine from A to B' at constant temperature Th. Since the energies of isothermal compression or expansion for equal pressure ratios are proportional to the absolute temperatures, the quantities of heat added and removed at Th and Tt, respectively, are in the same ratio and consequently, the efficiency is equal to that of the Carnot cycle (equation (1)).

Turbine Installation with Closed Circuit

The addition of heat along an isotherm naturally cannot be realized, however, in this uniform manner. It will have to take place, like the compression, in individual groups of stages (a) (fig. 3). It appears likely that operation and construction will demand further deviations from the ideal case. The number of groups of stages will have to be limited to two or three, in each of which there will be a considerable adiabatic change of pressure. It will also be necessary to heat the air upstream of the first turbine in an air heater (e), since the required full temperature increase in the heat exchanger (g) will not be realized. This makes it necessary to examine carefully what the effect of these deviations will be upon the efficiency; in particular, it will be necessary to study the magnitude of the individual losses.

In addition to purely thermodynamic merits of the hot-air unit as compared with steam installation, considera-
tion must be given to a large number of factors of practical importance. Elimination of the feed water and its purification equipment, and decrease of cooling-water requirements to a fraction of that required for steam condensation lead to essential simplification of operation.

LIMIT LOAD OF THE CLOSED-SYSTEM INSTALLATION

It is frequently stated that the gas turbine, perhaps on account of its high temperature and its relatively small number of auxiliary units, might be expected to have especially low unit weight. From this it is concluded that this might make it applicable for use in ships or airplanes. This is not necessarily the case for present-day installations operating intake and exhaust of the working fluid at atmospheric pressure. The low circumferential speed of the last stage which is limited by high final temperature and the low axial velocity necessitated by these low circumferential speeds, the small weight of gas handled and, above all, the small heat drop per unit weight - all of which are inherent in these gas or air cycles - contribute to render the limit load of a single-pass installation relatively low. It must not be forgotten that the power of the compressor is of the order of 1/3 to 1/2 of the turbine capacity (reference 5)* when computing the limit load of an open-system single-pass unit. On the basis of feasible assumptions it can be shown that great difficulties would be encountered to reach at 3000 rpm even a 10,000-kilowatt capacity. For central station capacities this would require numerous units. Even for ship installations the required number would be excessive.

Because of the rate of excess air required for internally fired gas turbines, large areas of exhaust pipes are required, which are most undesirable, particularly in nonstationary plants. Basically heated parts of large dimension should be avoided because they require large quantities of special materials. Space requirements are also quite important. For the open-system cycle an improvement can hardly be expected, but it appears quite possible that

*For the combustion turbine described by Stodola (reference 5), even 74 percent of turbine shaft output was required for the compressor.
for the closed cycle the limit load may be greatly extended by increase of the pressure. It can be seen from figure 2 that the efficiency is independent of the absolute pressure. This suggests that if the lowest absolute pressure which occurs in the cycle—hereafter referred to as the pressure level—be increased to several times its value, the absolute pressures will rise in the same ratio since steady-flow machines produce, or require, the same pressure ratio as long as the speeds are the same.

Effect of the Pressure Level upon the Dimensions

The following assumptions are made:

(a) The turbines as well as the compressors are designed geometrically similar (equal number of stages, blade angles, etc.).

(b) The temperatures at given points of the cycle are to be constant; likewise, the velocities at corresponding points are to be equal.

(c) Neglect the effect of Reynolds number upon the pressures of turbine and compressor.

Let the design of a compressor, air heater, turbine, and heat exchanger be given for a pressure level $p_0 = 1$ atmosphere absolute. Then let the pressure level be increased, for example, to $p^* = 9$ atmospheres absolute. Since the ratio of the weight of air handled by the compressor and turbine for equal inlet and outlet temperatures and equal velocities are proportional to $D^2 p$, we have

$$D^*^2 \ p^* = D_0^2 \ p_0$$

or

$$\frac{D^*}{D_0} = \sqrt{\frac{z}{1}}$$

provided the horsepower remains the same. $D^*$ and $D_0$ denote the diameters. For the following considerations let us assume a pressure ratio $z = \frac{p^*}{p_0} = 9$.

Since the blade velocity of the rotating parts must be the same, the rpm increase.
for $z = 9$, $n^* = 3 n_0$. The diameter of the inlet and outlet pipes for the axial length of the unit are decreased to $1/\sqrt{z} = 1/3$. The wall thicknesses of the pipes will increase because of higher pressures to $\sqrt{z}$ times the original, assuming equal stresses. Inasmuch as the diameter decreases to $1/\sqrt{z}$, the actual increase of wall thickness is $\sqrt{z}^3$. Consequently the weight of the pipe unit length remains the same. The width of the flanges are $\sqrt{z}$ times larger; their thicknesses remain the same. Consequently their weight does not change. For blind flanges the thickness does not change, hence the weight becomes $z$ times smaller. The torque of the machines decreases to $1/\sqrt{z}$ of the former, which would allow for a decrease of shaft diameter to $1/\sqrt{z} = 0.70$. Assuming perfect similarity, the weight of the runner would decrease to $(1/\sqrt{z})^3 = 1/27$, on account of the decrease of torque to $1/\sqrt{z}$ only the weight of the runner would actually be a little higher.

Comparative stresses from centrifugal force are for geometrically similar bodies and equal circumferential speeds the same; bending stresses of the buckets, however, are $z$ times higher since for strict similarity there corresponds to a bending moment decreased to $1/\sqrt{z}$ a moment of inertia diminished to $(1/\sqrt{z})^3$. It follows that if the bending stresses are considerable as compared with the centrifugal stresses, it will be necessary to strengthen the roots of the blades.

The thickness of casing walls must be increased by $z$; since the length of the casing, however, decreases, the weight goes down to approximately $1/\sqrt{z}$ of the original.

Since the critical speed of similar rotors increases as $\sqrt{z}$, the ratio of operating speed to critical speed according to equation (3), again is the same. It will be necessary, of course, to deviate from strict similarity in many details in consideration of blade stresses, seals, etc. Nevertheless, the basic fact remains as a conclusion
of the foregoing consideration that casing diameter and length may be reduced to a fraction of their dimension for the open cycle. This statement is of decisive importance in favor of design of air turbines of large capacity. For it is possible, instead of decreasing the dimensions for equal horsepower in comparison to an open installation, to affect a considerable increase of the limit load for given dimensions. Numerical computations show that an installation for as high as 50,000 kilowatts capacity at 3000 rpm can be built as single-pass units for safe operation. Since for double-pass arrangement, no insuperable difficulties appear to exist, it may be said that hot-air prime movers with a closed cycle, even for the largest capacities, will remain within conventional dimensions.

Effect of the Pressure Level upon the Efficiency

The efficiency of steady-flow machines generally increases with increase of Reynolds number. In the case under discussion, Reynolds number increases in spite of decrease of the size of the units. Since the kinematic viscosity \( \nu = \mu / \rho \) varies at constant temperatures inversely proportional to the density \( \rho \), (the absolute viscosity \( \mu \) being a function of the temperature only) and in this case, consequently, inversely proportional to the pressures, it follows:

\[
Re^* = \frac{w^*D^*}{\nu^*} = \frac{\nu_0 D_0}{\nu_0} \frac{1}{\sqrt{z}} = \frac{\nu_0 D_0}{\nu_0} \sqrt{z} \tag{4}
\]

where \( w^* \) and \( \nu_0 \) are velocities. In our case, consequently, Reynolds number \( Re^* \) is three times as large as \( Re_0 \).

Naturally, Reynolds number affects the internal efficiency \( \eta_1 \) only, which does not include the bearing friction. The internal efficiency combines the effect of many different losses. These cannot be analyzed in detail, but we have found in our experience the following method of estimation to lead to practicable results:

A part of the internal losses \( (1 - \alpha) \) will be referred to as "irreducible," - that is, independent of Reynolds num-
ber. Losses of this type are kinetic leaving losses, losses from major roughness, losses from variation in the distribution of circulation (induced losses), leakage losses, etc. The remainder a is "reducible" in the range of Reynolds number encountered in medium-size steady-flow machines for handling gases; these losses, caused by skin friction, vary approximately as the turbulent plate and disk friction coefficients, namely, $1/\sqrt{Re}$. From this is deduced the following relation for the internal efficiency:

$$\frac{1 - \eta_i^*}{1 - \eta_{i_0}} = (1 - a) + \frac{a}{10\sqrt{a}}$$ (5)

The more that steady-flow machines are improved, the larger will be the portion $a$ of the reducible losses. Consider an example:

$\eta_{i_0} = 0.86, a = 0.7, z = 9$

It follows that $\eta_i^* = 0.88$. Since it is well known that the overall efficiency of the installation depends greatly upon the efficiencies of the units, such gain naturally is most desirable. It is necessary, of course, that the clearances be chosen smaller to correspond to the smaller dimensions. Bearing friction decreases on account of the smaller rotor weights in spite of higher peripheral velocity.

Influence of the Pressure Level upon the Heat Exchanger

Of particular importance is the influence of the pressure upon the dimensions of the heat exchanger. On both sides of the exchanger surface there is gas, consequently both heat transfer coefficients increase as the pressure is increased. The coefficient of heat conduction of the metal, of course, is not affected. With the increase of mass density there occurs an increase of the pressure drop in the heat exchanger.

In connection with this aspect, it must be kept in mind that heat transfer and friction associated with the flow of gases through tubes are closely interrelated, inasmuch as the same turbulent process causes the transport of momentum as well as of energy (heat) (reference 6). Hence the tests have shown that the process takes place in the range of validity of the Blasius equation. It is
logical to use this equation and the heat-transfer equation resulting from it. The pressure drop is given by

\[ \Delta p = \frac{k_1}{Re^{0.25}} \frac{l}{D} \frac{\gamma}{2g} w^2 \]  

(6)

where

- \( k_1 \) is a constant
- \( l \) length
- \( D \) diameter
- \( \gamma \) specific weight
- \( g \) acceleration of gravity
- \( w \) velocity of flow

Using the analogy between shear stress near the wall and heat transfer, the heat transfer coefficient is given by

\[ a = k_2 \frac{\gamma w c_p}{Re^{0.25}} \]  

(7)

where \( k_2 \) is a constant and \( c_p \) is specific heat for constant pressure. The values relate to the mean portion of the heat exchanger.

In the case under discussion, a particularly simple solution is obtained for the case that Reynolds number is kept constant through decrease of the tube diameter. This requires decrease of the diameter of the tube to \( 1/z \) of their former size. This would make it so small that operation with clean air only is possible. Internal combustion therefore is out of the question. For equal velocity \( w \) the heat transfer coefficient \( a \) is proportional to the mass density and consequently also to the pressure. The total heat transfer surface thereby is reduced to \( 1/z \). The pressure drop is the same percentage of the pressure level, \( \epsilon = \Delta p/p = \) constant, provided \( l/D \) is kept the same, or provided the length of the tube is reduced to the \( 1/z \) portion of the former. The surface of the tube bundle headers is likewise reduced since the number of tubes per unit area increases as \( z^2 \). The diameter of the ex-
changer consequently is reduced to \(1/\sqrt{z}\). The wall thickness of the tubes must remain the same; since the pressure difference from inside to outside increases to \(z\) times the original, this requires slightly more space. Nevertheless, even so, the difference of dimension is striking. The result is a consequence of the increase of pressure as much as of the use of smaller tube diameters. The weight of tubing goes down to \(1/\sqrt{z}\) of the former because of reduction of surface. Such a great reduction, of course, might not be used in early installations, but the small tube diameters may be found suitable for nonstationary applications.

If the tube diameters are kept the same, the dimensions are not reduced to such great extent but even then the advantage of a higher pressure level remains considerable if the heat exchanger is varied in such a way that the tube diameters remain the same \((D^* = D_0)\) and, furthermore, if the percentage of pressure drop is kept the same \(\varepsilon = \Delta p/p\) and the temperature difference constant, the dimensions and numbers of tubes \(x\) are derived from the following relations:

1. **Constancy of the weight of gas flowing per unit time.** For equal pipe diameter, the weight of gas per unit time is proportional to the velocity \(w\), the number of tubes \(x\), and the specific weight \(\gamma\), hence proportional to the pressure \(p\). This is expressed through the relation

\[
\frac{w^* x^* P^*}{w_0 x_0 P_0} = \frac{1}{z}
\]

or

\[
\frac{w^* x^*}{w_0 x_0} = \frac{P_0}{P^*} = \frac{1}{z}
\]

2. **Constancy of the percentage of pressure drop** \(\varepsilon = \Delta p/p\). The Reynolds number \(Re = wd\mu\) is in our case proportional to the velocity \(w\) and the pressure \(p\); the percentage of pressure drop is obtained from equation (6)

\[
\varepsilon = \frac{k_1}{P_0} \left(\frac{\frac{1}{\gamma} \frac{\mu}{D^{0.25}}}{rac{1}{\gamma} \frac{\mu}{D^{0.25}}}\right) = \frac{P_0}{P^*} \cdot 0.25 w^{1.75} l^{0.25}
\]

This leads to the following condition for constant ratio \(\varepsilon\)

\[
P^* = 0.25 w^*^{1.75} l^* = P_0 - 0.25 w_0^{1.75} l_0
\]

or
3. Constancy of pressure difference $\Delta T$ for equal rate of heat transfer. From the condition $a F^* = a_0 F_0$, where $F^*$ and $F_0$ denote the areas of the heat exchanger and equation (7) lead to

$$p^* 0.75 \, w^* 0.75 \, l^* x^* = p_0 0.75 \, w_0 0.75 \, l_0 x_0$$

or

$$\left(\frac{w^*}{w_0}\right)^{0.75} \frac{l^* x^*}{l_0 x_0} = \left(\frac{p_0}{p^*}\right)^{0.75} = \frac{1}{z^{0.75}}$$

(10)

It follows from equations (9) and (10) that:

$$\frac{w^*}{w_0} \frac{x_0}{x^*} = z$$

and from equation (8)

$$w_0 = w^* \frac{x_0}{x^*} = \frac{x_0}{z}$$

(11)

It is seen that the number $x^*$ of the tubes is greatly diminished, but they must be longer than according to (9):

$$\frac{l^*}{l_0} = z^{0.25}$$

for $z = 9 \, l^* = 1.73 \, l_0$. Since the wall thickness, theoretically, must increase to $z$ times its former value, the ratio of weights would be $z^{0.25}$. Actually, of course, it is utterly impossible to build the exchanger for open-type installation with correspondingly thin walls. The wall thicknesses of commercial tubes are adequate also for higher pressure, consequently there would be a saving in weight when going to higher pressures with the assumptions made under 3.

Effect of the Pressure Level upon the Air Heater

The air heater is an additional piece of equipment which is required by the closed type of installation; it takes the place of the combustion chamber of the open-type installation. On the side on which combustion takes place there is atmospheric pressure unless the combustion chamber is supercharged through a unit consisting of compressor and exhaust-gas turbine similar to that used in the Velox boiler (reference 7). This variation of design, which perhaps is of importance for marine installation, will not be discussed further. The air tubes are arranged
around the combustion chamber and receive heat by means of radiation and boundary-layer heat transfer. An increase of combustion-chamber pressure would reflect favorably upon increase of the heat-transfer coefficient and decrease upon the mean tube temperature, particularly for radiation from high-temperature flames at heat-transfer coefficients on the fire side are very large. It is therefore very advisable that the heat-transfer coefficients for the inside of the tubes be increased by constant velocity as \( z^{0.75} \). In the limiting case of infinitely high heat-transfer coefficient on one side, the same relations apply for the decrease of dimensions as derived for the heat exchanger. This case obtains with close approximation for the intercoolers of the compressor which has much larger heat-transfer coefficients on the water side than on the air side. Aside from this, heat-transfer processes in the air heater are so complicated that little headway is made with simple similarity considerations. A detailed computation of air heaters of widely different capacity on the basis of data obtained from operation of the experimental installation shows clearly that it is possible through the use of a high-pressure level to arrive at smaller dimensions of steam generators for equal capacity.

Proper utilization of the heat content of the combustion gases leaving the heater, which on account of their high-inlet temperature of the air entering from the heat exchanger have themselves a very high temperature, is of prime importance. This may be done by means of an air preheater which operates approximately at atmospheric pressure on both sides, and therefore is not affected by any increase of pressure in the closed-type cycle. In the case of supercharging of the combustion chamber and expansion of the combustion gases in a special turbine, the size of the preheater would naturally be reduced. Basically, it would be possible to utilize the heat of the combustion gases down to the dew point. For low rates of excess air, which is always the case in the process under discussion, the dew point lies at about 40° to 50° C. This constitutes a loss of a few percent of the heat value of the fuel, so that the preheater efficiency is predicated essentially by incomplete combustion, radiation, and auxiliary machinery. Radiation losses cannot be neglected in comparison with steam installations on account of the appreciably higher temperatures. Fortunately, the use of higher pressure levels leads to such a reduction of the surface that radiation losses are diminished to an acceptable figure.
REGULATION OF OUTPUT BY MEANS OF VARIATION
OF PRESSURE LEVEL

One of the most difficult problems in development of a new prime mover is the design of good efficiencies at part load. In the open-type cycles of previous installations, this is rendered difficult because even moderate variations of the efficiencies of compressor and turbine for small reduction of the temperature result in a pronounced decrease of over-all efficiency. For constant speed, it is not possible to regulate the open-type machine without moving the point of operation along the characteristic performance curve. If it is located at full load in a high-efficiency region, it is apt to move into one of lower efficiency. The closed-type cycle renders possible a simple means of regulation through variation of the pressure level, which produces a variation of the weight of air flowing. Thereby the temperatures do not vary at any given place, all velocities remain the same, the point of operation remains fixed on the performance characteristic, and no change of the relative angles of flow through fixed and moving rows occurs. By this means, a regulation is obtained which produces high efficiencies even at small output. Naturally, for very small loads there is a preponderance of bearing losses and of the requirements of auxiliaries. Test results thus far obtained prove that the over-all efficiency of the installation is quite satisfactory at part loads at which efficiencies of open-type installations are no longer acceptable.

Regulation through variation of the pressure level, and at the same time of the weight of air in circulation, is inherently applicable for relatively slow variations of load. For short-time load variation, it is necessary to combine pressure regulation with temperature or throttle regulation. This combination presents a number of theoretical and practical problems which will be presented on a later occasion.

OPERATION WITH GASEOUS AND SOLID FUELS

Gaseous fuels when used in the open-type process, operating with internal combustion, require cooling and in some cases dust removal before their compression. In
closed-type installations with external combustion, no pre-cooling is required, nor is a gas compressor necessary. The installation is less sensitive to solid matter carried by the gas. Much more difficult is the use of solid fuels in gas turbines with internal combustion – for instance, pulverized coal; in this case, problems of deposit and of wear in turbines and in the heat exchangers must be met. As compared with oil-fired installations, the external combustion of pulverized coal presents new requirements. These are related to those from a steam-generator technique and can be met rather easily. If it becomes possible to utilize pulverized coal for the operation of aerodynamic prime movers at efficiencies pronounced as attainable in the following paragraphs, it is beyond doubt that this prime mover represents an exceedingly economical power source for large capacities. As an alternative, it is suggested that solid fuels be subjected to a cracking process in gas generators, and that the gases then be burned in the air heater.

DEVIATIONS FROM THE DOUBLE ISOThERMAL CYCLE

In investigations and comparative studies of gas-turbine processes, frequently ideal cycles only are studied without consideration of losses, or the losses are considered only in a cursory manner. Generally speaking, however, such losses as are caused by incomplete energy interference in the machines, pressure losses in the equipment and the pipes, heat losses through incomplete waste-heat utilization – these losses are of such importance that they become a decided factor for the choice of one or the other system.

We shall discuss, therefore, in the following paragraph also the cycles deviating from the double isothermal cycle. The magnitude of the individual losses will be chosen in accordance with the present state of development of engineering practice. It will be shown that the losses resulting from the flow of the working medium and the heat losses alter the situation, as compared with the ideal cycle, profoundly. Our present knowledge of pressure losses and of heat transfer derived from similarity relations (reference 8) relate to very reliable numerical results. It is of particular interest that the aerodynamic prime-mover installation is particularly easily analyzed theoretically in contrast to other prime movers (internal-combustion engine).
Cycles without Consideration of Losses

If heat is to be converted into mechanical energy in a continuous process taking place between two temperatures $T_1$ and $T_2$, and pressures $p_1$ and $p_2$, any of a number of cycles may be chosen, of which the most important ones are shown in figure 4. Cycles II, III, and IV represent intermediate solutions for the purpose of approximating the double isothermal cycle I. Let us assume an ideal working gas (air with constant specific heat $c_p$) complete waste-heat utilization through heat exchange, and neglect losses in the machines. For closed-type cycles $p_1$ and $p_2$ may have any value for open-type processes, $p_2$ is nearly equal to the atmospheric pressure.

Cycle I

Compression CD and expansion AB of the working medium between $p_1$ and $p_2$ takes place isothermally. No heat is removed during compression CD. Heat is added from the outside only during expansion AB in the turbine. Heating DA from $T_4 = T_3$ to $T_1$ takes place completely through heat exchange. As shown above, this cycle has the same efficiency as the Carnot cycle (equation (1)), hence the highest efficiency possible between two limits of temperature $T_1$ and $T_3$.

Cycle II

The working medium is adiabatically compressed in the compressor from $T_3$, $p_2$ to $T_4$, $p_1$, line CD. Through addition of heat from the outside the gas is heated from $T_4$ to $T_1$ along $p_2$; thereby the heat exchange DE with the stream of the working medium expanded to $p_2$ supplies the quantity of heat for the temperature rise from $T_4$ to $T_2$, and the heat added from the outside EA causes the temperature rise from $T_2$ to $T_1$. Adiabatic expansion from $T_1$, $p_1$ to $T_2$, $p_2$ takes place in the turbine along AB before entering the compressor; point C, the working medium, is cooled in a closed-type cycle from $T_4$ to $T_3$ along FC. For the open-type cycle, new air is taken in from the atmosphere.

*Instead of using an entropy diagram, use may be made to advantage of the thermodynamic potential according to Colombi (reference 9).*
Cycle III

Here the gas expands in the turbine along the isotherm AB, during which process heat is added from outside to the working medium. The temperature increase $\Delta A$ from $T_4$ to $T_1$ is accomplished completely through exchange of the heat rejected by the turbine. The compressor is the same as for cycle II. After expansion and after the exchange of heat $BE$ at $p_1$ has taken place, the gas is cooled from $T_4$ to $T_3$ along FC.

Cycle IV

The gas expands adiabatically from $T_1$, $p_1p$ to $T_2$, $p_2$ along AB. However, isothermal compression $CD$ from $p_2$ to $p_1$ takes place while heat is removed along $T_3$. Heat is exchanged along $BO$ and $DE$ between the streams of gas. Then follows the heating $EA$ through heat added from outside.

Figure 5 shows the thermal efficiencies $\eta$ of the four basic cycles as a function of the pressure ratio $p_1/p_2$. It is computed from the ratio of the useful work $AL$ to the heat added from outside $Q$, where $A$ is Joule's constant. The useful work is equal to the difference between turbine work $AL_T$ and compressor work $AL_I$. The heat added, $Q$, is for complete exchange of heat equal to the turbine work $AL_T$, consequently

$$\eta = 1 - \frac{AL_I}{AL_T} \quad (12)$$

The basic relations for the work of adiabatic compression reads:

$$AL_{V_{ad}} = c_P T_3 \left[ \left( \frac{P_1}{P_2} \right)^{\frac{K-1}{K}} - 1 \right] \quad (13)$$

and for the work of isothermal compression

$$AL_{V_{is}} = AR T_3 \ln \frac{P_1}{P_2} \quad (14)$$

for adiabatic turbine work.
}\begin{equation}
\Delta T_{\text{ad}} = c_p \frac{1}{T_1} \left[ 1 - \left( \frac{p_1}{p_2} \right)^{\frac{k-1}{k}} \right] \tag{15}
\end{equation}

\text{and for isothermal work}

\begin{equation}
\Delta T_{\text{is}} = \frac{R}{\kappa} T_1 \ln \frac{p_1}{p_2} \tag{16}
\end{equation}

where \( R \) is the gas constant and \( \kappa \) is the adiabatic exponent. The thermal efficiencies for the four basic cycles, according to figure 4, are derived from equations (12) to (16), as follows:

\begin{equation}
\eta_I = 1 - \frac{T_3}{T_1} \tag{17}
\end{equation}

\begin{equation}
\eta_{\text{II}} = 1 - \frac{T_3}{T_1} \left( \frac{p_1}{p_2} \right)^{\frac{k-1}{k}} \tag{18}
\end{equation}

\begin{equation}
\eta_{\text{III}} = 1 - \frac{T_3}{T_1} \left( \frac{p_1}{p_2} \right)^{\frac{k-1}{k}} \frac{\ln \frac{p_1}{p_2}}{\ln \frac{p_1}{p_2}} \tag{19}
\end{equation}

\begin{equation}
\eta_{\text{IV}} = 1 - \frac{T_3}{T_1} \left( \frac{p_1}{p_2} \right)^{\frac{k-1}{k}} \frac{\ln \frac{p_1}{p_2}}{\ln \frac{p_1}{p_2}} \tag{20}
\end{equation}

For cycles II, III, and IV, the efficiency depends, in addition to the temperatures, upon the pressure ratio only — not upon the absolute values of pressures. For cycle I, it is also independent of the pressure ratio — being equal to that of the Carnot-cycle efficiency \( \eta_c \).

In appraising the various cycles, it is not sufficient to consider the efficiency only, but it is important to know the value of useful load obtainable per unit weight of the circulating working medium, which varies from gas to gas. In consideration of the dimensions of machines and pipes, it is desirable to realize a high specific output per kilogram per second together with a high efficiency.
Figure 6 shows, for instance, the efficiency plotted against the available output per 1 kilogram of air per second. In cycle II, operating with adiabatic compression and expansion, 1 kilogram of circulating fluid per second produces only 95 kilowatts of useful work at \( \eta = 0.62 \), whereas cycles III and IV produce 206 and 184 kilowatts, respectively, that is, twice as much.

Cycle II deviates most from the double isothermal cycle, but it may be put into operation with the least expenditure of machines and equipment since neither cooling during compression, nor heating during expansion in the turbine need be provided. This process is approached in present-day constant-pressure gas turbines with internal combustion, open-type cycle, and heat exchange.

Multistage preheating of the working medium, as required when approximating cycles I and III, will encounter considerable structural difficulties. Particularly, for the open-type process, the admission and exhaust pipes of the hot gases for the individual turbine stages must handle very large volumes. The intermediate combustion chambers also require much room. The last highly stressed turbine stages with long blades are exposed to high temperatures, likewise the heat exchanger (fig. 12) which, therefore, must be constructed of special materials.

Compression by stages to approximate cycle IV by means of intermediate cooling at the lower temperature \( T_3 \) in intercoolers \( d \) (fig. 3) is comparatively much simpler. It is frequently used in industrial compressors (reference 10). The high temperatures are already reduced in the turbine; the heat exchanger \( g \), therefore, receives lower temperatures. Furthermore, cycle IV is not theoretically inferior to cycle III. Heating in the intermediate heater \( f \) during expansion in turbines \( b \) naturally can also be employed, for even one stage will produce a definite effect in approximating \( \eta \) to the flow \( \eta_c \) (fig. 5).

Cycle IV with Consideration of Losses

The basic cycle IV with compression \( CD \) closely approximating isothermal conditions and with adiabatic expansion \( AB \) (fig. 4) has been chosen because it offers a possibility of approaching most closely the double isothermal cycle I by means of methods of construction which
are already relatively well developed. Corresponding considerations may be applied to the analysis of any of the other basic cycles.

The ideal cycle IV with constant specific heat \( c_p \) has for the case of complete heat exchange the efficiency \( \eta_{IV} \), according to equation (20).

When considering losses in the machines by neglecting pressure and heat losses, the efficiency for complete heat exchange is

\[
\eta_0 = 1 - \frac{\frac{ALV}{\eta_T}}{\frac{AR}{\eta_T}} = 1 - \frac{\frac{AR \ln \frac{P_1}{P_2}}{\frac{AR}{\eta_T}}}{\frac{AR \ln \frac{P_1}{P_2}}{\frac{AR}{\eta_T}}} = 1
\]

(21)

where \( \eta_T \) is the efficiency of adiabatic expansion and \( \eta_V \) is the efficiency of isothermal compression. When substituting \( AR = c_p - c_v \) (reference 11) and \( \kappa = c_p/c_v \), where \( c_v \) is the specific heat for constant volume, we obtain from equations (20) and (21) the following:

\[
\eta_0 = 1 - \frac{1 - \eta_{IV}}{\eta_T \eta_V} \quad (22)
\]

The gas cycle for isothermal compression takes place actually as shown on figure 7.

On account of pressure losses \( \Delta p_2 \) on the low pressure side of the heat exchanger \( g \) and in the precooler \( c \) (fig. 3), and on account of pressure losses \( \Delta p_1 \) on the high-pressure side of the heat exchanger \( g \) and in the gas heater \( e \) as well as pressure losses during compression, the compressor has to operate between pressure \( p_2' = p_2 - \Delta p_2 \) and pressure \( p_1'' = p_1 + \Delta p_1 \). Furthermore, there is a temperature difference \( \Delta T \) between the two streams of air required at any point of the heat exchanger. If isothermal compression is to take place at \( T_3 \), then the low-pressure stream can be cooled down only to \( T_3 + \Delta T \), point \( C' \); on the other hand, the high-pressure stream can be heated only to \( T_2' - \Delta T \), point \( E' \). The

*The installation under consideration is without intercooler \( d \) and intermediate heater \( f \).
smaller $\Delta T$, the smaller is the heat loss associated with it (heat rejected in cooling water, excess heat required for external heating).

**Efficiency of Cycle IV**

A numerical estimate of the actual process with machine and exchanger losses now becomes possible. The overall turbine work is

$$\Delta L_T \eta_T = c_p (T_1 - T_2) \eta_T$$  \hspace{1cm} (23)

The compressor work

$$\Delta L_Y \frac{1}{\eta_Y} = \frac{AR}{\eta_Y} \frac{T_3}{\eta_Y} \ln \frac{p_1''}{p_2''}$$  \hspace{1cm} (24)

the heat added

$$Q = c_p \left[ \eta_T (T_1 - T_2) + \Delta T \right]$$  \hspace{1cm} (25)

The efficiency of the cycle with losses then is

$$\eta = \frac{c_p(T_1 - T_2) \eta_T - \frac{1}{\eta_Y} AR T_3 \ln \frac{p_1''}{p_2''}}{c_p \left[ (T_1 - T_2) \eta_T + \Delta T \right]}$$  \hspace{1cm} (26)

When introducing $\epsilon_1 = \Delta p_1/p_1$ and $\epsilon_2 = \Delta p_2/p_2$ and assuming pressure losses $\Delta p$ small compared with absolute pressure $p$, which assumption appears permissible:

$$\ln \frac{p_1''}{p_2''} = \ln \frac{p_1(1 + \epsilon_1)}{p_2(1 - \epsilon_2)} \approx \ln \frac{p_1}{p_2} + \epsilon_1 + \epsilon_2$$  \hspace{1cm} (27)

Furthermore, for the actual case, $\Delta T$ is small compared with the drop temperature in the turbine $(T_1 - T_2)\eta_T$. Let $\delta = \Delta T/(T_1 - T_2)\eta_T$; transform, using equations (21), (26), and (27), and obtain:

$$\eta = \frac{\eta_0}{1 + \delta} - \frac{(1 - \eta_0)(\epsilon_1 + \epsilon_2)}{(1 + \delta) \ln \frac{p_1}{p_2}} = \eta_0(1 - \xi)$$  \hspace{1cm} (28)
where

\[ \xi = \frac{\delta}{1 + \delta} + \frac{(1 - \frac{1}{\eta_0})(\epsilon_1 + \epsilon_2)}{(1 + \delta) \ln \frac{p_1}{p_2}} \]  

(29)

It follows from equations (22) and (29)

\[ \xi = \frac{\delta}{1 + \delta} + \frac{\epsilon_1 + \epsilon_2}{[\eta_T \eta_f/(1 - \eta_f) - 1]} (1 + \delta) \ln \frac{p_1}{p_2} \]  

(30)

The quantity \( \xi \) indicates by how much the value \( \eta_0 \) is decreased because of the over-all temperature and pressure losses. The first term stands for the effect of heat losses, the second term for the effect of pressure losses.

Equation (30) shows the magnitude of the various sources of losses. Numerical computation reveals that a nearly complete heat exchange with small temperature differences \( \Delta T \) is imperative for obtaining high over-all efficiencies.

Actually, \( \epsilon_0 \) for air varies, particularly with increasing temperature (reference 12). This has been taken into account in plotting figures 8 to 10. In these figures assumptions have been made in regard to pressure losses \( \epsilon \) and temperature losses \( \Delta T \), which are in accordance with feasible practical values. The thermal efficiencies of the actual cycle IV have been plotted for various pressure ratios \( p_1/p_2 \) for the case of two-stage intercooling in the compressor and complete expansion without intermediate heating in the turbine. Good internal efficiencies of the machines, \( \eta_{\text{ad}} = 88 \text{ percent} \), \( \eta_{\text{ad}} = 85 \text{ percent} \) for each group of stages have been assumed. The ideal cycle without losses II and IV shows a decreasing efficiency with increasing pressure ratio \( p_1/p_2 \) (fig. 5). The losses, however, change this situation in such a fashion that for increasing pressure ratio there is an increase of efficiency at first to a maximum (figs. 8 to 10). With further increase of the pressure ratio the efficiency drops steadily. In the region of maximum efficiency the curve is rather flat for practical conditions. In actual installations, operation would take place for pressure
ratios to the right of the maximum since very small values $p_1/p_2$ require too large quantities of air. The stipulated pressure ratios of approximately 2 to 4 are the very ones suitable for the design of the machines since they result in few stages and small dimensions.

The values of figures 8 to 10 do not include the losses for radiation and friction, waste gases of the air heater and the energy required for auxiliaries. They indicate, however, that the utilization of the fuel of such a plant can be higher than that of steam-power plants. For larger installations, where secondary losses become relatively small, they may approach the values of the Diesel engine.

As a matter of interest, an estimate is presented on the basis of designs which may be feasible in the near future and under considerations of improvements in design and in the technology of construction materials giving the fuel consumption for larger installations of 10,000 kilowatts and more. Table I contains figures on which this estimate is based.

**TABLE I**

*Estimated Performance of Installations of 10,000 Kilowatts and More on the Basis of Engineering Practice Feasible in the Near Future.*

<table>
<thead>
<tr>
<th>Compressor with Two-Stage Intercooling</th>
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<tbody>
<tr>
<td>Air temperature $t_1$ at turbine inlet, °C</td>
</tr>
<tr>
<td>Adiabatic turbine efficiency $\eta_{ad}$ per stage group, percent</td>
</tr>
<tr>
<td>Adiabatic compressor efficiency $\eta_{ad}$ per stage group, percent</td>
</tr>
<tr>
<td>Inlet temperature $t_2$ of the compressor, °C</td>
</tr>
<tr>
<td>Mechanical efficiency $\eta_m$ of the turbine and compressor, each, percent</td>
</tr>
<tr>
<td>Pressure ratio $p_1/p_2$</td>
</tr>
<tr>
<td>Temperature difference $\Delta T$ in the heat exchanger, °C</td>
</tr>
<tr>
<td>Pressure loss $\epsilon$</td>
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<tr>
<td>Factor $\lambda$ for excess air of combustion</td>
</tr>
<tr>
<td>Exhaust gas temperature, °C</td>
</tr>
<tr>
<td>Radiation losses in percent of fuel energy, percent</td>
</tr>
<tr>
<td>Auxiliary power in percent of shaft</td>
</tr>
</tbody>
</table>
The thermal efficiency of the complete installation (ratio of shaft energy to fuel energy) is $\eta = 41.6$ percent. Single-stage reheating, including the effect of high-pressure losses in the air heater, increases the efficiency to $\eta = 46.2$ percent. This would increase it to a heat consumption of 1860 kilogram-calories per kilowatt-hour.*

**DISCUSSION OF THE EXPERIMENTAL INSTALLATION**

The Escher Wyss Company, Zurich, has developed and constructed an experimental installation of an aerodynamic heat power plant with closed-type cycle, which at present is undergoing systematic investigations (fig. 11). Although the installation was designed to be applicable for industrial power generation, nevertheless a certain separation of the individual units and major parts of equipment has been effected without regard to space requirements for the purpose of facilitating the installation of measuring equipment. In this way the working of the cycle can be studied in detail and the operating performance of all units of the installation can be determined numerically.

The units are designed according to established practice in order to eliminate shutdowns as much as possible.

The working air is heated by means of oil burners and is expanded successively in a high-pressure turbine $c$ and a low-pressure turbine $b$ without intermediate re-heating. The former drives the compressor $d$, which has single-stage cooling; the latter drives directly a generator $a$ at 3000 rpm. The plant has been in operation many hundred hours at temperatures which are far above those encountered in steam-turbine operation, and no trouble was encountered.

Figure 12 shows the tube bundle of the heat exchanger.

*The minimum heat consumption of a steam plant attained in the Port Washington Station is 2750 kcal/kw-hr, cf. according to Karl Schröder the minimum heat consumption of a high-pressure steam plant for present-day practice is 2500 kcal/kw-hr, cf. 499.*
Construction Materials

The installation differs in a few aspects from conventional steam-turbine design, which will be discussed below. These deviations were necessary on account of the different properties of thermal expansion of high-alloy steel which were utilized in places where strength at high temperatures was required. The coefficients of thermal expansions are almost twice as high compared with ordinary steels, and the thermal conductivity is considerably less (reference 13). This calls for new principles of design. Fortunately, steels with high strength at elevated temperatures have also good welding properties (reference 14). This makes new types of construction possible.

Whereas only a few years ago an increase of working temperature above 500°C was considered a risk in compressor and turbine design (reference 15), it is possible today, on the basis of strength and fatigue tests at high temperature, to appraise with a high degree of accuracy the possibility of operation at considerably higher temperatures.

In order not to exceed permissible limits of fatigue strength of construction steels, the stresses must be very small as compared with present-day practice in heat-engine and steam-turbine construction. Values must be kept at from 5 to 10 kilograms per square millimeter at temperatures of 650°C, and at from 2 to 5 kilograms per square millimeter at approximately 750°C, for it is only within those limits that the rate of creep is low enough to keep permanent deformations, even within years, below fractions of a percent, consequently, within safe limits.

It appears from the investigations of this paper that the closed-type cycle, with external heating and with a pressure of the working medium higher than atmospheric, offers particular advantages for low material stresses. A particular advantage is offered because of the uniform heating of the air, which eliminates local peak temperatures, for instance, in the first turbine stages. In combustion turbines with combustion chamber located immediately upstream, the local temperature differences in the flame may, under certain circumstances, be high, and the heat stresses of the material consequently much higher than would be expected from the mean temperatures.

For the closed process, centrifugal forces are smaller on account of reduced dimensions of the machines; likewise,
the stresses form internal pressures. This obviates the necessity of using cooled blades, rotors, or casings (reference 16) at least, for units that might be considered at the present stage of development. The high rate of heat transfer on the inside of the heater tubes decreases the mean wall temperature. Therefore, tube bundles can be built with safety at the above stress figures.

It is to be noted that material stresses can result from two different sources, namely, from centrifugal forces and internal pressure on one hand, and from temperature variations on the other hand. The latter may disappear with time if the material is stressed by them to the limit of creep, in contrast to the former which always remain present. This is because the creep occurs at the point of temperature stress under the effect of excessive stresses without necessarily leading to fracture, since the strength at high temperature of alloyed steel under short-time stresses is several times the strength under continuous stress. The material deforms and thereby nullifies the temperature effect because this results in a decrease of stress. Recognition of these effects and previous experiences in gas-turbine design has tended to eliminate objections in regard to safe design and operation at higher temperatures (reference 17).

High-Temperature Air Conduits

Since the process takes place at high temperatures, radiation losses must be considered, particularly for small loads. Figure 13 shows the design of a pipe which has been found suitable for high-temperature and high-pressure gases; for instance, for line 1 (fig. 11). It consists of a thin-walled inner tube which is heat-resistant and serves merely to conduct the gas. It contains holes d connecting with the insulating space, and thus eliminating stresses from internal pressure. The pipe is surrounded by an insulating space c which in turn is placed inside of a thick-walled tube b made of conventional material. This outer tube takes up the pressure of the working medium since it is protected from temperature by the insulation, and is therefore always at low temperature. Provisions have been made, of course, to prevent insulating material from getting into the inner tube a. This design should effect a considerable saving of high-quality and expensive steel.
It is proposed to present test results, operating experiences, applications and problems of regulation in a future publication.

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REFERENCES


Excerpts by E. Eckert will appear in Z.V.D.I. subsequent to the May 31, 1941 issue.


Figure 1. - Carnot-cycle.

- AB: Isothermal process with heat added
- CD: Isothermal compression

Figure 2. - Double isothermal cycle with heat exchange BC and DA.

Figure 3. - Schematic arrangement for the approximated isothermal cycle according to figure 2.

- a: Group of compressor stages
- b: Group of turbine stages
- c: Precooler
- d: Intercooler
- e: Air heater
- f: Intermediate air heater
- g: Heat exchanger
- h: Generator
Figure 4. - Entropy diagram of four constant pressure basic cycles between the temperatures $T_1$ and $T_2$, the pressures $p_1$ and $p_2$ (ideal cycles).

I Double isothermal cycle
AB Isothermal expansion in the turbine
BC Heat exchange
CD Isothermal compression in the compressor
DA Heat exchange

II Double adiabatic cycle
AB Adiabatic expansion
BF Heat exchange
DE Heat exchange
EA Heat added in the heater

III Cycle with isothermal expansion AB and adiabatic compression CD, BF and DA heat exchange.

IV Cycle with adiabatic expansion AB and isothermal compression CD.

In the closed type cycle the working medium is cooled in a cooler along FC, in the open type cycle new air is taken in from outside before the adiabatic compression CD occurs.

Figure 5. - Efficiency $\eta$ of the four ideal cycles according to figure 4 for a lower temperature $t_3 = 20^\circ\text{C}$. The efficiency of cycle I is equal to the efficiency of the Carnot-cycle $\eta_C$.

I Double isothermal cycle.
II Double adiabatic cycle.
III Cycle with isothermal expansion and adiabatic compression.
IV Cycle with adiabatic expansion and isothermal compression.
Figure 6.— Specific capacity referred to 1 kg of gas circulating per second for the four ideal cycles according to figure 4.
Upper temp. $t_1 = 650^\circ\mathrm{C}$
Lower temp. $t_3 = 20^\circ\mathrm{C}$

Figure 7.— Entropy diagram of cycle IV with losses.

Figures 8 to 10.— Efficiency of cycle IV with losses as a function of the pressure ratio $p_1/p_2$ at turbine inlet and outlet. $\epsilon$, pressure loss; $\Delta T$, temperature loss; initial temperature $t_1 = 600, 700, 800^\circ\mathrm{C}$; final temperature $t_3 = 20^\circ\mathrm{C}$; turbine efficiency $\eta_{\text{Tad}} = 88\%$ per stage group; compressor efficiency $\eta_{\text{Vad}} = 85\%$ per stage group; two-stage intercooling, no intermediate heating.
Figure 11.- Photograph showing the complete installation of the aerodynamic heat power plant developed and built by Escher Wyss Company.

Figure 12.- Tube bundle of the heat exchanger.

Figure 13.- Pipe conduit for high temperature gases. Separation functions: Conduction of the air (inner tube a), strength against internal pressure (outer tube b) and insulation c; d shows pressure equalizing holes.