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THE ROLE OF INSTABILITY WAVES IN PREDICTING JET NOISE

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Abstract

Debate over whether linear instability waves play a role in the prediction of jet noise has been going on for many years. Parallel mean flow models, such as the one proposed by Lilley,² usually neglect these waves because they cause the solution to become infinite. The present paper solves the true non-parallel acoustic equations for a two-dimensional shear layer by using a vector Greens function and assuming small mean flow spread rate. The results show that linear instability waves must be accounted for in order to construct a proper causal solution to the problem.

Introduction

Current industrial noise prediction methods, such as the MGB approach,¹ are based on Lilley's² equation which involves a wave operator appropriate to sound propagation on a parallel mean flow. This equation possesses homogeneous solutions corresponding to spatially growing instability waves on that flow and its complete solution consists of a particular solution plus these homogeneous contributions. But the result is meaningless because the instability waves become unbounded (infinite) far downstream in the flow. The usual resolution to this dilemma is to completely neglect the contribution of the instability waves. Dowling, et al.³ attempted to justify this by invoking causality arguments.

A better approach is to begin with an equation appropriate to sound propagation on a non-parallel flow, say the actual mean flow in the jet. The most important difference between this approach and Lilley's parallel flow result is that the homogeneous solutions to the acoustic equations correspond to instability waves that grow and then decay on the diverging non-parallel base flow and therefore always remain bounded, which eliminates the dilemma alluded to above.

A vector Greens function approach is used in reference 4 to obtain a general solution to the non-parallel acoustic equations by assuming that the spread rate of the mean flow is small. The relevant Green's function, which must satisfy an appropriate causality condition, consists of two components—both of which act on the

same source term. The first corresponds to the usual Lilley equation solution but with slightly modified source terms. The second is associated with linear instability waves but is very different from conventional instability models since these waves are now continuously generated along the length of the jet and do not constitute separate sound sources. They merely serve as conduits to carry the sound to the far field.

The Greens Function can be thought of as a filter that only allows certain parts of the source to radiate. Each of its two components responds to different portions of the source spectrum causing the radiation field to exhibit a bi-model structure. The contribution of the second Greens function component is fairly small at subsonic Mach numbers but can be significant at supersonic speeds. The result is therefore somewhat different from the conventional parallel flow models.

The present paper applies the general result of reference 4 to a two-dimensional shear layer with fully developed similarity form, which greatly simplifies the final equations.

The Fundamental Equations

We let

$$h_o \equiv h + \frac{1}{2} v^2 \quad (1)$$

denote the stagnation enthalpy, h the enthalpy, t the time, $\mathbf{x} \equiv \{x_1, x_2, x_3\}$ the Cartesian coordinates, p the pressure, ρ the density, $\mathbf{v} = \{v_1, v_2, v_3\}$ the fluid velocity, σ_{ij} the viscous stress tensor, and q_i the heat flux vector. The flow variables are assumed to satisfy the ideal gas law

$$p = \rho RT, \quad h = c_p T \quad (2)$$

with $R = c_p - c_v$ being the gas constant, c_p and c_v the specific heats at constant pressure and volume and T the absolute temperature.

References 5 and 19 show that the Navier-Stokes equations can be recast into the convective form of the linearized Navier-Stokes equations by dividing the dependent variables

$$\rho = \bar{\rho} + \rho', \quad p = \bar{p} + p', \quad h = \bar{h} + h', \quad v_i = \bar{v}_i + v'_i \quad (3)$$

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as well as the viscous terms $\bar{\sigma}_{ij}$ and q_i , into their ‘base flow’ components $\bar{\rho}$, \bar{p} , \bar{h} , \bar{v}_i , $\bar{\sigma}_{ij}$, and \bar{q}_i and into their ‘residual’ components ρ' , p' , h' , v'_i , σ'_{ij} , and q'_i and requiring that the former satisfy the inhomogeneous Navier-Stokes equations

$$\frac{D_o}{Dt} \bar{\rho} = 0 \quad (4a)$$

$$\frac{D_o}{Dt} \bar{\rho} \bar{v}_i + \frac{\partial \bar{p}}{\partial x_i} = \frac{\partial}{\partial x_j} (\bar{T}_{ij} + \bar{\sigma}_{ij}) \quad (4b)$$

$$\frac{D_o}{Dt} \bar{\rho} \bar{h}_o - \frac{\partial \bar{p}}{\partial t} = \frac{D_o \bar{H}_o}{Dt} + \frac{\partial}{\partial x_j} (\bar{H}_j - \bar{q}_j + \bar{v}_i \bar{\sigma}_{ij}) \quad (4c)$$

along with an ideal gas law equation of state,

$$\bar{h} = c_p \bar{T} = \frac{c_p}{R} \frac{\bar{p}}{\bar{\rho}} \quad (5)$$

where the operator D_o/Dt is not the usual convective derivative but is defined by

$$\frac{D_o}{Dt} f \equiv \frac{\partial f}{\partial t} + \frac{\partial}{\partial x_j} (\bar{v}_j f) \quad (6)$$

for any function f ,

$$\bar{h}_o \equiv \bar{h} + \frac{1}{2} \bar{v}^2 \quad (7)$$

is the base flow stagnation enthalpy, and the ‘sources strengths’ \bar{T}_{ij} , \bar{H}_o , and \bar{H}_j , which are assumed to be localized, can be arbitrarily specified.

The residual variables are governed by the convective form of the linearized Navier-Stokes (LNS) equations

$$L_{\mu\nu} u_\nu = s_\mu, \quad \mu, \nu = 1, 2, \dots, 5 \quad (8)$$

where the linear operator $L_{\mu\nu}$, which is exactly the same as the operator that would be obtained by linearizing the Euler equations about the base flow (4), is written out in full in reference 5. The Greek indices range from 1 to 5, the Latin indices from 1 to 3, the Einstein summation convention is being used, the five-dimensional dependent variable vector u_ν is given by

$$\{u_\nu\} \equiv \left\{ \rho v'_\nu, \frac{p'_o}{\gamma-1}, \rho' \right\} \quad (9)$$

and the source term s_μ is given by

$$s_\mu \equiv \frac{\partial}{\partial x_j} (e'_{\mu j} - \tilde{e}_{\mu j}) + \delta_{\mu 4} (e'_{ij} - \tilde{e}_{ij}) \frac{\partial \bar{v}_i}{\partial x_j} \quad \text{for } \mu=1, \dots, 4 \quad (10)$$

where

$$p'_o \equiv p' + \frac{\gamma-1}{2} \rho v'^2 + (\gamma-1) \bar{H}_o \quad (11)$$

The source strengths $e'_{i\nu} - \tilde{e}_{i\nu}$ are given by

$$e'_{ij} \equiv -\rho v'_i v'_j + \frac{\gamma-1}{2} \delta_{ij} \rho v'^2 + \sigma'_{ij}, e'_{i4} \equiv -\rho v'_i h'_o - q'_i + \sigma_{ij} v'_j \quad (12)$$

$$\tilde{e}_{ij} \equiv \bar{T}_{ij} - \delta_{ij} (\gamma-1) \bar{H}_o, \tilde{e}_{i4} \equiv \bar{H}_i - \bar{T}_{ij} \bar{v}_j \quad (13)$$

where

$$h'_o = h' + \frac{1}{2} v'^2 \quad (14)$$

Equations (8) are exact consequences of the original Navier-Stokes equations, but they have been rearranged so that their left-hand sides are the same as the equations that would have been obtained by linearizing the convective form of the Euler equations about the base flow (4). *In other words, they are just the LNS equations with modified dependent variables and with the viscous stress perturbation replaced by the generalized Reynolds stress $e'_{ij} - \tilde{e}_{ij}$ and the heat flux perturbation replaced by the generalized stagnation enthalpy flux $e'_{i4} - \tilde{e}_{i4}$.* In the acoustic analogy approach the right hand sides would be treated as known source terms and the linearized Euler operators $L_{\mu\nu}$ would be inverted to solve for the dependent variables. The fact that they are non-linear causes no particular difficulty since one of them, p'_o , reduces to the linear pressure fluctuation in the far field.

The base flow equations (4) reduce to the usual Euler equations when the arbitrary source strengths \bar{T}_{ij} , \bar{H}_j ,

and \bar{H}_o and viscous terms $\bar{\sigma}_{ij}$ and \bar{q}_i are set equal to zero. A general class of solutions to these equations, which conveniently provide good approximations to the actual mean flow fields in jets and shear layers, is the *unidirectional transversely sheared mean flows*

$$\bar{v}_i = \delta_{i1} U(x_2, x_3), \quad \bar{p} = \text{constant}, \quad \bar{\rho} = \bar{\rho}(x_2, x_3) \quad (15)$$

The fifth equation (8) then decouples from the remaining four which become the inhomogeneous compressible Rayleigh equations⁶

$$\bar{\rho} \left(\frac{D_o u'_i}{Dt} + \delta_{i1} u'_j \frac{\partial U}{\partial x_j} \right) + \frac{\partial p'_o}{\partial x_i} = \frac{\partial}{\partial x_j} e'_{ij} \quad (16a)$$

$$\frac{D_o p'_o}{Dt} + \gamma \bar{p} \frac{\partial u'_j}{\partial x_j} = (\gamma-1) \left(\frac{\partial e'_{4j}}{\partial x_j} + e'_{1j} \frac{\partial U}{\partial x_j} \right) \quad (16b)$$

where $u'_i \equiv u_i / \bar{\rho}$ and D_o/Dt now reduces to the usual convective derivative

$$\frac{D_o}{Dt} = \frac{\partial}{\partial t} + U \frac{\partial}{\partial x_1} \quad (17)$$

It is well known (see chapter 1 of ref. 7) that the velocity-like variables u'_i can be eliminated between these equations (by taking the divergence of the first equation and the convective derivative of the second, subtracting the results and then using the first equation to eliminate the velocity fluctuation on the left-hand side) to obtain the Lilley² equation

$$L p'_o \equiv s \quad (18)$$

where

$$L \equiv \frac{D_o}{Dt} \left(\frac{\partial}{\partial x_i} \tilde{\mathcal{C}} \frac{\partial}{\partial x_i} - \frac{\bar{D}^2}{Dt^2} \right) - 2 \frac{\partial U}{\partial x_j} \frac{\partial}{\partial x_1} \tilde{\mathcal{C}} \frac{\partial}{\partial x_j} \quad (19)$$

is the variable density Pridmore-Brown⁸ operator,

$$\tilde{\mathcal{C}} = \gamma \bar{\rho} / \bar{\rho}(x_2, x_3) \quad (20)$$

is the square of the mean flow sound speed and the source term s is written out in reference 5. As noted in the Introduction, it possesses homogeneous solutions corresponding to spatially growing instability waves on the base flow (15). The complete solution to these equations consists of a particular solution plus contributions from these homogeneous solutions, but the result is meaningless because the instability waves become unbounded far downstream in the flow causing the solution to become infinite there. The usual resolution is to completely neglect the instability wave contribution.

A better resolution is to choose the base flow to be the actual mean flow of the jet. The over bars in the base flow equations then denotes the time average

$$\bar{\bullet} \equiv \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T \bullet(\mathbf{x}, t) dt \quad (21)$$

where the dot is a place holder for ρ , v_i , p , and h , and

$$\tilde{\bullet} \equiv (\bar{\rho} \bullet / \bar{\rho}) \quad (22)$$

denotes a Favre averaged quantity⁸ for all variables except \tilde{h}_o , which is defined by Eq. (7). Notice that Eq. (20) is completely consistent with the overall ideal gas law (2) when the tilde is defined in this fashion.

The time derivatives drop out of the base flow equations and their source strengths are given by

$$\tilde{T}_{ij} = -\bar{\rho} \widetilde{v'_i v'_j} \quad (23)$$

$$\tilde{H}_o = \frac{1}{2} \tilde{T}_{ii} \quad (24)$$

$$\tilde{e}_{4i} \equiv \tilde{H}_i - \tilde{T}_{ij} \tilde{v}_j = -\bar{\rho} \widetilde{h'_o v'_i} \quad (25)$$

They are now the ordinary RANS equations, which do not, of course, form a closed system. They are usually closed by assuming some sort of model relating the source terms to the mean flow variables \tilde{v}_i , $\bar{\rho}$, \bar{p} , and \tilde{h} and their derivatives, such as Boussinesq model^{8,9} for the Reynolds stresses and a similar model for \tilde{H}_j .

The most important difference between these results and the parallel flow result is that the homogeneous solutions to the LNS equations, which now correspond to instability waves growing and then decaying on the diverging non-parallel base flow, will always remain bounded. This eliminates the paradox alluded to above and the corresponding LNS equations can be used to calculate the radiated sound. The relevant solution consists of a particular solution that is driven by the sources (i.e. it satisfies causality and therefore the appropriate upstream boundary conditions) and a homogeneous solution that is driven by the upstream boundary conditions, say at the nozzle exit.

The Green's Function Solution

The particular solution that satisfies the appropriate outgoing wave boundary conditions can be written in terms of the causal vector Greens function for these equations $g_{\nu\sigma}(\mathbf{x}, t | \mathbf{x}', t')$, which satisfies¹¹

$$L_{\mu\nu} g_{\nu\sigma} = \delta_{\mu\sigma} \delta(\mathbf{x} - \mathbf{x}') \delta(t - t') \quad (26)$$

and leads to the following formula for u_ν

$$u_\nu(\mathbf{x}, t) = \int_{-\infty}^{\infty} \int_V g_{\nu\mu}(\mathbf{x}, t | \mathbf{x}', t') s_\mu(\mathbf{x}', t') d\mathbf{x}' dt' \quad (27)$$

where the symbol V denotes integration over all space.

The derivatives can be transferred to the Greens function (see reference 4) to obtain

$$p'_o = \int_{-\infty}^{\infty} \int_V \gamma_{j\mu}(\mathbf{x}, t | \mathbf{x}', t') \tau_{\mu j}(\mathbf{x}', t') d\mathbf{x}' dt' \quad (28)$$

where

$$\begin{aligned} \gamma_{j\mu} \equiv & -(\gamma-1) \left(\frac{\partial}{\partial x'_j} g_{4\mu} - \frac{\partial \tilde{v}'_\mu}{\partial x'_j} g_{44} \right) \\ & + \frac{(\gamma-1)^2}{2} \delta_{j\mu} \left(\frac{\partial g_{4\ell}}{\partial x'_\ell} - \frac{\partial \tilde{v}'_\ell}{\partial x'_\ell} g_{44} \right) \end{aligned} \quad (29)$$

accounts for the propagation effects and

$$\tau_{\mu j} \equiv - \left(\rho v'_\mu v'_j - \bar{\rho} \tilde{v}'_\mu \tilde{v}'_j \right) + \sigma'_{\mu j} \quad (30)$$

with

$$v'_4 \equiv h'_o \quad (31)$$

and

$$\sigma'_{4j} = -q'_j + \sigma_{j\ell} v'_\ell \quad (32)$$

when the bulk viscosity is zero. τ_{ν_j} , which accounts for the source structure, is just the generalized four-dimensional fluctuating Reynolds stress in the inviscid limit which is of primary interest here. Equation (28) therefore provides a direct linear relation between this quantity and the far field pressure fluctuations.

Small Spread Rate Approximation

Our interest here is in the two-dimensional shear layer with zero velocity in the upper stream. But the general solution to Eq. (26) is very complicated even for this simple case. However, high Reynolds number turbulent shear layers are nearly parallel and it is therefore appropriate to seek an asymptotic solution^{4,5} by expanding in the shear layer spread rate ε . Since the mean Reynolds stresses will certainly dominate over the viscous stresses in this case, we assume that $\bar{\sigma}_{ij} = \bar{q}_{ij} = 0$.

Then the mean cross flow velocity \tilde{v}_3 and the Reynolds stresses $\tilde{T}_{\mu 3}$ and $\tilde{T}_{3\mu}$ will vanish and the remaining mean flow variables will expand like

$$\tilde{v}_1 = U(X, x_2) + \varepsilon U^{(1)}(X, x_2) + \dots \quad (33)$$

$$\tilde{v}_2 = \varepsilon V(X, x_2) + \varepsilon^2 V^{(1)}(X, x_2) + \dots \quad (34)$$

$$\bar{p} = P(X, x_2) + \varepsilon P^{(1)}(X, x_2) + \dots \quad (35)$$

$$\bar{\rho} = \bar{R}(X, x_2) + \varepsilon R^{(1)}(X, x_2) + \dots \quad (36)$$

and

$$\tilde{T}_{j\mu} = T_{j\mu}^{(0)} + \varepsilon T_{j\mu}^{(1)} + \varepsilon^2 T_{j\mu}^{(2)} + \dots \quad (37)$$

with similar expansions for \tilde{h} , \tilde{H}_j , and \tilde{H}_o . Here $X \equiv \varepsilon x_1$ denotes the slow streamwise variable. Substituting these into the mean flow equations and assuming that all Reynolds stresses vanish in the free streams shows that $T_{12}^{(0)} = T_{21}^{(0)} = T_{24}^{(0)} = 0$, $P = T_{22}^{(0)}$, $P^{(1)} = T_{22}^{(1)}$ and that the lowest order mean flow equations reduce to the usual two-dimensional compressible turbulent boundary layer equations.

These results imply that the linear Euler operator $L_{\mu\nu}$ will expand like

$$L_{\mu\nu} = L_{\mu\nu}^{(0)} + \varepsilon L_{\mu\nu}^{(1)} + \dots \quad (38)$$

and therefore that the vector Greens function $g_{\nu\sigma}$ has an expansion of the form

$$g_{\nu\sigma} = g_{\nu\sigma}^{(0)} + \varepsilon g_{\nu\sigma}^{(1)} + \dots \quad (39)$$

where $g_{\nu\sigma}^{(0)}$ and $g_{\nu\sigma}^{(1)}$ are determined by

$$L_{\mu\nu}^{(0)} g_{\nu\sigma}^{(0)} = \delta_{\mu\sigma} \delta(\mathbf{x} - \mathbf{x}') \delta(t - t') \quad (40)$$

and

$$L_{\mu\nu}^{(0)} g_{\nu\sigma}^{(1)} = -L_{\mu\nu}^{(1)} g_{\nu\sigma}^{(0)} \quad (41)$$

The operators $L_{\mu\nu}^{(0)}$ and $L_{\mu\nu}^{(1)}$ are written out in detail in reference 4. The important thing is that the left hand sides of the first 4 components of these equations are formally the same as the left sides of the Rayleigh equations (16) and since their coefficients are independent of x_3 and t and depend on x_1 only through the slow streamwise variable X , a local (causal) solution can be constructed by taking Fourier-Laplace transforms to obtain

$$\begin{aligned} (\gamma-1) g_{4\sigma}^{(0)} = & - \int_{ic-\infty}^{ic+\infty} \int_{-\infty}^{\infty} \frac{\tilde{c}_o^2(x'_2)}{[k_1 U(x'_2) - \omega]^2} \\ & D'_\sigma e^{i[k_1(x_1-x'_1) + k_3(x_3-x'_3) - \omega(t-t')]} \bar{G}_o(x_2 | x'_2) d\omega dk \end{aligned} \quad (42)$$

where

$$\mathbf{k} = \{k_1, k_3\} \quad (43)$$

is the wave number vector in the plane of the shear layer

$$D'_\sigma \equiv \frac{\partial}{\partial x'_j} \quad \text{for } \sigma = j = 1, 2, 3 \quad (44)$$

$$D'_\sigma \equiv \frac{-(\gamma-1)}{\widetilde{c}_o^2(x'_2)} \left[i\omega + U(x'_2) \frac{\partial}{\partial x'_1} \right] \quad \text{for } \sigma=4 \quad (45)$$

and the one-dimensional self adjoint Greens function $\bar{G}_o(x_2|x'_2)$ satisfies

$$L_{k_1} \bar{G}_o = \frac{\delta(x_2 - x'_2)}{(2\pi)^3} \quad (46)$$

where

$$L_{k_1} \equiv \frac{\partial}{\partial x_2} \frac{\widetilde{c}_o^2}{(k_1 U - \omega)^2} \frac{\partial}{\partial x_2} + 1 - \frac{k^2 \widetilde{c}_o^2}{(k_1 U - \omega)^2} \quad (47)$$

with $k^2 = |\mathbf{k}|^2$ is the one-dimensional reduced Rayleigh's operator.

\bar{G}_o is therefore given by

$$\bar{G}_o = \frac{w_+(x_2) w_-(x'_2)}{\Delta (2\pi)^3} \quad \text{for } x_2 > x'_2 \quad (48)$$

where

$$L_{k_1} w_\pm = 0 \quad (49)$$

$$w_\pm \sim e^{\pm \lambda_\pm x_2} \quad x_2 \rightarrow \pm\infty \quad (50)$$

$$\lambda_+ = \sqrt{k^2 - \omega^2/c_+^2}, \quad \lambda_- = \sqrt{k^2 - (k_1 U_- - \omega)^2/c_-^2} \quad (51)$$

c_+ , c_- , and U_- are defined in the obvious ways and

$$\Delta \equiv W(x_2) \widetilde{c}_o^2(x_2) / [k_1 U(x_2) - \omega]^2 \quad (52)$$

where W is the Wronskian of ω_+ and ω_- is a constant independent of x_2 . The result for $x_2 < x'_2$ will not be needed.

Steady state solutions can only exist if the Laplace inversion contour (shown in fig. 1) can be continuously deformed onto the real axis. But $\bar{G}_o^{(0)}$ possess (usually simple) poles in the upper half k_1 -plane that cross the real k_1 -axis during this deformation (since the shear

layer is inviscidly unstable). The k_1 -integration contour c_{k_1} must therefore be deformed to lie below these poles (as shown in fig. 2) in order to obtain a continuous result.^{12,13} The poles correspond to the eigenvalues, say $k_1 = \kappa(\omega, k_3, X)$ of the Rayleigh operator (47). The integral over c_{k_1} can then be decomposed into an integral over the real k_1 -axis plus a contour integral that can be evaluated as the residue of the pole that crossed the real axis. The causal Greens function (43) can therefore be written as

$$\begin{aligned} (\gamma-1) g_{4\sigma}^{(0)} &= - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int \frac{\widetilde{c}_o^2(x'_2)}{[k_1 U(x'_2) - \omega]^2} \\ &D'_\sigma e^{i[k_1(x_1-x'_1) + k_3(x_3-x'_3) - \omega(t-t')]} \bar{G}_o(x_2|x'_2) d\omega d\mathbf{k} \\ &- 2\pi i \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int \frac{\widetilde{c}_o^2(x'_2) w_{o+}(x_2)}{[\kappa U(x'_2) - \omega]^2 \Delta'(\omega, k_3)} \\ &D'_\sigma e^{i[\kappa(\omega, k_3)(x_1-x'_1) + k_3(x_3-x'_3) - \omega(t-t')]} w_{o-}(x'_2) d\omega dk_3 \end{aligned} \quad (53)$$

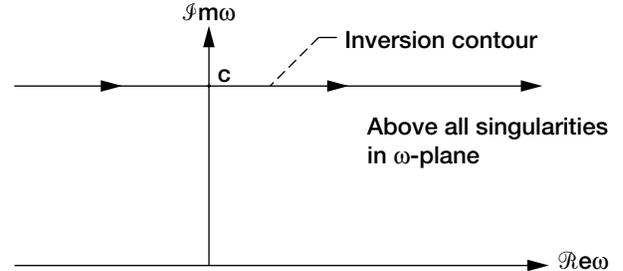


Figure 1.—Inversion contour for Laplace transform.

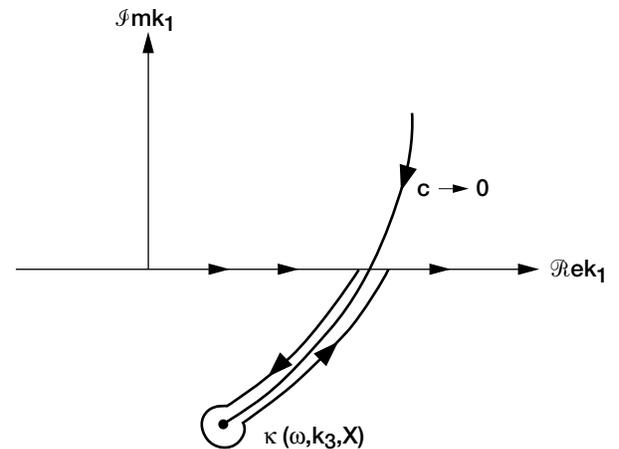


Figure 2.—Deformed k_1 -plane contour.

where $w_{0\pm}$, which are now linearly dependent, are the limits of w_{\pm} as $k_1 \rightarrow \kappa$ and

$$\Delta' \equiv \lim_{k_1 \rightarrow \kappa} \frac{d\Delta}{dk_1} \quad (54)$$

The first term in this result, which corresponds to the usual Lilley's equation solutions that appear in the literature, remains bounded and is a uniformly valid approximation to the analogous term in the true non-parallel flow result. But the second terms grows without bound as $x_1 - x'_1$ becomes large (since $\text{Im } \kappa < 0$) and therefore becomes invalid on the long streamwise length scale $X - X'$. It can, however, be rendered uniformly valid within the shear layer by using the method of multiple scales, which amounts to replacing

$$e^{i[\kappa(\omega, k_3)(x_1 - x'_1)]}$$

by

$$A(X|X') e^{i \left[\frac{1}{\varepsilon} \int_{X'}^X \kappa(\omega, k_3, X) dX \right]}$$

where the slowly varying amplitude function A , which satisfies the initial condition

$$A(X|X') \rightarrow 1 \text{ as } X \rightarrow X' \quad (55)$$

is determined by imposing a solvability condition on the next order solution. The detailed results are given in reference 4.

But, this solution is still not uniformly valid at large transverse distances where $Y \equiv \varepsilon x_2 = O(1)$, because the second order solution¹⁴ (which is a generalization of the second order instability wave solution considered by Tam and Morris) now decays more slowly with x_2 as $x_2 \rightarrow \pm \infty$ than the first order solution. But an "outer" solution can be constructed by using the WKBJ method to solve the reduced stationary (or uniformly moving) media wave equation that governs the flow in that region. This result can be used to form a "composite" solution¹⁷ that remains uniformly valid everywhere within and outside the shear layer. The details are given in reference 4. Our interest here is in the upper region and we therefore do not require uniformity below the shear layer.

The final uniformly valid Greens function $g_{4\sigma}$ can be written as

$$(\gamma - 1) g_{4\sigma}^{(0)} = \int_{-\infty}^{\infty} e^{-i\omega(t-t')} G_{4\sigma}^{(0)}(\mathbf{x}|\mathbf{x}'; \omega) d\omega \quad (56)$$

where

$$\begin{aligned} G_{4\sigma}^{(0)} = & - \int_{-\infty}^{\infty} \int \frac{\widetilde{c}_o^2(x'_2)}{[k_1 U(x'_2) - \omega]^2} \\ & D'_{\sigma} e^{i[k_1(x_1 - x'_1) + k_3(x_3 - x'_3)]} \bar{G}_o(x_2|x'_2) dk \\ & - \frac{i}{(2\pi)^2} \int_{-\infty}^{\infty} \frac{c_o^2(x'_2) A(X_o|X') w_{o+}(x_2)}{[\kappa U(x'_2) - \omega]^2 \Delta'(\omega, k_3) \sqrt{1 + \alpha'(X_o) Y}} \\ & D'_{\sigma} e^{i \left[\frac{1}{\varepsilon} \bar{\Theta}(X, Y) + k_3(x_3 - x'_3) \right]} w_{o-}(x'_2) dk_3 \end{aligned} \quad (57)$$

$$\bar{\Theta} \equiv \Theta - i \sqrt{\kappa^2(X) + k_3^2 - (\omega/c_+)^2} Y \quad (58)$$

$$\Theta = \int_{X'}^{X_0} \kappa(Z) dZ - \frac{i \left[\left(\frac{\omega}{c_+} \right)^2 - k_3^2 \right] Y}{\sqrt{\kappa^2(X_0) + k_3^2 - (\omega/c_+)^2}} \quad (59)$$

$$X_o \equiv X - \alpha(X_o) Y \quad (60)$$

$$\alpha \equiv \frac{\kappa}{i \sqrt{\kappa^2 + k_3^2 - (\omega/c_+)^2}} \quad (61)$$

with the arguments k_3 and ω suppressed. The functions \bar{G}_o and Δ' are defined by Eqs. (48), (52), and (54).

It is necessary to evaluate Eq. (57) in the region where

$$R^2 = \varepsilon^2 r^2 \rightarrow \infty \quad (62)$$

with

$$r^2 \equiv \sqrt{x_1^2 + x_2^2 + x_3^2} \quad (63)$$

in order to calculate the radiated sound at large distances from the shear layer. But applying the method of stationary phase and using Eqs. (48), (50), (51) along the asymptotic behavior of Θ and a shows that

$$\begin{aligned} G_{4\sigma}^{(0)} \rightarrow & - \frac{e^{i\omega r/c_+}}{(2\pi)^2 r} \frac{\omega}{c_+} \sin \theta \cos \varphi \\ & \left[G_{B\sigma}(\hat{\mathbf{x}}|\mathbf{x}') + G_{I\sigma}(\hat{\mathbf{x}}|\mathbf{x}') \right] \end{aligned} \quad (64)$$

when the azimuthal polar angle θ and the Mach number M are sufficiently large. The $G_{B\sigma}(\hat{\mathbf{x}}|\mathbf{x}')$ and $G_{I\sigma}(\hat{\mathbf{x}}|\mathbf{x}')$ are defined in reference 4, φ is the circumferential polar angle and $\hat{\mathbf{x}} = \hat{\mathbf{x}}(\theta, \varphi)$ denotes the unit vector in the \mathbf{x} -direction.

The Far Field Acoustic Spectrum

The quantity of interest is the mean square acoustic pressure

$$\overline{p^2} \equiv \frac{1}{2T} \int_{-T}^T p'_o(\mathbf{x}, t) p'_o(\mathbf{x}, t+t_o) dt \quad (65)$$

at large distances above the shear layer, where we have used the fact that $p'_o \rightarrow p'$ in this region (see Eqs. (11), (23), and (24)) and T denotes a large but finite time interval. Substituting Eq. (28) into this result and changing integration variables shows that

$$\overline{p^2}(\mathbf{x}, t_o) = \int_{-\infty}^{\infty} \int_V \bar{\gamma}_{j\sigma\mu\ell}(\mathbf{x}|\mathbf{x}'; \boldsymbol{\zeta}, t_o + \tau) \bar{\tau}_{\sigma j\mu\ell}(\mathbf{x}'; \boldsymbol{\zeta}, \tau) d\mathbf{x}' d\boldsymbol{\zeta} d\tau \quad (66)$$

where

$$\bar{\gamma}_{j\sigma\ell\mu} \equiv \int_{-\infty}^{\infty} \gamma_{j\sigma}(\mathbf{x}|\mathbf{x}', t_1 + t_o + \tau) \gamma_{\ell\mu}(\mathbf{x}|\mathbf{x}' + \boldsymbol{\zeta}, t_1) dt_1 \quad (67)$$

accounts for the acoustic propagation and mean flow interaction effects and

$$\bar{\tau}_{\sigma i\mu j} \equiv \frac{1}{2T} \int_{-T}^T \tau_{\sigma i}(\mathbf{x}', t') \tau_{\mu j}(\mathbf{x}' + \boldsymbol{\zeta}, t' + \tau) dt' \quad (68)$$

is the density weighted fourth order 2-point time delayed turbulent velocity/total enthalpy correlation.

Equation (66) provides a direct linear relation between the acoustic pressure and the fourth order correlations of the turbulent fluctuations in the shear layer. Unfortunately, the latter quantity is very difficult to determine—either experimentally or numerically—and it is, therefore, highly desirable to make the acoustic predictions as insensitive to its details as possible. It would be possible to make considerable progress toward achieving that objective if the propagation factor, $\bar{\gamma}_{j\sigma\mu\ell}$, were to vary only slowly over the correlation length of $\bar{\tau}_{\sigma j\mu\ell}$, because the former could then be treated as a constant relative to the

$\boldsymbol{\zeta}$ -integration and the result would then depend only on the temporal correlation of the turbulence at any given point in the shear layer. This is probably the true in the cross stream direction, but $\bar{\tau}_{\sigma j\mu\ell}$ is expected to decay much more slowly in the stream-wise direction—especially at the higher Mach numbers of technological interest. However, Lighthill¹⁵ pointed out that the ζ_1 -decay should be much more rapid in a reference frame moving with the convection velocity, U_c , of the turbulence and Ffowcs Williams¹⁶ showed that this idea can best be implemented by introducing the moving frame correlation

$$\tau_{\sigma j\mu\ell}^M(\mathbf{x}'; \boldsymbol{\xi}, \tau) \equiv \bar{\tau}_{\sigma j\mu\ell}(\mathbf{x}'; \boldsymbol{\zeta}, \tau) \quad (69)$$

where

$$\boldsymbol{\xi} \equiv \boldsymbol{\zeta} - \hat{i}U_c\tau \quad (70)$$

into Eq. (66) and changing the integration variable from $\boldsymbol{\zeta}$ to $\boldsymbol{\xi}$.

The result, which is much simpler when re-expressed in terms of the acoustic spectrum

$$I_{\omega}(\mathbf{x}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega t_o} \overline{p^2}(\mathbf{x}, t_o) dt_o \quad (71)$$

or better yet in terms of $I_{\omega}(\mathbf{x}|\mathbf{x}')$

$$I_{\omega}(\mathbf{x}) = \int_V I_{\omega}(\mathbf{x}|\mathbf{x}') d\mathbf{x}' \quad (72)$$

the acoustic spectrum at \mathbf{x} due to a unit volume of turbulence at \mathbf{x}' , is given by

$$I_{\omega}(\mathbf{x}|\mathbf{x}') = 2\pi \int_{-\infty}^{\infty} \int_V \Gamma_{j\sigma}(\mathbf{x}|\mathbf{x}'; \omega) \Gamma_{\ell\mu}^*(\mathbf{x}|\mathbf{x}' + \boldsymbol{\xi} + \hat{i}U_c\tau; \omega) e^{-i\omega\tau} \tau_{\sigma j\mu\ell}^M(\mathbf{x}', \boldsymbol{\xi}, \tau) d\boldsymbol{\xi} d\tau \quad (73)$$

where capital letters denote Fourier transforms (with respect to time) of the corresponding lower case letters, and the asterisk denotes complex conjugates.

When θ and M are sufficiently large, equation (64) and the form of $G_{B\mu}$ and $G_{I\mu}$ given in reference 4 imply that

$$\Gamma_{j\sigma} = -\frac{e^{i\omega r/c_+} \omega}{(2\pi)^2 c_+ r} \sin \theta \cos \varphi \left[\Gamma_{Bj\sigma}(\hat{\mathbf{x}}|\mathbf{x}') + \Gamma_{Ij\sigma}(\hat{\mathbf{x}}|\mathbf{x}') \right]$$

(74)

where

$$\Gamma_{Bj\sigma} \equiv \tilde{\Gamma}_{Bj\sigma}(\hat{\mathbf{x}}|\mathbf{x}'_{\perp}) e^{-i\frac{\omega}{c_+}x_1 \cos \theta} = \tilde{D}_{j\sigma\mu} G_{B\mu}(\hat{\mathbf{x}}|\mathbf{x}') \quad (75)$$

$$\Gamma_{lj\sigma} \equiv \tilde{\Gamma}_{lj\sigma}(\hat{\mathbf{x}}|\mathbf{x}'_{\perp}) e^{i\Theta_{\infty}(\theta, \varphi, X')} = \tilde{D}_{j\sigma\mu} G_{l\mu}(\hat{\mathbf{x}}|\mathbf{x}'_{\perp})$$

$$\Theta_{\infty} \equiv \int_{X'}^{\bar{\alpha}_0} \kappa(Z) dZ - \frac{\omega \bar{\alpha}_0}{c_+} \sqrt{1 - \sin^2 \theta \sin^2 \varphi \cos \theta} \quad (76)$$

$\bar{\alpha}_0$ is determined implicitly by

$$\kappa\left(\omega, \frac{\omega}{c_+} \sin \theta \sin \varphi, \bar{\alpha}_0\right) = \frac{\omega}{c_+} \cos \theta \quad (77)$$

$$\tilde{D}_{j\sigma\mu} \equiv -\delta_{\sigma\mu} \frac{\partial}{\partial x'_j} + \frac{\partial U}{\partial x'_j} \delta_{\sigma 1} \delta_{\mu\sigma} + \frac{(\gamma-1)}{2} \delta_{j\sigma} \frac{\partial}{\partial x'_\mu} \quad (78)$$

and $\mathbf{x}'_{\perp} = \{x'_2, x'_3\}$ is the cross flow source variable.

Since Θ_{∞} varies on the slow streamwise length scale X' , which is much longer than the correlation length of the turbulence, we can account for its variation over the range of integration in Eq. (73) by expanding it in a Taylor series to show that

$$\Theta_{\infty}(\hat{\mathbf{x}}, X' + \varepsilon(\xi_1 + U_c \tau)) + \Theta_{\infty}(\hat{\mathbf{x}}, X') - \varepsilon \kappa(X')(\xi_1 + U_c \tau) \quad (79)$$

with more than enough accuracy to evaluate this result. Using this in Eq. (76) and inserting the result along with Eqs. (74) and (75) into Eq. (73) shows that

$$I_{\omega}(\mathbf{x}|\mathbf{x}') = \left(\frac{\omega \sin \theta \cos \varphi}{c_+ r 2\pi} \right)^2$$

$$\int_V \left[\Gamma_{Bj\sigma}(\hat{\mathbf{x}}|\mathbf{x}') + \Gamma_{lj\sigma}(\hat{\mathbf{x}}|\mathbf{x}') \right]$$

$$\left[\Gamma_{B\ell\mu}^*(\hat{\mathbf{x}}|\mathbf{x}' + \boldsymbol{\xi}) H_{\sigma j\mu\ell}^*(\mathbf{x}'; \boldsymbol{\xi}, \omega(1 - M_c \cos \theta)) \right.$$

$$\left. + \Gamma_{l\ell\mu}^*(\hat{\mathbf{x}}|\mathbf{x}' + \boldsymbol{\xi}) H_{\sigma j\mu\ell}^*(\mathbf{x}'; \boldsymbol{\xi}, \omega - U_c \kappa^*(X', \omega)) \right] d\xi \quad (80)$$

where

$$H_{\sigma j\mu\ell}^*(\mathbf{x}'; \boldsymbol{\xi}, \omega) \equiv \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega\tau} \tau^M \tau_{\sigma j\mu\ell}^M(\mathbf{x}'; \boldsymbol{\xi}, \tau) d\tau \quad (81)$$

denotes the 2-point fourth order turbulence spectrum relative to the moving frame and the second terms in the square brackets are identically zero when ω is greater than the neutral frequency where $\text{Im} \kappa(\omega, x'_o) = 0$.

This result is nearly exact. The only significant approximation is that the mean flow spread rate, ε , is small. But $H_{\sigma j\mu\ell}$ is expected to vary much more rapidly with $\boldsymbol{\xi}$ than $\Gamma_{e\mu}$ and Eq. (80) can therefore be approximated by

$$I_{\omega}(\mathbf{x}|\mathbf{x}') = \left(\frac{\omega \cos \varphi \sin \theta}{c_+ r 2\pi} \right)^2$$

$$\left[\Gamma_{Bj\sigma}(\hat{\mathbf{x}}|\mathbf{x}') + \Gamma_{lj\sigma}(\hat{\mathbf{x}}|\mathbf{x}') \right]$$

$$\left[\Gamma_{B\ell\mu}^*(\hat{\mathbf{x}}|\mathbf{x}') \Phi_{\sigma j\mu\ell}^*(\mathbf{x}'; \omega(1 - M_c \cos \theta)) \right.$$

$$\left. + \Gamma_{l\ell\mu}^*(\hat{\mathbf{x}}|\mathbf{x}') \Phi_{\sigma j\mu\ell}^*(\mathbf{x}'; \omega - U_c \kappa^*(\omega, X')) \right] \quad (82)$$

where

$$\Phi_{\sigma j\mu\ell}(\mathbf{x}'; \omega) \equiv \int_V H_{\sigma j\mu\ell}(\mathbf{x}'; \boldsymbol{\xi}, \omega) d\boldsymbol{\xi} \quad (83)$$

is the single point fourth order turbulence spectral tensor at \mathbf{x}' .

This approximation cannot remain valid when $M_c \cos \theta = 1$ because the first spectral function in Eq. (82) does not $\rightarrow 0$ as $\omega \rightarrow \infty$ in this case, which causes the integral of Eq. (82) with respect to ω (and therefore the mean square pressure) to become infinite. Ffowcs Williams¹⁶ argued that this type of difficulty could be overcome by replacing the Doppler factor $(1 - M_c \cos \theta)$ with

$$1 - M_c \cos \theta \rightarrow \sqrt{(1 - M_c \cos \theta)^2 + (aM_c)^2} \quad (84)$$

where a is a small constant. While this result is not strictly valid in the present context, it may be useful for obtaining numerical results.

Notice that Eq. (82) can now be evaluated without performing any quadratures once the appropriate solutions to the Rayleigh equation (49) have been found. But this equation is still fairly complex and a great deal of modeling and/or computation is required to evaluate all of its terms. It is therefore worthwhile to

introduce some approximate results that can be used to get some insight into the important physical effects.

Some Approximate Results

It is well known²⁰ that the Reynolds stresses $T_{11}^{(0)}$ and $T_{12}^{(1)}$ that appear in the lowest order mean flow equations can be chosen so that these equations possess similarity solutions of the form

$$U = U(\eta), \quad \bar{R} = \bar{R}(\eta) \quad (85)$$

where

$$\eta \equiv x_2/x \quad (86)$$

and since this causes no particular inconsistency with the expansions (33) through (37) and agrees well with experimental observation we shall now assume this to be the case.

We also suppose that the observation point lies in the plane perpendicular to that of the shear layer, i.e. that the azimuthal angle $\varphi = 0$. Then we can evaluate all of the factors in Eq. (82) by solving the one-dimensional Rayleigh equation, i.e. by setting $k_3 = 0$ (or equivalently $k = k_1$) in Eq. (47) which now becomes

$$L_{k_1} \rightarrow \frac{d}{d\eta} \frac{\tilde{c}_o^2}{(\sigma U - 1)^2} \frac{d}{d\eta} + \bar{\omega}^2 \left[1 - \frac{\sigma^2 \tilde{c}_o^2}{(\sigma U - 1)^2} \right] \quad (87)$$

where

$$\bar{\omega} \equiv X\omega \quad (88)$$

and

$$\sigma \equiv k_1/\omega \quad (89)$$

is the reciprocal of the complex phase speed. The eigenvalue κ in Eq. (76) can now be replaced by

$$\sigma_o(\bar{\omega}) \equiv \frac{\kappa}{\omega} \quad (90)$$

i.e. it is a function of the single variable $\bar{\omega}$ which means that $\bar{\alpha}_o$ is now determined by

$$\sigma_o(\bar{\Omega}) = \frac{1}{c_+} \cos \theta \quad (91)$$

where $\bar{\Omega} \equiv \omega \bar{\alpha}_o$ is essentially the complex frequency eigenvalue of the Rayleigh problem corresponding to the specified real phase speed $c_+/\cos \theta$.

Additional simplification can be achieved by assuming that

$$\Phi_{\sigma j \mu \ell}(\omega) \approx \delta_{\sigma j} \delta_{\mu \ell} \Phi_o(\omega) \quad (92)$$

which implies, among other things, that $h'_o \approx 0$. For simplicity we also neglect the cross coupling between the two components of the Greens function and set $\tilde{c}_o^2 = c_+^2 = \text{constant}$. It then follows from Eqs. (45), (47), (65), (74) through (77) and (82) that

$$I_\omega(\mathbf{x}|\mathbf{x}') = \left[\frac{\omega^3 X'(\gamma+1)}{2\pi c_+^3 r} \right]^2 \left\{ \left| \frac{w_- \left(\eta', \bar{\omega}, \frac{1}{c_+} \cos \theta \right)}{\bar{\Delta} \left(\bar{\omega}, \frac{1}{c_+} \cos \theta \right)} \right|^2 \right. \\ \left. \Phi_o(\omega(1 - M_o \cos \theta)) + \left| \frac{w_{o-}(\eta', \bar{\omega}) A(\bar{\Omega}|X')}{\bar{\Delta}'(\bar{\omega})} \right|^2 \right. \\ \left. \frac{2\pi}{\varepsilon \sigma'_o} e^{-(2/\varepsilon) \text{Im} \Theta_\infty(\theta, \bar{\omega})} \text{Re} \Phi_o(\omega(1 - U_c \sigma_o(\bar{\omega}))) \right\} \quad (93)$$

where

$$\bar{\Delta} \equiv \frac{W_o}{(\sigma U - 1)^2} \quad (94)$$

$$\bar{\Delta}' \equiv \lim_{\sigma \rightarrow \sigma_o} \frac{\partial \bar{\Delta}}{\partial \sigma} \quad (95)$$

$$\sigma'_o \equiv \frac{d\sigma_o}{d\bar{\Omega}} \quad (96)$$

and W_o denotes the Jacobian with derivatives with respect to η .

Numerical Results

To compute the radiated sound field from Eq. (93) it is necessary to specify the turbulence spectra Φ_o . Recent numerical studies^{18,20} suggest that it should be of the Gaussian form

$$\Phi_o = \frac{2\pi (\rho_s u_s^2)^2 a_s}{\omega_s} \ell_s^3 e^{-(\omega/2\omega_s)^2} \quad (97)$$

where $\bar{\omega}_s = X'\omega_s$, $\bar{\ell}_s = \ell_s/X'$, ρ_s , and U_s are constants with obvious physical interpretation and a_s is an empirically determined scale factor. It is also necessary to specify the mean velocity profile $U(\eta)$ in the shear layer which is reasonably well represented by²⁰

$$U = \frac{1}{2}(1 - \tanh \eta) \quad (98)$$

The result is, as it should be, independent of the axial source location X' within the shear layer when expressed in terms of the scaled frequency $\bar{\omega} \equiv X'\omega$.

When $\text{Im } \Theta_\infty = O(1)$, the $1/\varepsilon$ factor in the exponent of Eq. (93) will cause the second term to be negligibly small when $\text{Im } \Theta_\infty > 0$ and very large (i.e. it will be dominant) when $\text{Im } \Theta_\infty < 0$. The ε in the denominator will cause his term to be large when $\text{Im } \Theta_\infty = O(\varepsilon)$ no matter what its sign.

Figure 3 is a plot of $\text{Im } \Theta_\infty$ as a function of θ for various values of the Mach number M and source frequency $\bar{\omega}$. The curves are truncated at small θ because the second term has no radiation field beyond this point, i.e. it becomes evanescent there. These results show that $\text{Im } \Theta_\infty$ is always positive at $\theta = 90^\circ$ but can change sign and become negative when M is sufficiently large and θ is sufficiently small.

The spectral shape at 90° is therefore produced by the first term in Eq. (93) which is primarily determined by the factor

$$\left(\frac{\omega}{c_\infty}\right)^6 \Phi_o(\omega(1 - M_c \cos\theta)) = \left(\frac{\omega}{c_\infty}\right)^6 \Phi_o(\omega)$$

at that angle. Refraction effects, which are accounted for by the first factor in this term, will cut off the high part of the source spectrum at small angles to the axis. But figure 3 shows that the second term can become dominant at these angles when M is sufficiently large. Its spectral shape, which is primarily determined by

$$\left(\frac{\omega}{c_\infty}\right)^6 \text{Re } \Phi_o(\omega - U_c \kappa(X', \omega)) H(-\text{Im } \kappa(X', \omega))$$

where $H(x)$ denotes the Heaviside function $H(x) = 1, 0$ $x \geq 0$, is relatively independent of angle and has a much narrower width than $\Phi_o(\omega)$ due to the relatively narrow band of unstable frequencies at X' .

Concluding Remarks

The Navier-Stokes equations, rewritten in the form of the LNS equations with externally applied stress and energy flux sources were solved for a two-dimensional shear layer by using a vector Greens function approach and assuming that the spread rate ε of the shear layer is small. The relevant Greens function has two components -one of which corresponds to the usual Lilley equation solutions that appear in the literature and the other associated with linear instability waves.

Numerical results obtained for a simplified source model show encouraging qualitative agreement with experimental observations. Observed differences in spectral shapes at small and large angles to the downstream axis that occur at the higher Mach numbers can, in part, be attributed to the fact that the first Greens function component is dominant at $\theta = 90^\circ$ while the second component is dominant at small θ .

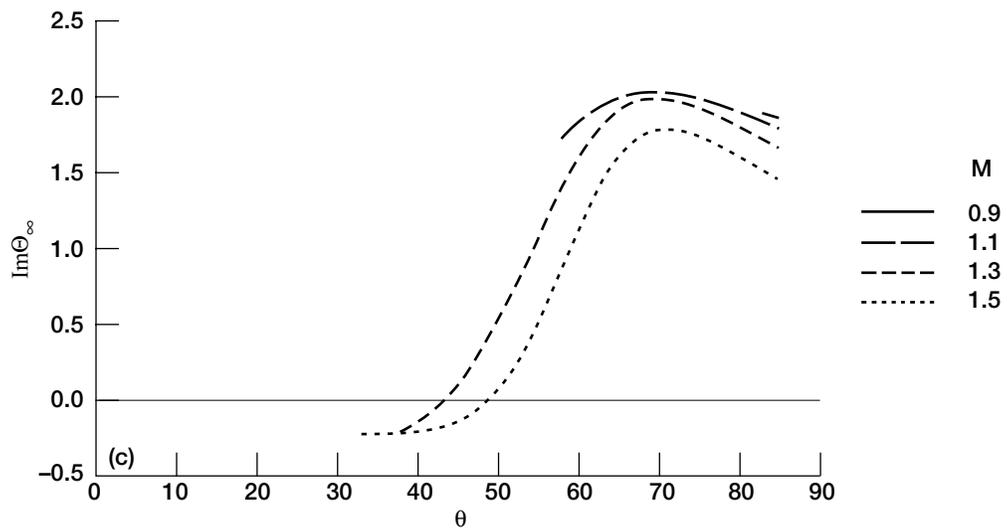
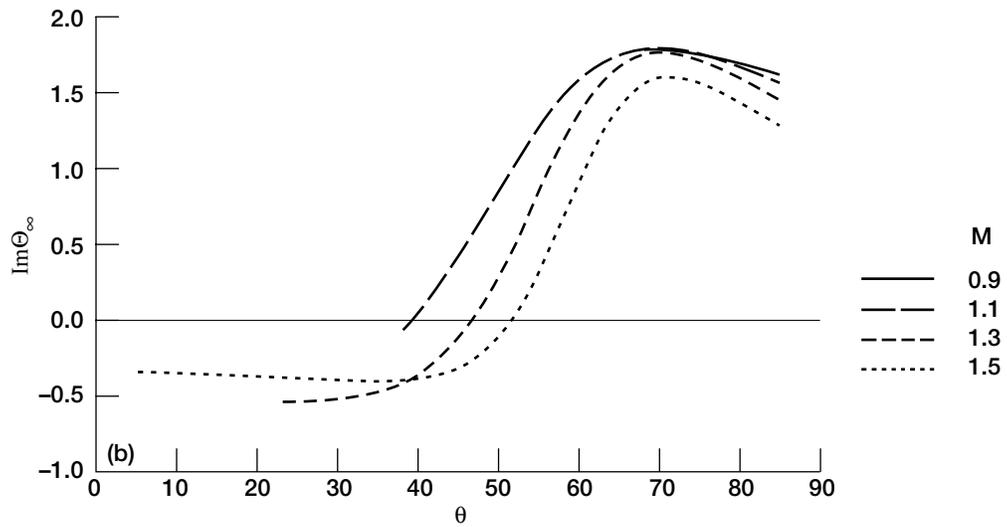
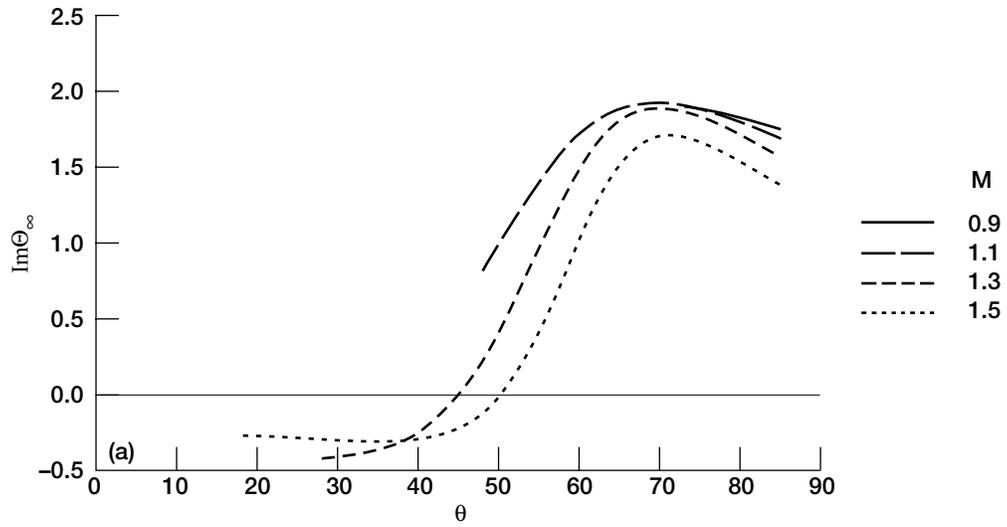


Figure 3.— $\text{Im}\Theta_\infty(\theta, \omega)$ vs θ . (a) $\bar{\omega} = 0.2$. (b) $\bar{\omega} = 0.1$. (c) $\bar{\omega} = 0.3$.

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