ROBUST CONTROL FOR MICROGRAVITY VIBRATION ISOLATION USING FIXED-ORDER, MIXED $H_2/\mu$ DESIGN

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Abstract

Many space-science experiments need an active isolation system to provide a sufficiently quiescent microgravity environment. Modern control methods provide the potential for both high-performance and robust stability in the presence of parametric uncertainties that are characteristic of microgravity vibration isolation systems. While $H_2$ and $H_\infty$ methods are well established, neither provides the levels of attenuation performance and robust stability in a compensator with low order. Mixed $H_2/H_\infty$ controllers provide a means for maximizing robust stability for a given level of mean-square nominal performance while directly optimizing for controller order constraints. This paper demonstrates the benefit of mixed norm design from the perspective of robustness to parametric uncertainties and controller order for microgravity vibration isolation. A nominal performance metric analogous to the $\mu$ measure for robust stability assessment is also introduced in order to define an acceptable trade space from which different control methodologies can be compared.

1 Introduction

Conducting research in a low-acceleration environment enables scientific investigations that are not possible on the surface of the earth. Entirely new research directions are being developed for pursuit in low earth orbit in disciplines such as life sciences, materials science, combustion, fundamental physics and fluid mechanics. When completed, the International Space Station (ISS) will be a unique research laboratory for state-of-the-art microgravity ($\mu g$) science investigations.

Yet due to a variety of vibro-acoustic disturbances on the ISS, the acceleration environment is expected to significantly exceed the requirements of many acceleration sensitive experiments. Figure 1 presents an estimate of the acceleration environment on the ISS at assembly complete, which exceeds the required acceleration levels for $\mu g$ science (from the ISS Microgravity Environment Specification\(^1\)) at virtually all frequencies. Mitigation of the excessive acceleration environment requires the implementation of vibration isolation systems at either the disturbance source or the science payload. While an effort is being made to limit the induced disturbances, the ubiquitous nature of disturbance sources necessitates the use of vibration isolation at the payload/rack locations.

By comparing the ISS acceleration requirement with the expected ISS acceleration environment, an isolation performance specification can be derived. True inertial isolation is neither desirable nor possible due to the large amplitude motion of the ISS from low frequency gravity gradient and aerodynamic torques. At these low frequencies, the isolated experiment must move with the ISS and the isolation system must directly transmit the very low frequency quasi-steady accelerations of the vehicle to the isolated assembly. Figure 1 implies that the isolation system must attenuate the ambient ISS accelerations by one order of magnitude at 0.1 Hz, which for a second order system implies a maximum break frequency of 0.01 Hz. This break frequency is constrained from below as well by the "rattle
Figure 1: Predicted RMS Acceleration Environment of the International Space Station in \( \mu g \)

space, an of relative motion between the isolated experiment and the moving ISS support structure. Hence the isolation system must pass through accelerations below 0.01 Hz and attenuate disturbances above 0.01 Hz. An isolation system that satisfies this attenuation function would reduce the ambient environment sufficiently to provide the required environment at the payload location.

Much like traditional flexible space structures, microgravity vibration isolation systems require high performance control of uncertain systems. Microgravity vibration isolation is a challenging structural control problem due to the precision of operation and the stringent performance requirements. The isolation system must sense and cancel accelerations with a magnitude of one-millionth the acceleration of gravity at the surface of the earth (1 \( \mu g \)) occurring over tens of seconds. This low frequency attenuation is typically accomplished with high gain acceleration feedback. Some payloads also require command tracking or cancellation of direct disturbance forces at frequencies of tens of Hz, thus necessitating high bandwidth control. Model uncertainties such as flexible utility umbilicals and payload structural dynamics add to the challenge by introducing the potential for interaction between control and payload isolation systems.

To date, most flight systems have used classical control methods.\(^2,3\) Limited applications of \( H_2 \) and \( H_\infty \) control methods to microgravity vibration isolation have been addressed in design studies. \( H_2 \) methods have been fruitfully applied for nominal performance\(^4-6\) while \( H_\infty \) methods have been used for robust stability to parametric uncertainties\(^6\) and high frequency unmodeled modes.\(^7\) However, previous multivariable designs did not meet performance requirements with acceptable stability properties, nor did they address implementation issues such as compensator dimension. This paper addresses mixed norm controllers that maximize robust stability while satisfying an \( H_2 \) nominal performance requirement.

\section{g-LIMIT System Description}

g-LIMIT (GLovebox Integrated Microgravity Isolation Technology) is a microgravity vibration isolation system designed and built by the NASA Marshall Space Flight Center to provide active vibration isolation for \( \mu g \) science payloads on the International Space Station.\(^8\) Shown in Figure 2, g-LIMIT is scheduled for launch on the ULF-1 mission and will commence characterization testing shortly thereafter.

\subsection{Hardware Description}

There are two main subsystems of g-LIMIT: the inertially isolated assembly to which an experiment is mounted and the base assembly which is rigidly attached to the ISS rack support structure. Control forces are applied to the isolated assembly using non-contact Lorentz force actuators. Inertial force-balanced proof mass accelerometers measure the inertial motion of the isolated platform and a unique patent-pending relative position sensing system measures the relative motion of the isolated
assembly with respect to the ISS-mounted base assembly. Three lock-down fasteners secure the isolator assembly to the base plate during transport, stowage, installation and idle times. Integrated into the base assembly and the isolated assembly is a snubber system, which provides mechanical rattle-space constraints with a maximum relative displacement of ten mm between the isolator assembly and the base assembly. As shown in Figure 2, the only mechanical connection between the isolated platform and the base assembly is the set of umbilicals that pass resources between the ISS and the experiment. Since the umbilicals are the load path for ISS vibration disturbances to the platform, the mechanical properties of these flexible umbilicals are the primary uncertainties in the dynamic model used for control design. g-LIMIT is approximately 13 inches high, 15 inches long at the base and 14 inches wide at the base.

2.2 Control System Architecture

In order to provide a quiescent acceleration environment to an experiment, an active isolation system must sense and cancel the acceleration of the experiment. g-LIMIT accomplishes inertial isolation using an acceleration feedback control system to sense the motion of the acceleration sensitive experiment and apply forces to reject the unwanted motion. An outer-loop (low frequency) position feedback controller is used to center the platform in the sway space while commanding the platform to follow the quasi-steady motion of the vehicle. By sensing relative position and absolute acceleration of the platform, the active control system forces the platform to follow the very-low-frequency motion of the base while attenuating the base motion at higher frequencies.

The inner loop/outer loop architecture for the g-LIMIT control system is chosen so that the position loop will issue acceleration commands to be tracked by the high gain acceleration loop. Because the accelerations are not always commanded to zero, the position loop actually disturbs the acceleration environment by forcing the platform to remain centered. However, the inner and outer loops are separated in the frequency domain such that the position control system only operates at very low frequencies. This architecture is used to reject acceleration sensor bias and umbilical force bias, both of which are not directly measurable but manifest a position error. Through integral action, the position control system rejects these biases and permits the acceleration control system to reject disturbances above 0.01 Hz. Because of the potential interaction between the position and acceleration control loops, the position loop must be considered in the analysis and design of the acceleration control loop. The baseline control approach for g-LIMIT uses PID controllers for both loops.

Another controller architecture permitted by g-LIMIT software is a fully centralized multiple-input/multiple-output (MIMO) controller that has as its input both acceleration and position measurements. The position sensor measurements are sampled at 25 Hz whereas the accelerations are sampled at 500 Hz, but this two-sample rate scenario can be adequately accommodated in a MIMO context by including the anti-aliasing filters on the relative position measurements. This is one means of frequency weighting that may be used to prevent interaction between the position and acceleration control in a fully centralized design.

However, the large dynamic range and large number of frequency weighting states required of a fully centralized design unnecessarily complicates the control design and could potentially lead to numerical issues. While modern control tools are well suited to frequency dependent designs of this nature, fully centralized control is unnecessary and undesirable in this context. In view of the simplicity and adequacy of a PID type position controller in the outer loop, this paper explores an architecture where a MIMO acceleration controller is designed for the inner loop while the baseline nominal PID position controller is implemented in the outer loop.

3 Control Design Approaches

Classical control methods are known to provide inherent robustness and simplicity of design and implementation. However, classical control methods are not well suited for multivariable (coupled) systems with parametric uncertainties. Such is the case with microgravity vibration isolation systems. Isolation systems cannot be tested in six degrees of freedom on the ground due to gravitational coupling, hence the dynamic properties of the system are often poorly known prior to on-orbit operation. In some cases, only a lower limit is known for the
structural frequencies of an experiment payload to be mounted to the isolation system. The combination of unknown payload characteristics and uncertain flexible umbilicals in the primary load path results in an uncertain multivariable dynamic system with stringent performance requirements.

3.1 Design for Nominal Performance

$H_2$ methods are often used when designing control systems to reduce the vibration response of a flexible structure. While $H_2$ design gives good nominal performance, the controllers are highly tuned to the design model and errors in the design model are not accounted for, typically inducing instability in the presence of slight parameter variations with high authority controllers. Controller order reduction is also not routinely possible because of the sensitivity of the control gains. As a result, the actual performance achievable is limited with $H_2$ designs. To achieve high levels of performance in the presence of uncertainties associated with umbilical and payload (isolated experiment) dynamics, robustness to model errors must be taken into account in the design process.

Another approach to design for nominal performance employs the $H_{\infty}$ norm, which can be interpreted as the gain of the system and is the worst-case amplification over all inputs $w(t)$ of unit energy. From a frequency domain perspective, the $H_{\infty}$ norm is defined as the maximum singular value of $T(s)$ over all frequencies, i.e.

$$\| T_{zw} \|_{\infty} = \sup_{\omega} \sigma(T_{zw}(j\omega))$$

where $z(t)$ represents the vector of performance variables. $H_{\infty}$ control design theory, based on Refs. 9 and 10, involves defining (possibly frequency dependent) weights on the inputs and outputs such that the performance objectives are satisfied by minimizing $\| T_{zw} \|_{\infty}$. Because the $H_{\infty}$ norm is defined with respect to the peak magnitude of the transfer matrix frequency response and the $H_2$ norm is defined by an integral square quantity (in time or frequency by Parseval’s Theorem), the respective closed loop systems typically have considerably different characteristics. With regard to mean-square performance requirements of microgravity vibration isolation, $H_2$ design is typically better suited for nominal performance than are $H_{\infty}$ designs. The significant benefit of $H_{\infty}$ theory however is that robustness to model errors is explicitly factored into the design process.

3.2 Design for Robust Stability

In addition to nominal performance, robust stability is an important design consideration. Robust stability requires the closed loop system to remain stable in the presence of bounded model errors. The uncertainty may be modeled in many forms such as multiplicative, inverse multiplicative, additive, parametric, etc and may be located at various points in the loop. Because previous research has shown that the g-LIMIT control system stability is most sensitive to variations in the umbilical stiffness, a parametric uncertainty model is used herein to account for variations in the stiffness and damping of one umbilical. Additional details on the uncertainty model may be found in Ref. 11.

By absorbing all of the scalings and weights into the plant $P$, the robust stability problem may be formulated as the Linear Fractional Transformation (LFT) shown in Fig. 3-a. The uncertainties are scaled so that $\Delta$ is the set of all stable perturbations such that $\| \Delta \|_{\infty} \leq \delta$. Assuming that $K(s)$ internally stabilizes the closed loop for $\Delta = 0$, then a sufficient condition for robust stability for all plants in the set formed by $\Delta \in \Delta_\delta$ is that

$$\| T_{zw}(K) \|_{\infty} \leq \frac{1}{\delta}$$

Thus like the nominal performance problem, robust stability is provided by minimizing the norm of a particular transfer function.

3.3 Design for Nominal Performance and Robust Stability

Whereas $H_{\infty}$ (and $\mu$-synthesis) methods provide both robust stability and robust performance in the presence of model errors, the performance is defined by an $\infty$-norm measure. A better approach from a mean square performance perspective is mixed $H_2/H_{\infty}$ optimization. The mixed $H_2/H_{\infty}$ design procedure has been developed to provide robust stability and nominal ($H_2$) performance by minimizing the $H_2$ norm for one set of inputs/outputs while satisfying an $\infty$-norm over-bound for another set of inputs/outputs. This mixed norm approach separates the optimization problem into two subproblems where the most appropriate norm is applied...
applied. For the inputs and outputs associated with performance, the $H_2$ norm is optimized while an upper limit on the $\infty$-norm is guaranteed for the inputs and outputs associated with the uncertainty model. With respect to Fig. 3-b, the objective is to satisfy

$$\min_K \| T_{2w2} \|_2$$

subject to

$$\| T_{z1w1} \|_\infty < \gamma$$

The foundational work in mixed $H_2/H_\infty$ design was done by Bernstein and Haddad who considered the case of one input and two outputs, with both full and fixed-order control. The first attempt at solving the general mixed $H_2/H_\infty$ problem was by Rotea and Khargonekar who allowed independent inputs and outputs for the two transfer functions and minimized the actual $H_2$ norm based on full state feedback. Ridgely extended the formulation to output feedback including the fixed order case with either regular or singular $H_\infty$ constraints and provided a numerical solution in Refs. 18 and 19. Another approach to the general mixed $H_2/H_\infty$ problem was developed by Sweriduk and Calise in Ref. 14, who used a differential games formulation to obtain fixed-order controllers. Ref. 15 presented a homotopy algorithm for the numerical solution of the necessary conditions of this formulation. The next section provides the mathematical framework for mixed $H_2/H_\infty$ control design based on the method in Refs. 14 -15. This approach is then applied to design mixed $H_2/H_\infty$ controllers for the g-LIMIT microgravity vibration isolation system.

4 Mixed $H_2/H_\infty$ Problem Formulation

The generalized plant of a standard control problem is given by

$$\dot{x} = Ax + B_p w_p + B_1 w + B_2 u$$

$$z_p = C_p x + D_{1p} u$$

$$z = C_1 x + D_{12} u$$

$$y = C_2 x + D_{2p} w_p + D_{21} w + D_{22} u$$

where $x \in R^n$ is the state vector, $w_p \in R^{nwp}$ and $z_p \in R^{nwp}$ are the inputs and outputs defining the $H_2$ subproblem, $w \in R^{nw}$ and $z \in R^{nz}$ are the inputs and outputs defining the $H_\infty$ subproblem, $u \in R^{nu}$ is the control vector, and $y \in R^{ny}$ is the measurement vector. Stabilizability, detectability, and standard rank conditions on the generalized plant are assumed.

To avoid the problem of overparametrization, a canonical form description for the controller can be used. It was shown in Ref. 20 that if either a controller or observer canonical form is imposed on the compensator structure, the number of parameters is reduced to its minimal number. The internal structure of the compensator is prespecified by assigning a set of feedback invariant indices $v_i$. In controller canonical form the compensator is defined as

$$\dot{x}_c = P^0 x_c + N^0 u_c - N^0 y$$

$$u_c = -P x_c$$

$$u = -H x_c$$

where $x_c \in R^{nc}$ and $u_c \in R^{nv}$. $P$ and $H$ are free-parameter matrices, and $P^0$ and $N^0$ are fixed matrices of zeros and ones determined by the choice of controllability indices $v_1$ as follows:

$$P^0 = \text{block diag}(P^0_1, \ldots, P^0_{nv})$$
\[ P_i^0 = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \ & \ddots & \vdots \\ \vdots & \ & \ & 1 \\ 0 & \cdots & \cdots & 0 \end{bmatrix}_{\nu_i \times \nu_i} \]  
\[ N^0 = \text{block diag}\{[0\ldots0\ldots0]_{\nu_i \times \nu_i}\} \]

where \( i = 1, \ldots, n_y \). The controllability indices must satisfy the following condition:

\[ \sum_i \nu_i = nc \quad i = 1, \ldots, n_y \]

which imposes the lower bound \( nc \geq n_y \) on the order of the compensator.

When the canonical realization of the compensator dynamics is employed, augmenting the compensator states with the plant states allows the system to be formulated as a static gain optimization problem. Let

\[ \bar{x} = \begin{bmatrix} x \\ x_c \end{bmatrix} \]

\[ \bar{u} = \begin{bmatrix} u \\ u_c \end{bmatrix} \]

The augmented system may be expressed as:

\[ \dot{\bar{x}} = \begin{bmatrix} A & 0 \\ -N^0C_2 & P^0 \end{bmatrix} \bar{x} + \begin{bmatrix} B_p \\ -N^0D_{22} \end{bmatrix} w_p + \begin{bmatrix} B_1 \\ -N^0D_{21} \end{bmatrix} w + \begin{bmatrix} B_2 \\ 0 \end{bmatrix} \bar{u} \\
\]

\[ = \bar{A}\bar{x} + \bar{B}_p w_p + \bar{B}_1 w + \bar{B}_2 \bar{u} \]

\[ z_p = \begin{bmatrix} C_p & 0 \end{bmatrix} \bar{x} + \begin{bmatrix} D_{1p} & 0 \end{bmatrix} \bar{u} \\
\]

\[ = \bar{C}_p \bar{x} + \bar{D}_{1p} \bar{u} \]

\[ z = \begin{bmatrix} C_1 & 0 \end{bmatrix} \bar{x} + \begin{bmatrix} D_{12} & 0 \end{bmatrix} \bar{u} \\
\]

\[ = \bar{C}_1 \bar{x} + \bar{D}_{12} \bar{u} \]

\[ \bar{y} = \begin{bmatrix} 0 & I \end{bmatrix} \bar{x} = \bar{C}_2 \bar{x} \]

\[ \bar{u} = -\begin{bmatrix} H \\ P \end{bmatrix} \bar{y} = -G\bar{y} \]

Eqs. 18-22 define a static gain output feedback problem where the compensator is represented by a minimal number of free parameters in the design matrix, \( G \).

Consequently, the closed loop system is given by

\[ \dot{\bar{x}} = \begin{bmatrix} \bar{A} - \bar{B}_2 \bar{G}_2 \bar{C}_2 & \bar{B}_p \bar{w}_p + \bar{B}_1 \bar{w} \end{bmatrix} \bar{x} + \begin{bmatrix} \bar{B}_p \bar{w}_p + \bar{B}_1 \bar{w} \end{bmatrix} \]

For the \( H_2 \) sub-problem, the objective is to minimize the \( H_2 \)-norm of the closed loop transfer function from disturbance inputs, \( w_p \) to performance outputs, \( z_p \)

\[ T_{zwp} = \bar{C}_p (sI - \bar{A})^{-1} \bar{B}_p \]

where the disturbances are confined to the set of signals with bounded power and fixed spectra. For the \( H_\infty \) sub-problem, the objective is to minimize the \( H_\infty \) norm of the transfer function from disturbance inputs \( w \) to performance outputs \( z \). The performance index for the mixed \( H_2/H_\infty \) problem is a weighted combination of the Lagrangian for the \( H_2 \) problem and the Lagrangian for the \( H_\infty \) problem and is given by:

\[ \mathcal{L} = \text{tr} \left\{ Q_\infty \bar{B} \bar{B}^T + (\bar{A}^T Q_\infty + Q_\infty \bar{A}) \left[ \bar{C}^T \bar{C} + \gamma^{-2} Q_\infty \bar{B} \bar{B}^T Q_\infty \right] + \lambda \bar{X} \bar{X}^T \bar{C}_p + (\bar{A}^T X + X \bar{A}^T) \right\} \]

The weight \( \lambda \) on the \( H_2 \)-norm allows a tradeoff between (\( H_2 \)) performance and the \( H_\infty \) norm. Matrix gradients are taken to obtain the first order necessary conditions:

\[ \frac{\partial \mathcal{L}}{\partial Q_\infty} = (\bar{A} + \gamma^{-2} \bar{B} \bar{B}^T Q_\infty) L + L(\bar{A} + \gamma^{-2} \bar{B} \bar{B}^T Q_\infty)^T + \bar{B} \bar{B}^T = 0 \]

\[ \frac{\partial \mathcal{L}}{\partial L} = \bar{X}^T L_p + L_p \bar{A} + \lambda \bar{C}_p \bar{C}_p = 0 \]

\[ \frac{\partial \mathcal{L}}{\partial X} = \bar{A}^T X + X \bar{A}^T + \bar{B}_p \bar{B}_p = 0 \]

\[ \frac{\partial \mathcal{L}}{\partial \bar{G}} = 2[d_{12}^T \bar{D}_{12}^T \bar{G}_2 \bar{C}_2 \bar{L}_2^T - d_{12}^T \bar{C}_1 \bar{L}_2^T] \]

A homotopy algorithm is presented in Ref. 15 which solves these necessary conditions and computes a family of mixed \( H_2/H_\infty \) compensators for varying weights \( \gamma \) and \( \lambda \). The homotopy algorithm for fixed-order mixed \( H_2/H_\infty \) design also generates
$H_2$, $H_2/H_\infty$, and $\mu$ controllers of fixed dimension as special cases. This homotopy algorithm is a two-parameter iterative scheme that effectively trades between robust stability and nominal performance by varying the over bound on the $\infty$-norm and the weight on the $H_2$ cost in the mixed norm cost functional. For a fixed $\gamma$ overbound, the weight on the $H_2$ cost, $\lambda$, is increased until either the $\infty$-norm constraint becomes an active, equality constraint or until the $H_2$ norm ceases to decrease. Solutions where the $\infty$-norm constraint is active (at which point the $H_2$ norm can no longer be reduced) are called boundary solutions, the set of which provides an explicit, Pareto optimal trade between nominal performance and robust stability. By incorporating the D-scales from $\mu$-synthesis into the $H_\infty$ subproblem, the structure of the uncertainty block may be accounted for, resulting in the fixed order mixed $H_2/\mu$ design procedure used in the following section.

5 Control Design Results

Several aspects of the g-LIMIT microgravity vibration control application make the fixed order mixed $H_2/H_\infty$ design procedure particularly apropos. The primary control objective for g-LIMIT is to satisfy the officially documented flight project performance requirement, which is shown below against the design results in Fig. 5. Robustness is an obvious design issue as well, one that must be addressed while meeting the performance requirement. The mixed $H_2/H_\infty$ design method provides just that: it maximizes stability robustness to bounded model errors for a given level of mean-square performance. Likewise, nominal performance can be maximized while guaranteeing a specified level of robust stability.

Controller state dimension is also a significant implementation issue for g-LIMIT. Standard $H_2$ and $H_\infty$ design approaches yield full-order compensators with the dimension of the generalized plant, including weighting function states and D-scale states for $\mu$-synthesis. Generalized plant dimension is especially significant in this case because of the controller architecture. Including the closed position loop in the generalized plant as shown in Figure 3, increases the dimension of the generalized plant and hence the dimension of the full order $H_2$ and $H_\infty$ controllers.

This is a good example of superfluous controller states arising from the methodology rather than the design process. With six PID position controllers in the inner loop, the full order MIMO acceleration controller will have 12 states due strictly to the architecture. The nominal plant dynamics has a minimum of 18 states (six position, six velocity, and six for acceleration output filters). Since the flight computer throughput limits the controller to 35 states, the remaining five states available for design weights is clearly insufficient if full order design is to be used. Stability and performance properties are not generally retained if the controller order must be reduced to a degree that can be implemented. That is especially true in this case because of the stringent performance requirements of microgravity vibration isolation. The fixed-order mixed norm method addresses this limitation of standard robust control by synthesizing MIMO acceleration controllers with a specified dimension that is an integer multiple of the number of measurements, e.g. 6, 12, 18, etc states.

By enforcing the order constraints in the fixed order mixed $H_2/H_\infty$ design procedure, robust stability can be maximized for a given level of mean-square performance with controllers that can be implemented in flight hardware. More than simply demonstrating the utility of this methodology, the microgravity vibration isolation application justifies the significance of this approach.

5.1 $H_2$ Designs

The generalized plant for control design is shown in Fig. 4. For $H_2$ design, the disturbance inputs include the three translational base accelerations, $wacc$; three directly applied forces, $wfd$; and six elements of the accelerometer noise vector, $nacc$. The performance outputs include weighted platform acceleration, $zacc$; weighted position control signal, $zpu$; and weighted acceleration control, $zu$. As a consequence of the control architecture described in the last section, the input to the control system is the acceleration error, which is the difference in platform acceleration measurement and the acceleration command generated by the baseline PID position controller. Six actuator forces comprise the control input vector. The g-LIMIT flight hardware includes analog anti-aliasing filters which are represented in this design as a second order filter with a 125 Hz break frequency on the acceleration mea-
measure and a single 10 Hz low pass filter on position measurements.

To assess nominal performance, a metric denoted “attenuation factor” is defined for each controller as the maximum value of the ratio of achieved performance to the required performance over the frequency range of interest:

\[
\text{Att. Factor} = \sup_{\omega_{\text{low}} < \omega < \omega_{\text{high}}} \left\{ \frac{\bar{\sigma}(T_p \hat{x}_b (j\omega))_{\text{dl}}}{\bar{\sigma}(T_p \hat{x}_b (j\omega))_{\text{req}}} \right\}
\]

where the “p” subscript indicates platform acceleration and the “b” subscript indicates the base (ISS) disturbance acceleration. Performance here is defined as the maximum singular value of the closed loop transfer function from ISS translational accelerations to the isolated platform translational acceleration. The frequency range of interest is not exactly arbitrary, but is chosen as the range from .01 Hz to 3 Hz (slight exceedances are allowed below 0.01 Hz and around 10 Hz, the latter due to limiting the controller state dimension). This range provides a consistent comparison across controllers designed with different methods. (Note that the results are slightly exaggerated for low authority controllers due to the resonant amplification at the break frequency.) Using this performance metric provides a scalar measure of nominal performance akin to the scalar \( p \) measure for robust stability analysis. Controllers with attenuation factors greater than one fail to meet the performance requirement while controllers with attenuation factors less than one exceed the requirement.

A set of \( H_2 \) control designs is designed for varying levels of performance and control authority by varying \( W_p \) in the acceleration performance weight

\[
W_{p, \text{accel}} = W_p \frac{1}{(0.0032s + 1)}
\]

which penalizes accelerations up to 50 Hz. Fig. 5 presents the nominal performance of the full-order, 54 state \( H_2 \) control designs at varying levels of attenuation performance. For each plot, the corresponding peak \( \mu \) measure is shown where robust stability is assessed with respect to 10% real parametric uncertainty in the umbilical stiffness. Nominal performance is defined by the attenuation of base accelerations on the isolated platform and is compared against the performance requirement shown in Fig. 5. Controller order reduction for these 54 state \( H_2 \) controllers was not possible: even slight reductions in controller order resulted in unstable controllers for each level of nominal performance.

![Figure 5: Nominal Attenuation Performance with \( H_2 \) Designs](image)

Robust stability substantially degrades as the attenuation performance improves with these designs. Fig. 5 indicates that the nominal performance requirement is met with a design that has a \( \mu \) measure of 12.76 (for 10% uncertainty), which indicates that a destabilizing perturbation in umbilical stiffness exists with a magnitude of less than 1%. Fig. 6 presents a surface of peak \( \mu \) values for each \( H_2 \) controller evaluated for real parameter variation in umbilical stiffness and damping varied up to 25%. Clearly demonstrated in Figs. 5 and 6 is the significant loss of robust stability as a function of uncertainty for a given \( H_2 \) controller and in general as a function of attenuation performance for the \( H_2 \) methodology. In general, only the lowest performing controllers (attenuation factors greater than 4) satisfy the robust stability test for the very low levels of parameter variation (5% or less).

### 5.2 Fixed-Order Mixed \( H_2/H_\infty \) Designs

Next, a set of 12th order mixed \( H_2/H_\infty \) controllers are synthesized for comparison to the 54 state full-order \( H_2 \) controllers. For this design, the \( H_2 \) optimization problem from the last section is augmented with an \( H_\infty \) sub-problem based on the umbilical stiffness and damping parametric uncertainty model. The two sub-problems of the mixed norm
design are defined by the inputs and outputs

\[ w_2 = \begin{bmatrix} \text{nacc} \\ \text{wacc} \\ \text{wfd} \end{bmatrix}, \quad z_2 = \begin{bmatrix} \text{zacc} \\ \text{zu} \\ \text{zpu} \end{bmatrix} \]

\[ w_\infty = \begin{bmatrix} \text{wunc} \end{bmatrix}, \quad z_\infty = \begin{bmatrix} \text{zunc} \end{bmatrix} \]

where the uncertain inputs and outputs, \( w_{\text{unc}} \) and \( z_{\text{unc}} \), are associated with the uncertain umbilical stiffness and damping. The mixed norm controllers are obtained by varying the \( H_2 \) performance weight in the same manner as the full order \( H_2 \) designs. The same weights were used for the \( H_2 \) sub-problem as for the full order \( H_2 \) designs with the exception of the performance weight range. Comparing the 54 state \( H_2 \) controller to the 12th order mixed \( H_2/H_\infty \) design for the same \( H_2 \) weights, the attenuation factor degrades from .95 for the \( H_2 \) design to 1.3311 for the mixed \( H_2/H_\infty \) design while the peak \( \mu \) measure improves from 12.76 to 0.8952. Whereas the performance only slightly decreases with the addition of robust stability and controller order constraints in the optimization process, the robust stability increases substantially. Hence slightly higher performance weights were needed to achieve the same level of nominal attenuation performance with the mixed norm designs. This loss of performance for a given set of weights is a direct result of the added constraints of the \( H_\infty \) cost and possibly the fixed controller order.

Attenuation performance for the full set of performance weights with the mixed norm design is similar to the nominal performance for the strict \( H_2 \) controllers shown in Fig. 5. However, the peak \( \mu \) values are much less through the full range of performance with the mixed \( H_2/H_\infty \) controllers than with the full order \( H_2 \) controllers for a given level of attenuation performance. This significant benefit of the mixed \( H_2/H_\infty \) method in comparison to \( H_2 \) design is revealed in Fig. 7. The improvement in robust stability (assessed for 10\% real
In addition to the real parameter uncertainty, a mixed $H_2/H_\infty$ controller was designed for additive uncertainty as well. In this case, an additive uncertainty model given by

$$W_{add} = 0.001 \frac{(1.2566s + 1)}{(0.0251s + 1)}$$

was included to provide robustness to unmodeled modes above 35 Hz (representing potential payload dynamics). The performance and stability of this controller is indicated on Fig. 7 along with the set of $H_2$ controllers evaluated for this parametric and additive uncertainty model.

This type analysis is important to determine the range of acceptable controllers in terms of nominal performance and robust stability. Figure 11 shows the region of acceptable controllers for a specified level of uncertainty (10% in this case). The vertical dashed line is the boundary of acceptable controllers in terms of stability robustness while the horizontal dashed line delineates the acceptable performance region. Controllers above the horizontal line fail the stability requirement while controllers to the right of the vertical line fail the attenuation performance criteria. Note however that even though the mixed $H_2/H_\infty$ designs are far superior to the $H_2$ designs, the nominal performance objective is not satisfied for any controller with a $\mu$ measure less than one. The performance requirement cannot be satisfied with guaranteed stability for 10% real parameter uncertainty with this design set.

6 Conclusions

Previous approaches to microgravity vibration isolation system designs have utilized strictly classical methods with the exception of a few studies using $H_2$ and $H_\infty$ methods. In this paper, $H_2$ and mixed $H_2/H_\infty$ methods have been shown to result in comparable levels of nominal performance, albeit at significantly different levels of robustness. An uncertainty model for microgravity vibration isolation controller design with respect to parametric uncertainties in umbilical stiffness was used for robust stability analysis and robust control design.

This paper presents a control design approach that is well suited to the stringent performance and stability requirements of microgravity vibration isolation. Standard full order $H_2$ and $H_\infty$ designs do
not simultaneously provide good mean-square performance and robust stability guarantees with controllers of low enough order to be implemented in flight hardware. It is important to note that in this application, the low compensator order was necessary due to implementation constraints. Incorporating D-scales resulted in a fixed order mixed $H_2/\mu$ design approach for which controllers are synthesized and analyzed to determine the maximum robust stability attainable subject to a nominal performance requirement. Using the fixed order mixed $H_2/H_\infty$ design approach resulted in controllers of low order that achieved much improved levels of robust stability for a given level of performance when compared to the standard $H_2$ designs.

7 References


17. Rotea, M.A. and Khargonekar, P.P., “$H_2$ Optimal Control with an $H_\infty$ Constraint: The


