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***Project Title:*** Point-to-Point Control of the Hydrogen Mixer

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# Point-to-Point Control of the Hydrogen Mixer

Final Report

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### Abstract

We continue to build on the theoretical modelling of a liquid hydrogen (LH2) and gas hydrogen (GH2) mixer subsystem. The mixer described in this work is responsible for combining high pressure LH2 and GH2 to produce a hydrogen flow that meets certain thermodynamic properties. The desired properties are maintained by precise control of the LH2 and GH2 flows. The mixer is modelled as a general multi-flow lumped volume for single constituent fluids using density and internal energy as states. The set of nonlinear differential equations is modelled in the SIMULINK environment including a table look-up feature of the fluid thermodynamic properties. A small-signal (linear) model is developed based on the nonlinear model and simulated as well. Pulse disturbances are introduced to the valve positions and the quality of the linear model is ascertained by comparing its behavior against the nonlinear model simulations. Valve control strategies that simulate an operator-in-the-loop scenario are then explored demonstrating the need for automatic feedback control. Finally, optimal single-output and multi-output Proportional/Integral controllers are designed based on the linear model and applied to the nonlinear model with excellent results to track simultaneous, constant setpoint changes in desired exit flow, exit temperature, and mixer pressure, as well as to reject unmeasurable but bounded additive step perturbations in the valve positions.


# 1 Introduction

The Test and Engineering Directorate at NASA John C. Stennis Space Center (SSC) continues its efforts to assemble a software simulation package that captures the static and dynamic characteristics of modern and future thermodynamic systems. The package is foreseen to fulfill the need for an accurate and verifiable thermodynamic system simulation environment [3, 4, 5, 6].

In this work, we build mainly upon a previous report [6] and focus on the point-to-point tracking control problem for the $LH_2$ and $GH_2$ mixer subsystem and associated flow controllers. The relevant components of the mixer subsystem are shown in the flow schematic of Figure 1. The $LH_2$ valve controls the flow of high pressure liquid $H_2$ from a pressurized tank; the $GH_2$ valve controls the flow of gaseous $H_2$ from a set of high pressure bottles; and the output valve controls the flow of $H_2$ into the test article. The primary objective of the mixer and control valves is to regulate the thermodynamic properties of the out flow of $H_2$ for optimal performance in spite of the steady depletion of the source bottles, measurement errors, modelling imperfections, and other uncertainties and perturbations.

# 2 Background

- Some application examples of the use of a related software package called EASY/ROCETS are presented in [3] including FORTRAN listings of several modules and a brief user's manual.
- The report [4] performs a detailed derivation of the model of the RUN-TANK module as well as its implementation within EASY/ROCETS. The low pressure and high pressure LOX

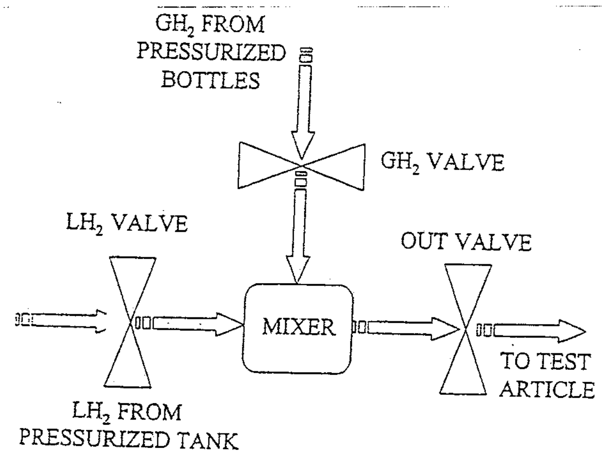


Figure 1: Diagram of HPH2 Mixer and Control Valves.

and hydrogen subsystems are simulated with EASY/ROCETS and compared against previous computer simulations with good results. In addition, various control strategies were simulated for the high pressure hydrogen subsystem. The objective being the temperature regulation of the output stream. *At present however, control designs are done in an ad-hoc manner due to the lack of a suitable control-oriented mathematical model of the mixer.*

- In [5], several upgrades and enhancements to EASY/ROCETS were reported including the NIST-12 thermodynamic database.
- Most recently, Victor Marrero, one of the SSC 2001 Summer Undergraduate students, reported his work on validating the EASY/ROCETS models of several components of the high-pressure hydrogen system with real test data. The results are very encouraging and prompts for continued analytical studies such as the one presented in this report.
- In [6], we developed an analytical model of the mixer and control valves suitable for dynamic analysis and control design in the vicinity of a thermodynamic equilibrium point. An optimal linear-quadratic regulator was designed and simulated using Matlab to illustrate the effectiveness of such a controller in compensating against perturbations that result in small deviations around the desired equilibrium. A simulation was included that indicates how one could study which valve or combination of valves is more effective in achieving a desired performance. In essence, this is a study on control authority or relegation.

## 2.1 Results

- Based on the work by H. Richter [7], the nonlinear mixer equations were simulated in the SIMULINK environment.

- As discussed in [6], there was a need to describe the gas valve using a compressible flow model. This is now incorporated in this report and the corresponding changes in the small-signal model were derived. The details are in Appendix A.
- Also based on the work by H. Richter [7], the NIST-12 property data is now incorporated in the model. The program MIPROPS was used to generate the necessary tables which are then continuously accessed via Matlab functions.
- My student, Ms. Jamie Granger, as part of her Summer LAMP program with Tulane University, verified the accuracy of the data in the table and also combined two routines into one.
- The small-signal model developed in [6] requires the numerical computation of several partial derivative terms which in turn depend on the equilibrium point. Since there are no analytical expressions that would allow the direct calculation of some of these partial derivatives, a Matlab routine was written to compute these terms.
- Open-loop control strategies were investigated mainly to test that the model does provide reasonable results. In the future, we hope to validate the model when real test data becomes available.
- Closed-loop control strategies were investigated including: 1) an optimal multi-input/single-output (MISO) PI controller to track the exit flow; and 2) an optimal multi-input/multi-output (MIMO) PI controller to track the exit flow, the mixer temperature, and the exit temperature simultaneously and in the presence of step perturbations in the valve positions.

# 3 Updated Mixer and Valve Models

The mixer has been modelled as a general multi-flow lumped volume for single constituent fluids using density and internal energy as states [3, 6]. The model was independently derived in [7]. In this report, we refer to the mixer's internal energy and density as the "state", not to be confused with the definition of state of a system as described by its thermodynamic properties. Letting $z_1(t)$, $z_2(t)$ denote the mixer's specific internal energy and density, respectively, the two dynamic equations are given by [2]

$$\dot{z}_1 = \frac{1}{Vz_2}\left[E_{net} - W_{net}z_1 + \dot{Q}\right] \tag{1}$$

$$\dot{z}_2 = \frac{1}{V}W_{net} \tag{2}$$

where $E_{net}$ is the net (input minus output) energy into the mixer, $W_{net}$ is the net (input minus output) flow, and $\dot{Q}$ is the total heat transfer rate. The net flow is simply

$$W_{net} = w_g + w_l - w_e$$

[2] In [6] the mixer states were its internal energy and its mass ($Vz_2$).

where the subscripts stand for "gas", "liquid", and "exit", respectively. In this study, only positive flows are considered, that is, the gas and liquid flows are into the mixer and the output flow is out of the mixer. In essence, normal operation requires that each valve always experience a positive pressure difference across it thus disallowing flow reversal. The net input energy term is then given by

$$E_{net} = h_g w_g + h_l w_l - h_v w_e$$

where $h_g$, $h_l$, $h_v$ denote the enthalpy of the $GH_2$ stream, $LH_2$ stream, and volume block (mixer), respectively.

The valve block calculates the *incompressible* fluid flow value $w_f$ given the inlet $P_i$ and outlet $P_o$ pressures, the density of the inlet stream $\rho_i$, and the valve flow coefficient $C_{vf}$, as follows:

$$w_f = \sqrt{\rho_i\, \rho_w (P_i - P_o)}\, C_{vf} = \alpha_f C_{vf} \tag{3}$$

where $\rho_w$ is the density of water (62.4 $lbm/ft^3$), and we have assumed that $P_i > P_o$. As pointed out in [6] the model of the gas valve had to be modified to simulate *compressible* flows. According to [7], the following expression models the compressible mass flow through a valve:

$$w_f = \begin{cases} 2.857 \times 10^{-2} \frac{\sqrt{T_i}\rho_i}{P_i} \sqrt{P_i^2 - P_o^2}\, C_{vf}, & P_o < P_i < 2P_o; \\ 2.423 \times 10^{-2} \sqrt{T_i}\rho_i C_{vf}, & P_i \geq 2P_o. \end{cases} \tag{4}$$

when the mass flow is in $lbm/s$, the pressure in $psi$, the density in $lbm/ft^3$, and the temperature $T$ in °$R$. By design and under normal operation, the properties of the inlet hydrogen gas and liquid are such that $P_l,\ P_g > P_o = P_v$ and also $P_v > P_e$; thus, the possibility of reverse flow is not considered in this study but can certainly be added if so required.

## 3.1 Small-Signal Model

Substituting the expressions for the valve flows and the net internal energy and net flow into equations (1)-(2), and treating the valve coefficients as inputs to be regulated, the dynamic model of the mixer is a 2-state, 3-input system of nonlinear differential equations of the form

$$\dot{z}_1 = F_1(z_1,\ z_2,\ C_{vg},\ C_{vl},\ C_{ve}) \tag{5}$$

$$\dot{z}_2 = F_2(z_1,\ z_2,\ C_{vg},\ C_{vl},\ C_{ve}) \tag{6}$$

where $F_1(\cdot)$ and $F_2(\cdot)$ are nonlinear functions of the state and valve coefficients.

For the remainder of the report, we will denote constant or equilibrium values of any variable by an upper bar $(\bar{\cdot})$. Given constant values of valve flow coefficients $\bar{C}_v = [\bar{C}_{vg}\ \bar{C}_{vl}\ \bar{C}_{ve}]^T$ (superscript $T$ denotes transposition) and constant fluid properties, the state of the model

$$z(t) = \begin{bmatrix} z_1: & \text{Internal Energy} \\ z_2: & \text{Density} \end{bmatrix}$$

reaches a constant equilibrium point $\bar{z}$. Next, consider perturbing such an equilibrium by small signals $x(t)$ and $u(t)$ so that

$$z(t) = \bar{z} + x(t) \qquad \text{and} \qquad C_v(t) = \bar{C}_v + u(t)$$

where $u(t)$ denotes a small, valve-coefficient correction/regulation signal. Then, a standard linearization of equations (5)-(6) results in the small signal model

$$\dot{x} = Ax + Bu \tag{7}$$

where $x(t)$ is the small perturbation state vector, $u(t)$ is the small perturbation control signal, and the two-by-two matrix $A$ and two-by-three matrix $B$ are given by

$$A = \begin{bmatrix} \frac{\partial F_1}{\partial z_1} & \frac{\partial F_1}{\partial z_2} \\ \frac{\partial F_2}{\partial z_1} & \frac{\partial F_2}{\partial z_2} \end{bmatrix}; \qquad B = \begin{bmatrix} \frac{\partial F_1}{\partial C_{vg}} & \frac{\partial F_1}{\partial C_{vl}} & \frac{\partial F_1}{\partial C_{ve}} \\ \frac{\partial F_2}{\partial C_{vg}} & \frac{\partial F_2}{\partial C_{vl}} & \frac{\partial F_2}{\partial C_{ve}} \end{bmatrix}$$

where the partial derivatives are evaluated at the equilibrium state $\bar{z}$ and constant valve flow coefficient vector $\bar{C}_v$.

Typically, in the control literature an output equation of the form

$$y = Cx + Du$$

where matrices $C$ and $D$ are appropriately dimensioned is appended to the model (7) to account for the measurement of certain variables such as temperature, pressure, or flow. An output of interest is the exit flow $w_e$. Using equation (3) and linearizing around a chosen equilibrium point, we obtain the linear approximation

$$w_{elin} = \left[\frac{\partial \alpha_e}{\partial z_1} \quad \frac{\partial \alpha_e}{\partial z_2}\right]_{eq} x + [\alpha_e]_{eq}\, u_{ve} = C_{vflow}x + D_{flow}u_{ve}$$

A second output of interest is the mixer pressure $P_v$. Taking this pressure to be a function of the mixer internal states, that is,

$$P_v = f_P(z_1(t), z_2(t))$$

then, the linearization of this function around the chosen equilibrium gives

$$P_{vlin} = \left[\frac{\partial P_v}{\partial z_1} \quad \frac{\partial P_v}{\partial z_2}\right]_{eq} x = C_{pv}x$$

A third output of interest is the exit temperature $T_e$. Taking this temperature to be a function of the mixer internal states, that is,

$$T_e = f_T(z_1(t), z_2(t))$$

then, the linearization of this function around the chosen equilibrium gives

$$T_{elin} = \left[\frac{\partial T_e}{\partial z_1} \quad \frac{\partial T_e}{\partial z_2}\right]_{eq} x = C_{te}x$$

The indicated partial derivative terms are calculated numerically using Matlab routines.

It is not reasonable to assume that the mixer states be measurable. However, the *state postulate* [3] implies that the state $z(t) = \bar{z} + x$ is directly available for control design if at least two independent thermodynamic properties were measured. Therefore,

- assume that real-time measurements of temperature $T_e$ and pressure $P_e$ are available at the exit point. Then. using the thermodynamic table, one finds the exit flow enthalpy $h_e$.
- Assuming a lossless valve, then enthalpy across the valve is preserved so that $h_v = h_e$, where $h_v$ is the mixer (volume) enthalpy.
- Using $h_v$ and $T_v$ (measured) and the table, one finds the mixer pressure $P_v$.
- From $h_v$ and $P_v$ and the table, one finds the mixer states (energy, density). Or, using the measured exit flow $w_e$, the known exit pressure $P_e$, the computed $P_v$, and the exit valve coefficient, one solves Equation (3) for the density $\rho_v$.

The above steps were verified using the program MIPROPS. In summary then, it is reasonable to assume that

$$y = x = z - \bar{z} \tag{8}$$

is available or that it can be deduced from other real-time measurements. This may be called a "static observer". An alternative is to design a so called *dynamic observer* to provide a real-time, optimal estimate of the states. This is beyond the scope of this report.

The calculation of the equilibrium points and the elements of the matrices $A$ and $B$ was done in [6]. The relevant results are compiled in Appendix A of this report. The only difference here is that the model of the flow in the gas valve is for compressible fluids.

The evaluation of $\frac{\partial P_v}{\partial z}$ is accomplished numerically via a Matlab routine. The evaluation of $\frac{\partial h_v}{\partial z}$ on the other hand, is done by differentiating the expression

$$h = E + C_{units}\frac{P}{\rho}$$

where $C_{units} = 1$ in the $SI$ units or $C_{units} = 0.1852$ when $h$, $E$ are in $BTU/lbm$, $P$ in $psi$, and $\rho$ in $lbm/ft^3$. This results in

$$\frac{\partial h_v}{\partial z_1} = 1 + \frac{C_{units}}{z_2}\frac{\partial P_v}{\partial z_1}$$

$$\frac{\partial h_v}{\partial z_2} = C_{units}\left[\frac{1}{z_2}\frac{\partial P_v}{\partial z_2} - \frac{P_v}{z_2^2}\right]$$

The relations between pressure $P$ and internal energy or density are essentially linear in the vicinity of an equilibrium point. This verifies the work reported in [6]. Therefore, now we have the capability to compute the small-signal model at any desired equilibrium point.

[3]For a simple compressible substance, any thermodynamic property is at most a function of two other independent thermodynamic properties.

## 3.2 Purpose of the Small-Signal Model

The model (7) describes the dynamics of the mixer model in the vicinity of the equilibrium under consideration. Ideally, both $x(t)$ and $u(t)$ are zero; in practice, the state $z(t)$ deviates from the desired $\bar{z}$ thus necessitating a corrective action by $u(t)$. The model is suitable for a variety of analysis studies such as stability, controllability, and regulation/tracking of the equilibrium. The stability study is relevant since it reveals an important characteristic of the equilibrium $\bar{z}$ under no control, that is, with $u(t) = 0$. Appropriate action must be taken by the control $u(t)$ in the event that $\bar{z}$ be unstable (worst case) or even "poorly behaved". The notion of controllability reveals whether a suitable control $u(t)$ exists such that the perturbation state $x(t)$ may be steered to zero. Moreover, it is possible to study which valve or combination of valves is most effective [6]. Finally, the equilibrium regulation/tracking study refers to the actual control design to regulate $x(t)$ to zero or to track a desired equilibrium trajectory as a function of time.

One strategy that is used to control systems whose linear model changes from one equilibrium point to the next is known as *gain scheduling*. Basically, one designs a controller for each small-signal model in the vicinity of each desired equilibrium. An outer-loop controller or scheduler performs the decision as to when to switch from one controller to the next.

## 3.3 A Numerical Example

It is to be noted that the thermodynamic tables used by the Matlab routines were generated with the program *MIPROPS* and therefore contain an inherent error because of the resolution used. For example, the step size in the pressure intervals is 100 *psi*. However, linear interpolation is used to calculate values not found in the tables which reduces the error. Table 1 lists the data for a typical equilibrium point $EQ_i$ corresponding to a given set of outflow requirements. A Matlab program [7] calculates the required valve positions. Later in this report we will simulate the model when the objective of a controller is to steer the system from $EQ_i$ to $EQ_f$ in a smooth manner.

The next step is to obtain the linearized model at a chosen equilibrium point. Consider the linear model (7) valid in the neighborhood of the equilibrium $EQ_i$ listed in Table 1. The matrices $A$ and $B$ are evaluated to be

$$A = \begin{bmatrix} -17.52 & -3393.08 \\ -0.1597 & -43.45 \end{bmatrix}; \qquad B = \begin{bmatrix} 25.11 & 332.74 & -42.01 \\ 0.651 & 0.6612 & -0.4919 \end{bmatrix}$$

For an output $y = w_e$, the linear approximation is

$$w_{elin} = C_{vflow}x + D_{flow}u_{ve} = [0.2386 \quad 66.84]\,x + [1.23]\,u_{ve}$$

In the figures that follow, we present the behavior of the mixer obtained from the *nonlinear* model. In the absence of real data, we shall refer to the nonlinear SIMULINK simulation as the *real system*. Several simulations were performed to evaluate the linear model. However, one has to keep in mind that the linear model is only valid in the vicinity of the point where the linearization was performed. In other words, the linear model's approximation gets worse as the state deviates from the linearization point, and it is actually an invalid representation of the true behavior. The linear model will also be used later in the report to design linear controllers.

- Disturbance Effect - No Control. We ran the model under a 20 percent disturbance in the gas and liquid valves that take the state away from the initial equilibrium point. When the disturbance disappears, the mixer returns to the equilibrium. It was observed that the linear model captures the important dynamic characteristics of the system very accurately.
- Tracking and Disturbance Rejection - Open Loop Control. The behavior of the mixer model was evaluated as the valves are steered in an *open-loop* fashion from the initial to the final positions. These were generated to simulate the manner in which an operator might open/close the valves in response to a visual monitoring of pressure, temperature, or flow sensors. The simulations included a 20 percent perturbations on the liquid and gas valve positions. This simulation run gave us a clear reminder that the linear model is only good in the vicinity of the linearization point.

# 4 Point-to-Point Tracking Control Design

As discussed in [6], the mixer-valve system is a multi-input, multi-output system thus having a transfer function for each input-output pair. It is in general difficult to design PID-style controllers for each loop because of the interactions between channels. The approach chosen in this work is based on the state-space model (7) rather than on the individual transfer functions.

## 4.1 Single Output Tracking

Consider the problem of exit flow tracking and define a tracking error

$$e(t) \;=\; w_{er} \;-\; w_e(t) \tag{9}$$

where the reference or desired *constant* exit flow is $w_{er}$. We seek to design a controller that steers the error signal signal to zero, that is, $e(t) \to 0$ as $t \to \infty$. To that end, define an augmented state vector

$$X = \begin{bmatrix} \dot{x} \\ e \end{bmatrix} \tag{10}$$

that satisfies the differential equation

$$\dot{X} \;=\; \begin{bmatrix} A & 0 \\ -C_{flow} & 0 \end{bmatrix} X \;+\; \begin{bmatrix} & B & \\ 0 & 0 & -D_{flow} \end{bmatrix} \dot{u} \;=\; \bar{A}X \;+\; \bar{B}\dot{u} \tag{11}$$

Given the system (11), a performance criterion of the form

$$J \;=\; \int_0^{\infty} \Big( X^T(t)QX(t) \;+\; \dot{u}^T(t)R\dot{u}(t) \Big)\, dt$$

is minimized. For technical reasons, the matrix $Q$ must be positive-semidefinite and penalizes how far the state $X(t)$ deviates from the origin. Similarly, the matrix $R$ must be positive-definite and penalizes how much control effort is used. The resulting optimal controller is given by the linear full-state feedback law

$$\dot{u}(t) = -KX(t) \tag{12}$$

where the matrix $K$ is *constant* and is found *a-priori* by solving the steady-state Riccati Equation

$$\bar{A}^T P + P\bar{A} - P\bar{B}R^{-1}\bar{B}^T P + Q = 0 \qquad \Longrightarrow K = R^{-1}\bar{B}^T P$$

The closed-loop system

$$\dot{X} = [\bar{A} - \bar{B}K]\,X$$

is guaranteed to be asymptotically stable, that is, $X(t) \to 0,\ t \to \infty$. Note that driving $X$ to zero is equivalent to driving the error $e$ to zero as required, and the mixer state $x$ to a constant. Finally, integrating both sides of (12) leads to the *optimal* multi-input/single-output proportional-integral (MISO-PI) controller expression

$$u(t) = -K_P x(t) - K_I \int_0^t [w_{er} - w_e(\tau)]\, d\tau \tag{13}$$

For example, selecting the following weighting matrices:

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 20 \end{bmatrix}; \qquad R = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$$

so that it is equally important to steer $x_1$ and $x_2$ to zero, but it is 20 times more important to drive the exit flow error to zero. Similarly, the liquid and exit valves have 10 times more authority than the gas valve in achieving the stated goal. The optimal gain matrix is found to be

$$K = \begin{bmatrix} 0.4679 & 22.5556 & -12.1274 \\ 0.8992 & -9.3271 & -0.2184 \\ -0.8653 & 13.2205 & -7.2421 \end{bmatrix}$$

from which

$$K_P = \begin{bmatrix} 0.4679 & 22.5556 \\ 0.8992 & -9.3271 \\ -0.8653 & 13.2205 \end{bmatrix}; \qquad K_I = \begin{bmatrix} -12.1274 \\ -0.2184 \\ -7.2421 \end{bmatrix}$$

The final equilibrium point was selected to be that corresponding to a 20 percent increase in exit flow, from $w_e = 37\ lbm/s$ to $w_e = 44.4\ lbm/s$. The resulting valve, mixer states, flows, and linear model performance was evaluated by simulations. There are (small) step changes that appear in the temperature and pressure plots caused by the resolution of the thermodynamic table and not by the controller. The PI controller successfully takes the exit flow to its final required value.

Simulation results under the same PI controller and with the added perturbations on the liquid and gas valves show that the controller recovers from these perturbations. This behavior could not be achieved under open-loop schemes. However, the mixer states, and the valves do not go to their unique, pre-calculated final values as would be given by a steady-state equilibrium calculation. This is because the controller is designed to track the exit flow step reference only. This is corrected in the next section.

## 4.2 Multi Output Tracking

Next, we design an optimal multi-input/multi-output PI (MIMO-PI) controller that simultaneously regulates the exit flow, the mixer pressure, and the exit temperature to arbitrary constant set points $w_{er}$, $P_{vr}$, $T_{er}$, respectively. The new augmented vector is

$$X = \begin{bmatrix} \dot{x} \\ e \end{bmatrix} \doteq \begin{bmatrix} \dot{x} \\ w_{er} - w_e(t) \\ P_{vr} - P_v(t) \\ T_{er} - T_e(t) \end{bmatrix}$$

that satisfies the differential equation

$$\dot{X} = \begin{bmatrix} A & 0 & 0 & 0 \\ -C_{flow} & 0 & 0 & 0 \\ -C_{pv} & 0 & 0 & 0 \\ -C_{te} & 0 & 0 & 0 \end{bmatrix} X + \begin{bmatrix} & B' & \\ 0 & 0 & -D_{flow} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \dot{u} = \bar{A}X + \bar{B}\dot{u} \tag{14}$$

Minimizing the quadratic cost introduced in the previous section leads to the linear $MIMO-PI$ controller

$$u(t) = -K_P x(t) - K_I \int_0^t \begin{bmatrix} w_{er} - w_e(\tau) \\ P_{vr} - P_v(\tau) \\ T_{er} - T_e(\tau) \end{bmatrix} d\tau \tag{15}$$

As a simulation example, select the following weighting matrices:

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 20 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 200 \end{bmatrix}; \qquad R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

so that it is equally important to steer $x_1$, $x_2$, and the mixer pressure error to zero, it is 20 times more important to drive the exit flow error to zero, and it is 200 times more important to drive the exit temperature error to zero. Several simulations indicated that the temperature tracking is the slowest of the three and hence the more stringent requirement. This time, all valves have equal authority in achieving the stated goals. The resulting optimal gain matrix is found to be

$$K = \begin{bmatrix} -0.0719 & 46.93 & -1.8728 & -0.8897 & 2.5733 \\ 0.9454 & -9.8472 & -0.384 & -0.1607 & -13.91 \\ -0.0439 & -11.455 & -4.0429 & 0.4274 & 0.1296 \end{bmatrix}$$

The set point requirements are a 20 percent increase in exit flow from 37 to 44.4 $lbm/s$; a 100 $psi$ increase in mixer pressure from 6000 $psi$; and a 2 $°F$ increase in exit temperature from $-240$ $°F$. These are introduced as step commands at time $t = 0.3$ $sec$. The valve disturbances are set on and a rate limiter with a slew rate at 50 is put in series with each valve to include another level of realism in the simulations. The simulation plots are in Figure 2 through 5. The effect of the rate limiter is an increase in overshoots and reduced overall performance quality while precluding unrealistically fast valve commands rises especially at the start of the command. The outputs are controlled as required. It is expected that real valves will not only exhibit rate limiting but other linear and nonlinear characteristics. These will be described in future reports.

# 5 Conclusions and Further Research

An analytical small-signal model of the high-pressure $H_2$ mixer and control valves has been derived. The model is suitable for a variety of analysis and control designs valid in the vicinity of an operating or equilibrium point. A classical approach to control design would involve the consideration of a total of nine single-input, single-output transfer functions. Unless the control loops are weakly coupled, such a design is difficult to complete because of the interaction among the control channels. Instead, a state-variable control design has been pursued that results in an optimal multi-input/multi-output proportional/integral law that can be implemented in real-time. As illustrated in the report, it is straightforward to design the controller and it is possible to assign different authorities to each control valve. Precise and simultaneous tracking of constant setpoint changes in exit temperature, exit flow, and mixer pressure were achieved in the presence of constant and unmeasurable additive valve position perturbations.

Topics for further research include:

- Introduce a valve model that exhibits valve dynamics, and other nonlinearities such as friction and backlash. The linear model has to be recomputed because the valve introduces additional states.
- Investigate the combined tank/mixer control problem. This is a six-state and four control-valve problem. Clearly, the quality of the out-flow properties is affected by the precise regulation of both the tank and mixer internal dynamic states.
- Use heuristics-based set-point relegation to determine control valve authority.
- *Last but definitely not least, we must validate these findings with real data.*

| | $\bar{P}$ ($psi$) | $\bar{T}$ ($F$) | $\bar{\rho}$ ($lbm/ft^3$) | $\bar{h}$ ($BTU/lbm$) | $C_{vopen}$ | $\%OPEN$ | $\bar{w}$ $lbm/s$ | $\bar{z}_1$ | $\bar{z}_2$ |
|---|---|---|---|---|---|---|---|---|---|
| $GH_2$ IN | 13500 | 90 | 2.91 | 2113.6 | 230 | 0.96 | - | - | - |
| $GH_2$ OUT | 6804 | 130.4 | 1.702 | 2113.6 | - | - | 3.72 | - | - |
| $LH_2$ IN | 8500 | -340 | 5.042 | 329 | 115 | 17.74 | - | - | - |
| $LH_2$ OUT | 6804 | -326 | 4.519 | 329 | - | - | 33.21 | - | - |
| Mixer | 6804 | -273.32 | 3.84 | 508 | - | - | - | 181.39 | 3.84 |
| Outflow | 5533 | -266.21 | 3.404 | 508 | 270 | 11.12 | 36.93 | - | - |

Table 1: Initial Equilibrium Data. Underlined Values are Computed. (IN=Into Valve; OUT=Out of Valve). $z_1$ in $BTU/lbm$: $z_2$ in $lbm/ft^3$.

# References


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[3] R. P. Taylor and R. F. Follett, "Technical Support for Dynamic Fluid Flow Simulation Capabilities for Ground Test Facility Modeling", *Final Report, Contract Number NAS13-564, Delivery Order 16*, May 1994.

[4] R. F. Follett and R. P. Taylor, "CTF Dynamic Fluid Flow Modeling Program", *Final Report, Contract Number NAS13-564, Delivery Order 83*, December 1996.

[5] R. F. Follett and R. P. Taylor, "E1 Dynamic Fluid Flow Model Update - EASY/ROCETS Enhancement and Model Development Support", *Final Report, Contract Number NAS13-564, Delivery Order 143*, July 1998.

[6] E. Barbieri, "Small Signal Modelling and Control of the Hydrogen Mixer for Facility E1", *Final Report, ASEE Summer Faculty Fellow*, August 2001.

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# Appendix A

Given a set of required exit mass flow, pressure, and temperature $\bar{w}_e$, $\bar{P}_e$, $\bar{T}_e$, then in steady-state [6]

$$\bar{C}_{vg} = \frac{\bar{h}_v - \bar{h}_l}{\bar{h}_g - \bar{h}_l}\frac{1}{\bar{\alpha}_g}\bar{w}_e \implies \bar{w}_g = \frac{\bar{h}_v - \bar{h}_l}{\bar{h}_g - \bar{h}_l}\bar{w}_e \tag{16}$$

$$\bar{C}_{vl} = \frac{\bar{h}_g - \bar{h}_v}{\bar{h}_g - \bar{h}_l}\frac{1}{\bar{\alpha}_l}\bar{w}_e \implies \bar{w}_l = \frac{\bar{h}_g - \bar{h}_v}{\bar{h}_g - \bar{h}_l}\bar{w}_e \tag{17}$$

To determine the linear perturbation model (7), it is necessary to find the analytic expressions of the indicated partial derivatives. For simplicity we include only those terms that do not evaluate to zero at the equilibrium point. Omitting the details, these are found to be:

$$\left[\frac{\partial F_1}{\partial z_1}\right]_{eq} = \frac{1}{V\bar{z}_2}\left[(h_g - z_1)\,C_{vg}\frac{\partial \alpha_g}{\partial z_1} + (h_l - z_1)\,C_{vl}\frac{\partial \alpha_l}{\partial z_1} - (h_v - z_1)\,C_{ve}\frac{\partial \alpha_e}{\partial z_1} - w_e\frac{\partial h_v}{\partial z_1}\right]_{eq}$$

$$\left[\frac{\partial F_1}{\partial z_2}\right]_{eq} = \frac{1}{V\bar{z}_2}\left[(h_g - z_1)\,C_{vg}\frac{\partial \alpha_g}{\partial z_2} + (h_l - z_1)\,C_{vl}\frac{\partial \alpha_l}{\partial z_2} - (h_v - z_1)\,C_{ve}\frac{\partial \alpha_e}{\partial z_2} - w_e\frac{\partial h_v}{\partial z_2}\right]_{eq}$$

$$\left[\frac{\partial F_2}{\partial z_1}\right]_{eq} = \frac{1}{V}\left[C_{vg}\frac{\partial \alpha_g}{\partial z_1} + C_{vl}\frac{\partial \alpha_l}{\partial z_1} - C_{ve}\frac{\partial \alpha_e}{\partial z_1}\right]_{eq}$$

$$\left[\frac{\partial F_2}{\partial z_2}\right]_{eq} = \frac{1}{V}\left[C_{vg}\frac{\partial \alpha_g}{\partial z_2} + C_{vl}\frac{\partial \alpha_l}{\partial z_2} - C_{ve}\frac{\partial \alpha_e}{\partial z_2}\right]_{eq}$$

$$\left[\frac{\partial F_1}{\partial C_{vg}}\right]_{eq} = \frac{\bar{\alpha}_g}{V\bar{z}_2}\left(\bar{h}_g - \bar{z}_1\right) ; \qquad \left[\frac{\partial F_1}{\partial C_{vl}}\right]_{eq} = \frac{\bar{\alpha}_l}{V\bar{z}_2}\left(\bar{h}_l - \bar{z}_1\right)$$

$$\left[\frac{\partial F_1}{\partial C_{ve}}\right]_{eq} = -\frac{\bar{\alpha}_e}{V\bar{z}_2}\left(\bar{h}_v - \bar{z}_1\right) ; \qquad \left[\frac{\partial F_2}{\partial C_{vg}}\right]_{eq} = \frac{1}{V}\bar{\alpha}_g$$

$$\left[\frac{\partial F_2}{\partial C_{vl}}\right]_{eq} = \frac{1}{V}\bar{\alpha}_l ; \qquad \left[\frac{\partial F_2}{\partial C_{ve}}\right]_{eq} = -\frac{1}{V}\bar{\alpha}_e$$

where

$$\left[\frac{\partial \alpha_g}{\partial z_1}\right]_{eq} = \begin{cases} -2.857\times 10^{-2}\frac{\sqrt{T_g}\rho_g}{P_g}\frac{P_v}{\sqrt{P_g^2 - P_v^2}}\frac{\partial P_v}{\partial z_1} & P_v < P_g < 2P_v; \\ 0, & P_g \geq 2P_v. \end{cases}$$

$$\left[\frac{\partial \alpha_l}{\partial z_1}\right]_{eq} = \frac{-1}{2}\left[\frac{\rho_l\rho_w}{\alpha_l}\frac{\partial P_v}{\partial z_1}\right]_{eq}$$

$$\left[\frac{\partial \alpha_e}{\partial z_1}\right]_{eq} = \frac{1}{2}\left[\frac{\rho_v\rho_w}{\alpha_e}\frac{\partial P_v}{\partial z_1}\right]_{eq} ;$$

$$\left[\frac{\partial \alpha_g}{\partial z_2}\right]_{eq} = \begin{cases} -2.857\times 10^{-2}\frac{\sqrt{T_g}\rho_g}{P_g}\frac{P_v}{\sqrt{P_g^2 - P_v^2}}\frac{\partial P_v}{\partial z_2} & P_v < P_g < 2P_v; \\ 0, & P_g \geq 2P_v. \end{cases}$$

$$\left[\frac{\partial \alpha_l}{\partial z_2}\right]_{eq} = \frac{-1}{2}\left[\frac{\rho_l\rho_w}{\alpha_l}\frac{\partial P_v}{\partial z_2}\right]_{eq} ; \qquad \left[\frac{\partial \alpha_e}{\partial z_2}\right]_{eq} = \frac{1}{2}\left[\frac{\rho_w}{V\alpha_e}\left(P_v - P_e + z_2\frac{\partial P_v}{\partial z_2}\right)\right]_{eq}$$

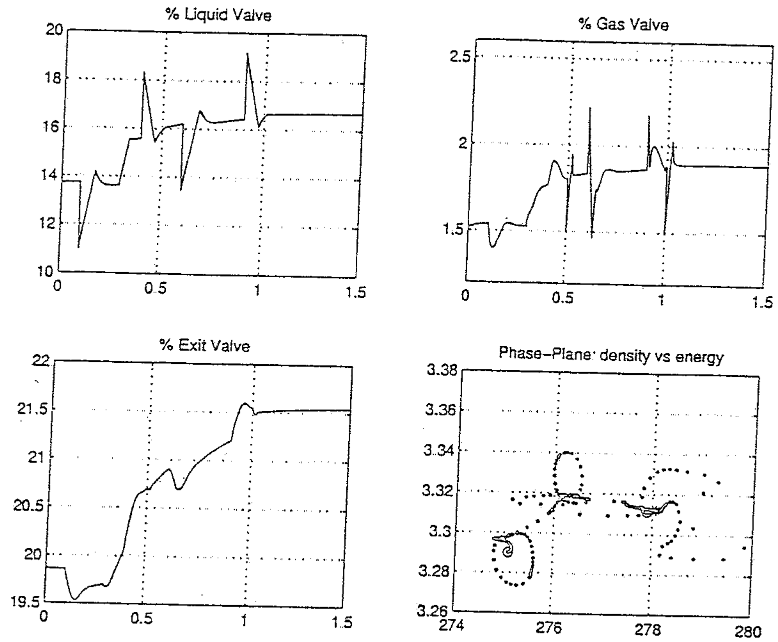


Figure 2: MIMO PI Control vs Time (sec) (with valve perturbations). Bottom right plot is the resulting State Trajectory: Start at $\circ$; End at $\times$.

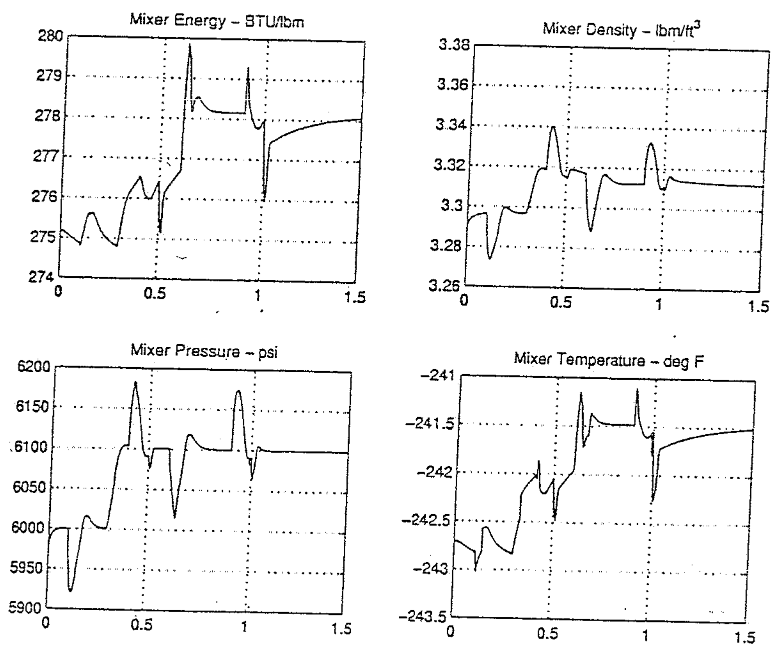


Figure 3: Mixer Behavior vs Time (sec). MIMO PI Control (with valve perturbations)

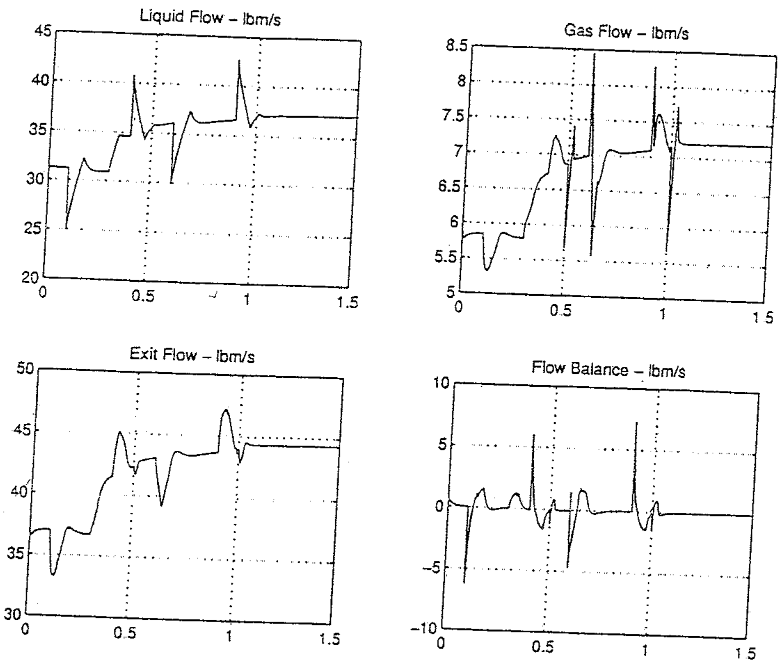


Figure 4: Input and Output Flows with MIMO PI Control vs Time (sec) (with valve perturbations). Flow Balance is $w_g + w_l - w_e$.

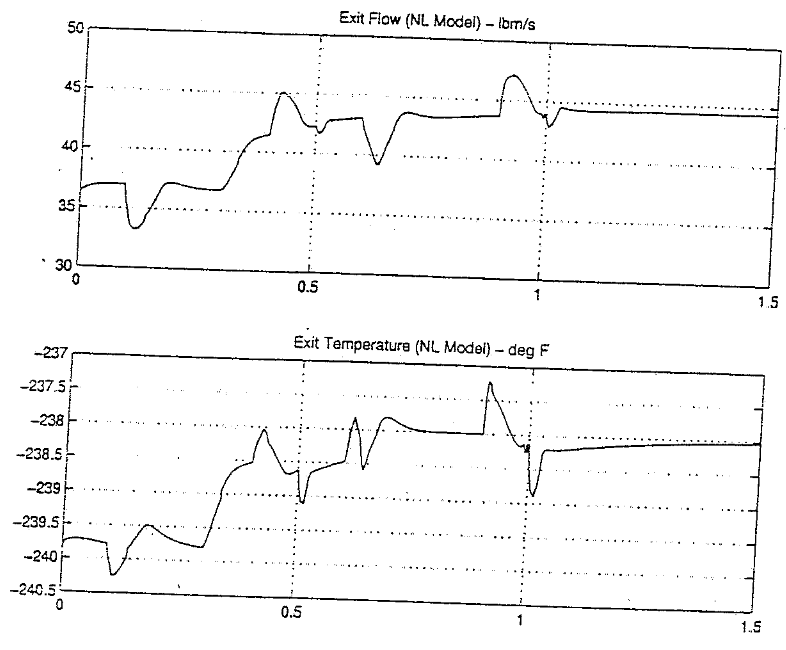


Figure 5: Exit Flow and Temperature Error vs Time (sec). (MIMO PI Control with valve perturbations.)