Gauging the Nearness and Size of Cycle Maximum

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NOMENCLATURE

\[ Ap \] geomagnetic index

\[ cl \] confidence level

\[ d_{\text{max}}(A) \] maximum daily value of sunspot number

\[ E(RM) \] epoch of conventional sunspot maximum

\[ E(Rm) \] epoch of conventional sunspot minimum

\[ f \] number of individual spots

\[ g \] number of groups

\[ k \] correction coefficient

\[ R \] relative sunspot number

\[ R(A) \] annual value of sunspot number

\[ Rm \] smoothed sunspot minimum (marks conventional sunspot minimum)

\[ RM \] smoothed sunspot maximum (marks conventional sunspot maximum)

\[ r \] linear correlation coefficient

\[ r^2 \] coefficient of determination

\[ r_{\text{max}}(A) \] maximum monthly mean sunspot number

\[ se \] standard error of estimate
GAUGING THE NEARNESS AND SIZE OF CYCLE MAXIMUM

1. INTRODUCTION

Sunspot cycles usually are described in terms of the variation of their annual or, more often, smoothed monthly mean sunspot number—this latter quantity being the average of two consecutive 12-mo averages of monthly mean sunspot number (the 12-mo moving average, or so-called 13-mo running mean).\textsuperscript{1–3} When viewed in this fashion, it is quite apparent that the variation of sunspot number over time is systematic rather than random, having a minimum and one or more maximums every 11 yr or so.\textsuperscript{4–6}

Sunspot cycles are conventionally reckoned from minimum to minimum rather than from maximum to maximum, and this length is called the “period” of the cycle. The elapsed time from minimum to primary maximum, or “maximum amplitude,” denotes the conventional “ascent” duration of the cycle, and the elapsed time from primary maximum to subsequent minimum denotes the conventional “descent” duration.

Although sunspots have been viewed telescopically for nearly 400 yr, it has only been $\approx160$ yr since Samuel Heinrich Schwabe first reported the spottedness of the Sun to vary in cyclic fashion, waxing and waning over an interval of about a decade in length,\textsuperscript{7–9} deduced by him on the basis of observations of “clusters of spots” and “number of spotless days.” Soon thereafter (in 1848), Rudolf Wolf introduced his now familiar notion of the “relative sunspot number ($R$),” defined by him as $R=k(10g+f)$, which today is routinely employed to describe the timing and strength of a sunspot cycle. In the equation, $g$ is the number of groups, $f$ is the number of individual spots, and $k$ is a correction coefficient dependent upon the qualities of the observer, the observing site, the telescope, etc.

Because sunspot number is progressively less reliable before the mid-1800’s\textsuperscript{1,3,5,8–10} than afterward, those cycles occurring since the mid-1800’s are referred to as the “modern era” sunspot cycles. Individually, cycles are numbered sequentially following Wolf’s reconstruction of the sunspot record, where the cycle currently in progress is denoted cycle 23, having had its minimum in 1996 and primary maximum in 2000 on the basis of annual averages. In this study, the maximum monthly mean sunspot number, the maximum value of the 2-mo moving average of monthly mean sunspot number, and the maximum daily value of sunspot number are examined as monitors for gauging the nearness and size of conventional cycle maximum (maximum smoothed monthly mean sunspot number). Indeed, these measures are found to provide a timely and clear indication of the nearness and size of conventional cycle maximum for an ongoing sunspot cycle.
2. RESULTS

Previously, Wilson\textsuperscript{11} showed that the first spotless day during the decline of an ongoing sunspot cycle serves as a useful predictor for establishing approximately when subsequent cycle minimum should be expected. The establishment of cycle minimum is crucial for the accurate prediction of the timing and size (the maximum amplitude, of a sunspot cycle).\textsuperscript{12–19}

Examination of a variety of data indicated that minimum for cycle 23 occurred sometime in 1996, although the specific month designating minimum was somewhat ambiguous.\textsuperscript{20–23} For example, based merely on the behavior of smoothed monthly mean sunspot number (the conventional method), the choice is May 1996, although such a choice makes cycle 23 peculiar in that it now is the first cycle on record to have had its first-appearing, high-latitude spot group (latitude $\geq$25°) occurring simultaneously with cycle minimum. In the past, the first-appearing, high-latitude spot group of a new cycle had always preceded conventional cycle minimum by at least 3 mo.\textsuperscript{13} A slightly later date of September–October 1996 was indicated on the basis of the number of spotless days and the behavior of the 2-mo moving average of sunspot number, although new cycle dominance (in terms of spot area) did not occur until early 1997. Curve fitting\textsuperscript{18,23} suggested July 1996 as the official start for cycle 23, giving a minimum in September 1996.

Figure 1 displays the annual averages (the thick, bottom line, $R(A)$) of sunspot number for the interval of 1856–2001, spanning cycles 10 through a portion of cycle 23 (the so-called modern era of sunspot observations). Also plotted are the maximum monthly mean sunspot number (the middle line, $r_{\text{max}}(A)$) and the maximum daily value of sunspot number (the thin, upper line, $d_{\text{max}}(A)$) for each year with individual cycle numbers delineating each specific cycle being shown at the bottom of the chart. Cycle minimum is best viewed as the 2- to 3-yr interval when sunspot number is at its lowest levels. Similarly, cycle maximum can be regarded as the 2- to 3-yr interval when sunspot number is at its highest levels. For the remainder of the time (outside these intervals of minimum and maximum), sunspot number is found to be in transition, progressing either from minimum to maximum levels or from maximum to subsequent minimum levels.

Because of their strong similarity in relative strengths and timing signatures, direct comparison of $R(A)$ against $r_{\text{max}}(A)$ or $d_{\text{max}}(A)$ obviously yields quite strong linear correlations. In particular, the scatterplot (not shown) of $R(A)$ versus $r_{\text{max}}(A)$ has a linear correlation coefficient, $r=0.979$, inferring that 95.8 percent of the variance in $R(A)$ can be simply explained by the variation of $r_{\text{max}}(A)$ alone, and the regression equation can be written as $y=-9.019+0.786 x$, where $y$ is $R(A)$ and $x$ is $r_{\text{max}}(A)$, having a standard error of estimate, $se=9.1$ units of sunspot number. Similarly, the scatterplot (not shown) of $R(A)$ versus $d_{\text{max}}(A)$ has a linear correlation coefficient $r=0.960$, inferring that 92.1 percent of the variance in $R(A)$ can be explained by the variation of $d_{\text{max}}(A)$ alone, and the regression equation can be written as $y=-24.323+0.553 x$, where $y$ again is $R(A)$ and $x$ now is $d_{\text{max}}(A)$, having a standard error of estimate, $se=12.8$ units of sunspot number. On the basis of the highest values of $r_{\text{max}}(A)$ and $d_{\text{max}}(A)$ that were seen in 2000, which are the highest values that were seen during cycle 23, respectively, 170.1 (July) and 246
Figure 1. Annual values of sunspot number $R(A)$, the maximum monthly mean sunspot number for the year $r_{\text{max}}(A)$, and the maximum daily sunspot number for the year $d_{\text{max}}(A)$ for the interval 1856–2001, spanning solar cycle 10 through a portion of cycle 23.

(July), the 90-percent confidence level of $R(A)$ for the year 2000 could have been estimated as $124.7 \pm 15.0$ based on $r_{\text{max}}(A)$ and $111.8 \pm 21.1$ based on $d_{\text{max}}(A)$, suggesting that $R(A)$ for the year 2000 would lie somewhere between 109.7 and 132.9 (the overlap). As it turned out, $R(A)$ for cycle 23 occurred in 2000 and measured 119.6 on the basis of annual averages.

A bivariate fit of $R(A)$ versus $d_{\text{max}}(A)$ and $r_{\text{max}}(A)$ does not appreciably improve the estimate. For example, the bivariate fit can be written as $y = -12.567 + 0.107 x_1 + 0.641 x_2$, where $x_1$ is $d_{\text{max}}(A)$ and $x_2$ is $r_{\text{max}}(A)$, having a correlation coefficient $r=0.980$, inferring that 96 percent of the variance in $R(A)$ can be explained using both $d_{\text{max}}(A)$ and $r_{\text{max}}(A)$, and a standard error of estimate, $se=9.0$ units of sunspot number. Thus, the estimate of $R(A)$ for the year 2000, known as early as the end of July 2000, was $122.8 \pm 14.9$, very close to its actual value of 119.6.

Figure 2 compares cycle 23 (the filled circles) to that of the mean (thick line) and envelope (thin lines) of cycles 10–22, based on the technique of epoch analysis, using cycle minimum as the epoch of comparison. Panel (a) depicts the comparison against $R(A)$, panel (b) depicts the comparison against $r_{\text{max}}(A)$, and panel (c) depicts the comparison against $d_{\text{max}}(A)$. Clearly, for all three measures, cycle 23 is found to closely match that of the mean cycle during its rise from minimum to maximum, being slightly larger than the mean at cycle maximum, and its maximum amplitude is observed to have occurred in year 4 postminimum (in the year 2000). Because at no time during the modern era has a maximum amplitude ever been observed to have occurred at greater than year 5 postminimum (corresponding to the year 2001; see paragraph below), the obvious downturn in activity after the year 2000 signified that primary maximum for cycle 23 indeed occurred in the year 2000.

Also depicted in figure 2 (panel (a)) are the occurrences and values for individual cycles 10–22, demonstrating the Waldmeier effect (the strong tendency for fast-rising cycles to also be cycles of larger maximum amplitude and slow-rising cycles to also be cycles of smaller maximum amplitude). Relative to other cycles of the modern era, cycle 23 is noted to have had an ascent of 4 yr (the range of ascents for the modern era spans 3–5 yr) and a maximum amplitude of 119.6, a value below that for cycle 11 (ascent 3 yr) but above that for cycle 17 (ascent 4 yr), both cycles having a maximum amplitude larger than the mean cycle.
Figure 2. Comparison of cycle 23 (denoted by filled circles) to the mean of cycles 10–22 for \( R(A) \), \( r_{\text{max}}(A) \), and \( d_{\text{max}}(A) \). In each panel, the thick line is the mean and the thin lines represent the upper and lower envelope values for the parameters. For \( R(A) \), the relative ascent durations and maximum amplitudes for cycles 10–22 are given.

Figure 3 displays the smoothed monthly mean sunspot number (panel (a)) for January 1996–July 2001, the monthly mean and maximum daily value for each month (maximums of these measures yielding the \( r_{\text{max}} \) and \( d_{\text{max}} \) for cycle 23 (panel (b)), and the 2-mo moving average of monthly mean sunspot number (panel (c)). While conventional minimum occurred in May 1996 (smoothed monthly mean sunspot number equal to 8.0), clearly, on the basis of \( r_{\text{max}} \), \( d_{\text{max}} \), and the 2-mo moving average of monthly mean sunspot number, cycle 23’s cycle minimum seems more likely to have occurred slightly later in time, about September–October 1996, in agreement with the results of Harvey and White.\(^{21,23}\)

During each year of its rise to cycle maximum, at least one or two strong bursts of activity are indicated, with the strongest burst having occurred in July 2000. Following this, values of sunspot number appreciably dropped and essentially flattened for several months prior to a smaller secondary burst of activity about September 2001. Because cycle maximum usually persists 2 to 3 yr, and occasionally 4 yr, it seems likely that these values would be in general decline after 2001, inferring that cycle maximum for cycle 23 indeed occurred in the year 2000 on the basis of annual averages and, conventionally, in April 2000 on the basis of smoothed monthly mean sunspot number.
Figure 3. Solar activity associated with cycle 23 for January 1996–January 2002. Shown are smoothed sunspot number (panel (a)), used for marking the epochs of conventional sunspot minimum ($E(Rm)$) and maximum ($E(RM)$); the maximum daily values of sunspot number (the thin line) and the monthly mean sunspot number $r$ (the thick line) for each month (panel (b)), used for determining $d_{\text{max}}$ and $r_{\text{max}}$ for cycle 23; and the 2-mo moving average of monthly mean sunspot number (panel (c)), used for determining the maximum value of the 2-mo moving average for cycle 23.

Figures 4 and 5 depict the scatterplots of maximum amplitude $RM$ (the maximum value of smoothed monthly mean sunspot number), against the maximum values of $r_{\text{max}}$ and $d_{\text{max}}$ (fig. 4(a) and (b), respectively) for cycles 10–22 and $RM$ against the maximum value of the 2-mo moving average (fig. 5). Each correlation is statistically very important (confidence level, $cl>99.9$ percent), and the observed
maximum values for cycle 23 of \( r_{\text{max}} \) (170.1), \( d_{\text{max}} \) (246), and the 2-mo moving average (148.9), as shown in figure 3, are such that \( RM \) for cycle 23 very probably should have been expected to lie in the upper-rightmost quadrant of each figure, indicating an \( RM \geq 110.6 \). In particular, the observed values indicated that maximum amplitude for cycle 23 would be \( \approx 124.5 \) (the average of the three estimates). Also, because sunspot cycles found in the upper-rightmost quadrant almost always have been short-period cycles (denoted by the filled inverted triangles) as opposed to long-period cycles (denoted by filled circles), cycle 23 is expected to be a cycle of shorter period (having a length less than \( \approx 132 \) mo).\textsuperscript{24} From figure 4 (panel (a)), it is found that six of seven cycles having an \( r_{\text{max}} \geq 154.5 \) have been shorter period cycles and five of seven cycles having a \( d_{\text{max}} \geq 237 \) have been shorter period cycles, while from figure 5 it is found that six of seven cycles having a maximum 2-mo moving average \( \geq 139.6 \) have been shorter period cycles.

Figure 4. Scatterplots of \( RM \) versus \( r_{\text{max}} \) (panel (a)) and \( RM \) versus \( d_{\text{max}} \) (panel (b)) for cycles 10–22. The thin vertical and horizontal lines in each panel are the medians and the diagonal line is the inferred regression. Identified for each is the regression equation, coefficient of correlation, coefficient of determination, the standard error of estimate, and the confidence level at which the regression is inferred to be statistically significant. Short-period cycles are identified using filled inverted triangles and long-period cycles are identified using filled circles.
Figure 5. Scatterplot of $RM$ versus the maximum 2-mo moving average of monthly mean sunspot number for cycles 10–22. The construction follows that of figure 4.
3. DISCUSSION AND CONCLUSION

In section 2, the conventional maximum amplitude of a sunspot cycle was shown to be strongly related to the maximum daily value of sunspot number, the maximum monthly mean sunspot number, and the maximum value of the 2-mo moving average of monthly mean sunspot number during the cycle. Also shown was that, statistically speaking, there is a strong tendency for large-amplitude cycles to be of faster rise and shorter period and small-amplitude cycles to be of slower rise and longer period. Because of these strong relationships, it is apparent that by monitoring the sizes of the aforementioned parameters during the critical rising and maximum phases of a sunspot cycle, accurate gauging of the nearness and size of the ongoing cycle and, perhaps, even its length can be accomplished.

As an example, from figure 2 it was found that for the modern era of sunspot cycles, maximum amplitude has always occurred in years 3, 4, or 5 postminimum. From figure 3, it is seen that year 3 postminimum for cycle 23 corresponds to the year 1999. During that year, two major bursts of activity occurred, the first in June and the second in November. The June burst had a peak daily sunspot number equal to 195, a monthly mean of 137.7, and a 2-mo moving average of 123.8, while the November 1999 burst had a peak daily sunspot number of 206, a monthly mean of 133.2, and a 2-mo moving average of 116.9. If the June burst truly represented the actual peak for the cycle, then maximum amplitude for cycle 23 would have been expected to measure only ≈97.3 (as compared to the expected value of ≈96.2 using the November data), a value considerably below that which had been predicted for it (160±30).25 The lack of a strong peak in year 3 postminimum plainly suggested that cycle 23’s peak would occur later, either in 2000 (year 4 postminimum) or in 2001 (year 5 postminimum), and that its size would consequently be closer to the mean. As noted in the previous section, it is now known that the real maximums for the aforementioned parameters occurred in July 2000 and, together, they suggested a maximum amplitude of ≈124.5 for cycle 23, peaking near July 2000 (±5 mo).

Because the estimated value for cycle 23’s maximum amplitude was larger than that of the mean cycle, it is further noted that, statistically speaking, cycle 23 should be expected to have a period shorter than 132 mo. If true, then this certainly has interesting ramifications for cycle 24, the next sunspot cycle. For example, cycles of shorter length typically are followed by cycles of larger than average maximum amplitude.23 Thus, if cycle 23 indeed turns out to be a cycle of shorter period, then cycle 24 should be expected to have an early onset (before 2007) rather than a late onset (2007 or later), and it should also be a cycle of larger than average maximum amplitude. Furthermore, because of the strong statistical relationship between the size of a sunspot cycle and the number of disturbed days (Ap ≥25) during the preceding cycle,17,18 the years 2004 through 2006 must see a substantial increase in the number of geomagnetically disturbed days.

Concerning the preceding, it should be noted that Schatten26 has predicted cycle 24 to have a smoothed sunspot number of 120±40, peaking in April 2011, on the basis of certain timing predictors, whereas Duhau27 has predicted a value of 87.5±23.5, on the basis of an extrapolation of a nonlinear coupling function between sunspot maxima and geomagnetic minima modulations. More recently, on
the basis of a strong statistical relationship between maximum amplitude of a cycle and the strength of
the drift velocity at cycle maximum two cycles preceding, it was determined that maximum amplitude
for cycle 24 should be larger than average, peaking sometime in 2010 or 2011, a prediction supported
by the present analysis reported here. Thus, at the present time, some several years before the start of
cycle 24, unless cycle 24 turns out to be a statistical outlier, it should be a cycle of larger than average size and
cycle 23 should be a cycle of shorter than average length.
REFERENCES


### Abstract

A simple method for monitoring the nearness and size of conventional cycle maximum for an ongoing sunspot cycle is examined. The method uses the observed maximum daily value and the maximum monthly mean value of international sunspot number and the maximum value of the 2-mo moving average of monthly mean sunspot number to effect the estimation. For cycle 23, a maximum daily value of 246, a maximum monthly mean of 170.1, and a maximum 2-mo moving average of 148.9 were each observed in July 2000. Taken together, these values strongly suggest that conventional maximum amplitude for cycle 23 would be $\approx 124.5$, occurring near July 2002 $\pm 5$ mo, very close to the now well-established conventional maximum amplitude and occurrence date for cycle 23—120.8 in April 2000.

### Subject Terms

Sun, sunspot cycle, solar cycle, solar cycle prediction