RANGE OF DENSITY VARIABILITY FROM SURFACE TO 120 KM ALTITUDE

by

Orvel E. Smith
and
Halsey B. Chenoweth

LIBRARY COPY

JAN 26 1967

MANNED SPACECRAFT CENTER
HOUSTON, TEXAS

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON

July 1961
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Summary</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>Statement of the Problem</td>
<td>1</td>
</tr>
<tr>
<td>A Proposed First Approach</td>
<td>2</td>
</tr>
<tr>
<td>A Proposed Second Approach</td>
<td>3</td>
</tr>
<tr>
<td>A Proposed Third Approach</td>
<td>3</td>
</tr>
<tr>
<td>SUMMARY OF THE THERMODYNAMIC RELATIONSHIPS IN THE ATMOSPHERE</td>
<td>3</td>
</tr>
<tr>
<td>Thermal Model of the Atmosphere</td>
<td>3</td>
</tr>
<tr>
<td>Electron Density Model of the Atmosphere</td>
<td>4</td>
</tr>
<tr>
<td>Composition Model of the Atmosphere</td>
<td>5</td>
</tr>
<tr>
<td>Radiation and its Control of the Atmosphere</td>
<td>7</td>
</tr>
<tr>
<td>VARIABILITY OF DENSITY RELATIVE TO A STANDARD DENSITY PROFILE - FIRST APPROACH</td>
<td>8</td>
</tr>
<tr>
<td>Choosing a Standard Density Profile</td>
<td>8</td>
</tr>
<tr>
<td>Range of Density Over the Northern Hemisphere Relative to the ARDC 1959 Model Atmosphere</td>
<td>8</td>
</tr>
<tr>
<td>VARIABILITY OF DENSITY WITH LATITUDE, SEASONS, AND TIME OF DAY - SECOND APPROACH</td>
<td>10</td>
</tr>
<tr>
<td>DENSITY MEASUREMENTS AND PREDICTION OF DENSITY - THIRD APPROACH</td>
<td>13</td>
</tr>
<tr>
<td>Observational Methods for Determining Density</td>
<td>13</td>
</tr>
<tr>
<td>Prediction or Forecast for Density</td>
<td>13</td>
</tr>
<tr>
<td>CONCLUSIONS AND RECOMMENDATIONS</td>
<td>15</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>22</td>
</tr>
</tbody>
</table>
SUMMARY

A re-entry space vehicle development program, such as Project Apollo, requires a knowledge of the variability of atmospheric density from the surface of the earth to re-entry altitude (120 km). This report summarizes the data on density given in the most recent literature on the subject. The range of atmospheric density with respect to the ARDC 1959 Model Atmosphere is determined and shown graphically.

From the surface to 30 km altitude abundant information on density is available. From 30 to 90 km altitude the summarized reports of observations made at a limited number of stations have been used. Between 90 and 120 km altitude the density is somewhat speculative, there being but few measurements available. Therefore, the qualitative values for the variability of density above 30 km must be considered tentative.

Variations of atmospheric density by latitude and seasons made it necessary to develop a family of curves rather than a single profile. Three curves are presented to show the range of density deviation versus altitudes with respect to the ARDC 1959 Model Atmosphere. Each curve is used for a specific latitude range and season.

INTRODUCTION

Statement of the Problem

There exists a need to know the variation of atmospheric density with respect to a standard density profile for the purpose of flight mechanical and aerodynamic heating studies in relation to Project Apollo and other re-entry bodies. While it would be most desirable to state the variations of density in terms of confidence limits relative to a standard density profile, this is not possible. Statistical methods do not permit a valid determination of variances using a heterogeneous sample, such as would result by combining frequency distributions of density from different locations taken at different times. Furthermore, sufficient data have not been accumulated to determine
variances for a great number of locations above 30 km. Therefore, another approach to the problem must be made.

A Proposed First Approach

A proposed approach is to determine the range of density variability with respect to a standard density profile (i.e., ARDC 1959). This approach is attractive because of certain restrictive conditions imposed. The restrictive conditions applicable to Project APOLLO are as follows (ref. 36):

a. Re-entry occurs at an altitude of approximately 120 km.

b. The re-entry body performs a maneuver in the 60 to 80 km altitude regions. It descends from 120 km to 60 km altitude, ascends to 80 km, and then proceeds to impact.

c. The range from re-entry to impact is equivalent to about 34 degrees of latitude.

d. The orientation of the flight path relative to geographical coordinates of the earth is unknown.

e. The coordinates of the impact are unknown.

f. The time of year, month, and day for the operation are unknown.

The above conditions make it necessary to determine a density profile that is representative for any latitude, longitude, altitude, season, and time of day. The magnitude of the range of density variability must also be determined with respect to a representative atmospheric density profile.

This approach will be developed, since it is the only available practical method.

A Proposed Second Approach

A proposed second approach is to establish reference density profiles for latitudes, seasons, and time of day. A more realistic density profile and variability than established by the first approach could be achieved if the restrictions d, e, and f in paragraph above could be removed. It may be possible from the preliminary evaluations based on the density profile in the first approach to determine the approximate altitude range and flight mechanical relationships. This information, together with a removal of restrictions d, e, and f, would make it possible to present a more refined density altitude and density variability relationship for a particular operation.
LIST OF TABLES

Table | Page
-----|-----
I Thermal Model of the Atmosphere | 4
II Ionosphere Model | 5
III Number Densities for Analytical Model of Thermosphere | 6
IV Typical Low and High Latitude Summer and Winter Density Profiles | 11
V High Altitude Measurement of Atmospheric Parameters | 14
VI Density Deviation Summary | 15

LIST OF ILLUSTRATIONS

Figure | Page
------|-----
1 Comparison of ARDC 1956 and 1959 Model Atmospheres | 17
2 Range of Density Deviation from ARDC 1959 Model Atmosphere for Northern Hemisphere | 18
3 Idealized Range of Density Deviations from ARDC 1959 Model Atmosphere | 19
4 Maximum and Minimum Density Variability Inferred from Thermoelectric Properties | 20
5 Amplitude of Diurnal Density Variation, Day Positive-Night Negative | 21
This second approach will not be developed fully in this report, since it is interdependent on many factors in the first approach. However, the second approach will be scrutinized and developed in a general form. A specific form would involve study inconsistent with the preliminary nature of this report.

A Proposed Third Approach

A third approach is to establish a density profile by utilizing observational and prediction techniques. To be useful in Project Apollo, the prediction must be valid over a five-day period. This requirement presents difficulties. Our present knowledge of the atmosphere above 30 km altitude is based upon extrapolation of a tremendous amount of data in the first 30 km with very little data above that level. The principal sites at which atmospheric density measurements above 30 km have been made and which are available for reference, are limited to:

a. Fort Churchill, Canada
b. Wallops Island, Virginia
c. White Sands Missile Range, New Mexico
d. Guam
e. Woomera, Australia
f. Cape Canaveral, Florida
g. Fort Greeley, Alaska

Limitations of measuring accuracy, frequency of measurement, altitude range, sampling area, and coordination of measurements prevent the third approach from being practical at this time.

SUMMARY OF THERMODYNAMIC RELATIONSHIPS IN THE ATMOSPHERE

Thermal Model of the Atmosphere

Before discussing the observed variability of ambient density and the possible mechanism whereby the density changes with time, it appears appropriate first to review the nomenclature of an atmospheric model based on temperature and ion concentration. The thermal model summarized in table I is the model considered most descriptive for the purpose of this report. The data in table I were based on Nicolet's work (ref. 21).

It should be noted that there is a wide difference of opinion on the altitude regions which comprise the stratosphere and mesosphere.
For example, Chapman (ref. 32), in measurements based on temperature, restricts the stratosphere to the layer between 8 and 22 km and the mesosphere to the region between 22 and 90 km.

**TABLE I**

**THERMAL MODEL OF THE ATMOSPHERE**

<table>
<thead>
<tr>
<th>Region</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth's surface</td>
<td>Temperature $275 \pm 20$ K</td>
</tr>
<tr>
<td>Troposphere</td>
<td>Temperature decreases with altitude. Altitude, 0 to $13 \pm 5$ km</td>
</tr>
<tr>
<td>Tropopause</td>
<td>Temperature minimum, $210 \pm 20$ K Altitude, $13 \pm 5$ km, decreasing from equator to poles</td>
</tr>
<tr>
<td>Stratosphere</td>
<td>Temperature increases with height. Altitude, $13 \pm 5$ to $50 \pm 5$ km</td>
</tr>
<tr>
<td>Stratopause</td>
<td>Temperature maximum, $273 \pm 20$ K Altitude, $50 \pm 5$ km</td>
</tr>
<tr>
<td>Mesosphere</td>
<td>Temperature decreases with height. Altitude, $50 \pm 5$ to $85 \pm 5$ km</td>
</tr>
<tr>
<td>Mesopause</td>
<td>Temperature minimum, $190 \pm 25$ K Altitude, $85 \pm 5$ km</td>
</tr>
<tr>
<td>Thermosphere</td>
<td>Temperature increases with height. Altitude, $85 \pm 5$ to $175$ km with the upper limit not firmly established</td>
</tr>
<tr>
<td>Thermopause</td>
<td>The beginning of an isothermal layer at 175 km altitude</td>
</tr>
</tbody>
</table>

**Electron Density Model of the Atmosphere**

The ionospheric region is generally considered to begin at 60 km altitude and to extend upward to 600 km altitude. However, due to the wide variation of the heights of the layers of maximum ion concentration (denoted as D, E, F₁, F₂, and a suggested G region), it is difficult to establish fixed altitude limits for these regions within the ionosphere. The model given in table II is taken from Murgatroyd (ref. 1)
TABLE II
IONOSPHERE MODEL

<table>
<thead>
<tr>
<th>Layer (Region)</th>
<th>Height (km)</th>
<th>Electron Densities cm(^{-3})</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>60 - 100</td>
<td>1.5 x 10(^4)</td>
</tr>
<tr>
<td>E</td>
<td>90 - 120</td>
<td>1.5 x 10(^5)*</td>
</tr>
<tr>
<td>F(_1)</td>
<td>160 - 220</td>
<td>2.5 x 10(^5)</td>
</tr>
<tr>
<td>F(_2)</td>
<td>250 - 500</td>
<td>1.5 x 10(^6)</td>
</tr>
<tr>
<td>&quot;G&quot;</td>
<td>about 600</td>
<td>3.0 x 10(^6)</td>
</tr>
</tbody>
</table>

*The high electron density of about 3.0 x 10\(^6\) electrons cm\(^{-3}\), known as "Sporadic E-Region," often exists locally in patches near 120 km altitude.

Composition Model of the Atmosphere

The ARDC 1956 and 1959 Model Atmospheres are based on the assumption that the air is homogenous in composition up to 90 km altitude. Above 90 km the molecular scale temperature is used to define the ideal gas relationship. However, the text for the ARDC 1959 Model Atmosphere (ref. 17), suggests that the mean molecular weight used for the 1956 Model Atmosphere is probably more representative of the actual atmosphere for low latitudes. This would be in agreement with Nicolet (ref. 20) who concludes that dissociation over Fort Churchill appears to begin near 80 km in July and 96 km in March. As of December 1959, no direct measurement for temperature had been made above 80 km (ref. 20), and pressure had not been measured above 120 km. Only density has been measured above 120 km. Therefore, no atmospheric model yet exists which is related by observed values of temperature, pressure, density, and composition for altitudes between 80 to 120 km and above.

Bates (ref. 2) has generated values of density in the altitude region 120 to 80 km from analytically derived values for the composition. In the equations defining composition he assumes for 120 km diffusive equilibrium, a given temperature, and a vertical temperature gradient. The resulting density profile is in reasonable agreement with satellite data. From these and other assumptions, Bates gives the number densities of O, O\(_2\), and N\(_2\) for altitude and temperature as shown in table III.
### TABLE III

NUMBER DENSITIES FOR ANALYTICAL MODEL OF THERMOSPHERE

<table>
<thead>
<tr>
<th>Altitude Z (km)</th>
<th>Temperature T(Z) (K)</th>
<th>n(O) (cm$^{-3}$)</th>
<th>n(O$_2$) (cm$^{-3}$)</th>
<th>n(N$_2$) (cm$^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>380</td>
<td>$1.8 \times 10^{11}$</td>
<td>$3.95 \times 10^{11}$</td>
<td>$2.4 \times 10^{11}$</td>
</tr>
<tr>
<td>140</td>
<td>776</td>
<td>$4.7 \times 10^{10}$</td>
<td>$5.6 \times 10^9$</td>
<td>$4 \times 10^{10}$</td>
</tr>
<tr>
<td>160</td>
<td>938</td>
<td>$2.6 \times 10^{10}$</td>
<td>$2 \times 10^9$</td>
<td>$1.6 \times 10^{10}$</td>
</tr>
<tr>
<td>180</td>
<td>1003</td>
<td>$1.7 \times 10^{10}$</td>
<td>$9 \times 10^8$</td>
<td>$7.8 \times 10^9$</td>
</tr>
<tr>
<td>200</td>
<td>1031</td>
<td>$1.1 \times 10^{10}$</td>
<td>$4.4 \times 10^8$</td>
<td>$4.1 \times 10^9$</td>
</tr>
<tr>
<td>250</td>
<td>1048</td>
<td>$4.9 \times 10^9$</td>
<td>$7.9 \times 10^7$</td>
<td>$9.2 \times 10^8$</td>
</tr>
<tr>
<td>300</td>
<td>1050</td>
<td>$2.1 \times 10^9$</td>
<td>$1.5 \times 10^7$</td>
<td>$2.2 \times 10^8$</td>
</tr>
<tr>
<td>350</td>
<td>1050</td>
<td>$9.4 \times 10^8$</td>
<td>$3 \times 10^6$</td>
<td>$5.2 \times 10^7$</td>
</tr>
<tr>
<td>400</td>
<td>1050</td>
<td>$4.2 \times 10^8$</td>
<td>$6 \times 10^5$</td>
<td>$1.3 \times 10^7$</td>
</tr>
<tr>
<td>450</td>
<td>1050</td>
<td>$1.9 \times 10^8$</td>
<td>$1.2 \times 10^5$</td>
<td>$3.2 \times 10^6$</td>
</tr>
<tr>
<td>500</td>
<td>1050</td>
<td>$8.8 \times 10^7$</td>
<td>$2.6 \times 10^4$</td>
<td>$8.2 \times 10^5$</td>
</tr>
<tr>
<td>600</td>
<td>1050</td>
<td>$1.9 \times 10^7$</td>
<td>$1.2 \times 10^3$</td>
<td>$5.7 \times 10^4$</td>
</tr>
<tr>
<td>700</td>
<td>1050</td>
<td>$4.4 \times 10^6$</td>
<td>$6.4 \times 10^1$</td>
<td>$4.3 \times 10^3$</td>
</tr>
<tr>
<td>800</td>
<td>1050</td>
<td>$1.0 \times 10^6$</td>
<td>$3.6$</td>
<td>$3.5 \times 10^2$</td>
</tr>
</tbody>
</table>
Radiation and its Control of the Atmosphere

Theoretically it is possible to determine the thermal structure of the earth's atmosphere, and thus infer the density structure, if one can obtain satisfactory solutions to three physical equations. These equations, as stated by Schocken (ref. 26) are as follows:

a. An equation of state:
   \[ \rho RT = MP. \]

b. An equation of hydrostatic equilibrium:
   \[ \frac{dP}{dz} = -g \rho. \]

c. An equation of radiative transfer:
   \[ \frac{1}{K_\nu \rho} \frac{dI_\nu}{ds} = I_\nu - J_\nu. \]

where:

- \( P \) = pressure
- \( R \) = universal gas constant
- \( M \) = molecular weight
- \( T \) = temperature
- \( g \) = acceleration of gravity
- \( z \) = geometric height
- \( K_\nu \) = mass absorption coefficient
- \( I_\nu \) = intensity of pencil of radiation
- \( J_\nu \) = source function
- \( S \) = thickness of atmospheric layer in direction of propagation
- \( \rho \) = density
- \( \nu \) = frequency of pencil of radiation

At the present time three factors preclude satisfactory solutions to these equations. These factors are (1) insufficient basic data (2) observational heterogeneity of the data and, therefore, (3) a lack of knowledge of the distribution functions and boundary values involved in the differential equations.
Choosing a Standard Density Profile

The principal reference sources of atmospheric information for the upper stratosphere and higher, are the ARDC 1956 and ARDC 1959 Model Atmospheres. These are in great disagreement for altitudes above 50 km. The density data for the ARDC 1956 and ARDC 1959 Model Atmospheres are compared in figure 1 by expressing the difference percentage deviation from the ARDC 1959 values. For example, the ARDC 1959 density is 189% higher than that of the ARDC 1956 at 120 km altitude and becomes 90% lower at 300 km altitude. Discrepancies between satellite data and the ARDC 1956 values resulted in the 1959 revision. The relative differences between the ARDC 1956 and the ARDC 1959 values for temperature, pressure, mean molecular weight, and density are shown in figure 1.

It is now the consensus of opinion by authorities* in the field that the isothermal layer for both the ARDC 1956 and the ARDC 1959 models beginning at 11 km (geopotential kilometers) is too thick, that the temperature at 32 km is too low, and that the stratopause (50 km) is about 10K too high. These opinions are derived from IGY rocket data analysis.

Plans are under way to make appropriate revisions of the ARDC 1959 Model Atmosphere to conform to the latest findings derived from the observational data for all altitudes above 11 geopotential kilometers.

Range of Density Over the Northern Hemisphere Relative to the ARDC 1959 Model Atmosphere

For the purpose of this report, the ARDC 1959 Model Atmosphere has been adopted to compare the range of density variability over the Northern Hemisphere. The principal sources used in obtaining the range of density variation over the Northern Hemisphere relative to the ARDC 1959 Model Atmosphere are as follows:

a. 0 to 30 km References 25, 27, and 29
b. 30 to 90 km References 1, 11, 19, and 22
c. 90 to 200 km References 7, 8, 10, 12, 16, 17, 23, and 31

From the surface to 30 km altitude there is abundant atmospheric density information available for the purposes of this report. Of particular benefit were the reports by Vaughan (ref. 29) and Sissenwine (ref. 27),

* Conference of U.S. Committee on Extension of the Standard Atmosphere Working Group, January 16-17, 1961, Massachusetts Institute of Technology.
which present for selected stations the monthly mean deviation of density from the ARDC 1959 Model Atmosphere and the variance of density from the monthly values.

In the 30 to 90 km altitude region, individual density observations are seldom reported in the literature and one must rely on the summarized reports of other investigators. For this report, the individual density observations for Fort Churchill (ref. 1) and for White Sands Missile Range (ref. 32) were used. It was from these data that the range of the density deviation from the ARDC 1959 Model Atmosphere was determined, as depicted by the solid curve in figure 2. The curves labeled (1), (2) and (3) in figure 2 represent the density deviations from the ARDC 1959, derived from Nordberg's "typical" low latitude density profiles and "typical" high latitude winter and summer profiles (ref. 22). Note, however, only the modulus of these deviations is illustrated. The high latitude "typical" profiles are based on 10 rocket grenade firings at Fort Churchill. It must be considered that there is some dispersion in the density about these "typical" winter profiles, producing an even larger range of density variations about the ARDC 1959 density profile. As a conservative estimate of the upper limit of the density variability with respect to the "typical" winter profile, the amplitude of the seasonal variation was added to this mean and is illustrated as curve 4, in figure 2. Since no evaluation was possible to assess the errors in the individual density measurements and the density information was extracted from many sources, the resulting range of density relative to the ARDC 1959 Model Atmosphere must be considered tentative between 30 and 100 km altitude, and in the altitude region between 100 and 200 km the density deviations are somewhat speculative. The isolated parts at 110 km and 200 km altitudes are taken from Horowitz (ref. 8) and Nicolet (ref. 20), respectively.

For the purpose of application of the range of density deviation versus altitudes with respect to the ARDC 1959 Model Atmosphere, an empirical formula has been devised such that a family of curves giving the deviations versus altitude can be determined by arbitrarily selecting different values for only one parameter. This equation is:

\[ \Delta \rho_{\text{rel}} = \pm \sqrt{a} \left(e^{by}\right) \]

where:

- \( a = 100, 225, \) and \( 400 \)
- \( b = 0.01842 \text{ km}^{-1} \)
- \( y = \text{altitude in km, } 0 = y \leq 125 \text{ km} \)
- \( \Delta \rho_{\text{rel}} = \text{range of density from ARDC 1959 Model Atmosphere} \)
Of course, the resulting deviations determined by the above equations are idealized. By comparing figures 2 and 3 it can be seen that the curve for $\sqrt{a} = 20\%$ over-estimates the range of the density deviations in the first 30 km altitude, probably under-estimates the deviation from 40 to 70 km, and then exceeds the probable range of deviations above 90 km. Some qualifying remarks concerning the range of deviation for $\sqrt{a} = \pm 10\%, \pm 15\%$, and $\pm 20\%$ (hereafter referred as 10, 15, and 20% curves) could prove helpful in using these values for space re-entry problems. The 20% curve should represent the range of density deviation for the entire Northern Hemisphere from the ARDC 1959 density profile. The 15% curve should represent the range for density for mid-latitudes, and the 10% curve for low latitudes. The 10% curve must be used with some reservation for the low latitudes in the altitude region from 0 to 30 km, for it is known that the density deviation at 15 km altitude over Guam can be as much as 22% higher than the ARDC 1959 density value (ref. 32). As another example, the annual mean density over Patrick Air Force Base at 13 km is 13% higher than the ARDC value. In both cases it is the bias that contributes mainly to the variance of the total range of density deviation from the ARDC density in the upper tropospheric level for these low latitudes. The 20% curve should more closely approximate the upper limit of the range from ARDC 1959 at high latitudes in winter than for any other season or location.

Unless one recognizes there are altitudes at which the variation of density is a maximum and a minimum, then there is no basis for making extrapolations or interpolations for the variance of density versus altitude into those regions not supported by observational data. This phenomenon is widely observed and is supported by theory for the first 30 km altitude, and to a more limited extent above this altitude. By way of an analogy with temperature, radiation absorption, electron density, and from the limited observed density data, the altitudes at which the density variation is a maximum and minimum are estimated to be as illustrated in figure 4.

VARIABILITY OF DENSITY WITH LATITUDE, SEASONS, AND TIME OF DAY - SECOND APPROACH

A reference atmosphere is a special purpose atmospheric model based on empirical observations and expressed in an analytical form such that the parameters are inherently consistent with both the observed quantities and rigorously connected by physical equations. This concept of a reference atmosphere was presented by Smith (ref. 28) in a report entitled "A Reference Atmosphere for Patrick Air Force Base, Florida." It is possible, at least in principle, to establish a reference atmosphere for space vehicle re-entry which would represent the mean atmosphere for a given re-entry flight path. Present basic atmospheric information would limit the mean density to:
a. Latitudinal variation only for altitudes above 30 km.

b. Seasonal reference periods for altitudes above 30 km.

c. A maximum altitude of 90 km.

d. Only inferred information concerning the variance with respect to the mean density for all altitudes.

It is anticipated that the outcome of the first approach as presented in section III, and with additional knowledge gained of the characteristics of the space re-entry body as the program progresses, will determine the feasibility of developing a reference atmosphere for a particular operation.

As an interim measure it may be well to consider taking a preview of what the expected reduction of the bias in the mean density would be by selecting a reference density profile over that of the ARDC 1959 density.

Nordberg (ref. 22) concludes that the variation of the mean density from month to month is small in the altitude region from 30 to 90 km for low latitudes (35 degrees north and 35 degrees south). Therefore, an annual mean density profile would represent only a slight bias in the monthly mean density for this region. Using Nordberg's "typical" low latitude, "typical" high latitude summer and winter density profiles, comparisons are made as shown in table IV.

### TABLE IV

**TYPICAL LOW AND HIGH LATITUDE SUMMER AND WINTER DENSITY PROFILES**

<table>
<thead>
<tr>
<th>Altitude (km)</th>
<th>Low Latitude</th>
<th>High Latitude</th>
<th>Seasonal Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta \rho_1$ (%)</td>
<td>$\Delta \rho_2$ (%)</td>
<td>$\Delta \rho_3$ (%)</td>
</tr>
<tr>
<td>30</td>
<td>-10</td>
<td>-2</td>
<td>-22</td>
</tr>
<tr>
<td>40</td>
<td>-5</td>
<td>-1</td>
<td>-28</td>
</tr>
<tr>
<td>50</td>
<td>-8</td>
<td>2</td>
<td>-38</td>
</tr>
<tr>
<td>60</td>
<td>-12</td>
<td>-4</td>
<td>-50</td>
</tr>
<tr>
<td>70</td>
<td>-15</td>
<td>-6</td>
<td>-55</td>
</tr>
<tr>
<td>80</td>
<td>-15</td>
<td>.4</td>
<td>-48</td>
</tr>
</tbody>
</table>

+ = Summer  
- = Winter
Relative to ARDC 1959

\[
\Delta \rho_1 (%) = \left( \frac{\rho_{\text{low latitude}} - \rho_{\text{ARDC 1959}}}{\rho_{\text{ARDC 1959}}} \right) \times 100
\]

\[
\Delta \rho_2 (%) = \left( \frac{\rho_{\text{High latitude, summer}} - \rho_{\text{ARDC 1959}}}{\rho_{\text{ARDC 1959}}} \right) \times 100
\]

\[
\Delta \rho_3 (%) = \left( \frac{\rho_{\text{High latitude, winter}} - \rho_{\text{ARDC 1959}}}{\rho_{\text{ARDC 1959}}} \right) \times 100
\]

\[
\Delta \rho_4 (%) = \left( \frac{\rho_{\text{High latitude}} - \bar{\rho}_{\text{summer + winter}}}{\bar{\rho}_{\text{summer + winter}}} \right) \times 100
\]

It is readily noted that the high latitude summer density profile agrees with the ARDC 1959 to within ± 5% for the altitude region 30 to 80 km. The high latitude winter density is from 22 to 55% lower than the ARDC 1959. Furthermore, the summer density is from 20 to 50% higher than the winter density. There would be a bias in the mean density toward lower values of density from 20 to 50% in winter by using the ARDC 1959 density profile. This bias would be reduced to that of the seasonal amplitude by choosing an annual reference density profile. This reduction in the bias is not great, but coupled by a reduction in the variance which could be determined with additional observations to permit further sub-division of the reference density to summer and winter should yield a substantial reduction in the range of the density variation.

It is generally expected that the random variation of density at low latitudes exceeds the seasonal variation, and in the high latitudes the random variations in winter should exceed the seasonal variation. This makes it difficult from the limited observations to describe the latitudinal variation in density for different seasons. The gradient of the density is positive from south to north in the troposphere and lower stratosphere. This is verified by observation, but whether the gradient remains positive for altitudes up to 200 km is as yet unknown. However, one may hypothesize that the relative amplitude of the seasonal density variations is negative in summer from the surface to the upper troposphere and then becomes positive for all altitudes above this level, and increases in relative magnitude from south to north with increasing altitudes up to 125 km. For reason of completeness it is noted that recent evaluations of satellite data, Priester (ref. 24) reveals a seasonal variation in density in the 200 to 600 km region. The amplitude of this variation is only slightly less than the diurnal variation of density reported by Jacchia (ref. 9). The day density is higher than the night density, and the summer density as inferred by Priester is also greater than winter density for these altitudes. The large diurnal variation in density is the predominant observed characteristic of the atmosphere above 400 km altitude (see figure 5).
For altitudes up to radiosonde ceiling of about 30 km, the diurnal variation is small and can be neglected for most purposes.

**DENSITY MEASUREMENTS AND PREDICTION OF DENSITY - THIRD APPROACH**

Observational Methods for Determining Density

The conventional method of making atmospheric measurements for the first 30 km is the radiosonde. The world-wide network of regularly scheduled radiosonde measurements taken over the past two decades has made it possible to present the main features of the atmosphere structure, up to the lower stratosphere. With the ever-increasing demands imposed by the development of missiles and satellites there has been a rapid development of methods and techniques that extend the measurement of atmospheric quantities to higher and higher altitudes. The IGY has produced emphasis on coordinated scheduling and increasing the frequency of upper atmospheric measurements through rocket experiments. Some of the more recent developments in atmospheric measurements are summarized in table V.

In addition to the methods presented above, the observations of meteors by radar, doppler methods using pulse or continuous wave techniques, and radio propagation studies as well as visual and photographic observations have produced information on temperature, wind, turbulence, diffusion, and density in the altitude region from 80 to 100 km. Greenhow reports measurements for density (ref. 5), turbulence (ref. 6), and diffusion (ref. 4) using meteor tracer technique. This method yields large errors in the individual observation such that a large sample is required in order to reduce the physical measurement by statistical techniques.

It is noted that each of the techniques listed in table V is limited in altitude range, and accuracy of measurement, or both.

There must be an improvement in observational techniques to include measuring accuracy, altitude range, frequency of observations, and coordinated schedule of observations for a larger number of locations before an adequate description of the structure and behavior of the atmosphere can be made. These requirements are a prerequisite for prediction of atmospheric parameters.

**Prediction or Forecast for Density**

The ultimate goal would be a prediction technique that would be superior to that of established statistical means and variances obtained from historical data. The facts are that even with the present world-wide network of radiosonde observations a five-day forecast for specific atmospheric parameters is far from being a reality. The
<table>
<thead>
<tr>
<th>Basic Methods and Measurement</th>
<th>Primary Data Sought</th>
<th>Error</th>
<th>Secondary Data</th>
<th>Range of Altitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure gauge (side of rocket) (nose cone)</td>
<td>Pressure</td>
<td>Below 75 km; error less than 10% Above 75 km; error may be 100%</td>
<td>Density Temperature</td>
<td>30-120 km</td>
</tr>
<tr>
<td>Pressure stagnation (nose tip of rocket)</td>
<td>Density</td>
<td>Error less than 20%</td>
<td>Pressure Temperature</td>
<td>30-100 km</td>
</tr>
<tr>
<td>Supersonic flow around a right circular cone</td>
<td>Temperature</td>
<td>Below 50 km, ± 5K Above 50 km, ± 7-15K</td>
<td>Pressure Density</td>
<td>30-90 km</td>
</tr>
<tr>
<td>Pressure modulations on rolling rocket</td>
<td>Density</td>
<td>Errors varied between 20% and 100%</td>
<td></td>
<td>above 120 km</td>
</tr>
<tr>
<td>Rocket grenades (transit time of sound wave)</td>
<td>Temperature Winds Speed Direction</td>
<td>Less than 3% (± 5K) +10 m/sec +18 degrees</td>
<td></td>
<td>30-80 km 30-80 km</td>
</tr>
<tr>
<td>Falling sphere drag acceleration</td>
<td>Density</td>
<td>Less than 10%</td>
<td>Temperatures Pressure</td>
<td>30-100 km</td>
</tr>
<tr>
<td>Anomalous sound; sound wave refraction</td>
<td>Winds Temperature</td>
<td>±5 K</td>
<td></td>
<td>25-50 km</td>
</tr>
<tr>
<td>Searchlight-probing</td>
<td>Density</td>
<td>1 to 5%</td>
<td>Temperature Pressure</td>
<td>10-65 km</td>
</tr>
</tbody>
</table>

NOTE: Rocket experimental data may contain one or more errors due to: wind, yaw, outgassing, and spin.

(Ref. 32)
major contribution a forecast for the atmospheric density to the space vehicle re-entry problems would be to reduce the variance and bias which are inherent in the proposed second approach, i.e., the establishment of reference atmospheres.

It is in the high latitudes and in the altitude region near 50 km that the largest random* variation of density is known to occur. (At satellite altitudes variations in density have been associated with variations of solar flux (ref. 9), but the magnitude of the variations relative to those at 50 km has not been established.) There are occasions in late winter and early spring during which a large organized warm mass of air moves from east to west at high latitudes in the upper stratosphere. This phenomenon is referred to as abrupt warming, and was first reported by Sherhag in 1951 and 1952. One report (ref. 32) describing the occurrence of stratospheric warming over Fort Churchill during a two-day period, January 27-29, 1958, reports that the density at 50 km altitude increased 79% over that of January 27. An order of magnitude for the temperature variation during periods of stratospheric warmings is suggested to be 47K by Nordburg (ref. 22). It is premature to suggest that the magnitude and periods of these outbreaks could be predicted.

CONCLUSIONS AND RECOMMENDATIONS

The range of density deviation versus altitudes with respect to the ARDC 1959 Model Atmosphere can be summarized from figures 2 and 3 for specific altitudes as shown below in table VI.

Table VI

<table>
<thead>
<tr>
<th>Altitude</th>
<th>$\sqrt{a} = 10%$</th>
<th>$\sqrt{a} = 15%$</th>
<th>$\sqrt{a} = 20%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Km</td>
<td>%</td>
<td>%</td>
<td>%</td>
</tr>
<tr>
<td>0</td>
<td>10</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>20</td>
<td>15</td>
<td>22</td>
<td>29</td>
</tr>
<tr>
<td>50</td>
<td>25</td>
<td>32.5</td>
<td>50</td>
</tr>
<tr>
<td>80</td>
<td>44</td>
<td>66</td>
<td>87</td>
</tr>
<tr>
<td>120</td>
<td>92</td>
<td>137</td>
<td>182</td>
</tr>
</tbody>
</table>

*The reference to random variation as used here is to infer that the quantity is not sequentially predictable for a long period of time.
a. The 10% curve for range of density should be used for low latitudes below 35 degrees N both summer and winter, and for high latitudes above 55 degrees N in summer.

b. The 15% curve for range of density should be used for mid-latitudes summer and winter.

c. The 20% curve for range of density should be used for high latitudes in winter and for the entire Northern Hemisphere.

It is recommended that these idealized density curves be used to depict the amplitude of the density range relative to the ARDC 1959 Model Atmosphere for space re-entry studies until more precise knowledge of the structure of atmospheric density becomes available.

The fact that there are two altitude regions of maximum wind should be considered as a potential problem area relative to the space vehicle re-entry studies. These altitude regions for the Northern Hemisphere are 5 to 15 km, and 50 to 75 km. In these two regions, the mean wind for some latitudes can be in the order of magnitude of 50 m/sec and 100 m/sec, respectively.
Figure 1. Comparison of ARDC 1956 and 1959 Model Atmospheres

\[ \Delta X_{\text{rel}} \% = \frac{(X_{\text{ARDC 56}} - X_{\text{ARDC 59}}) \times 100}{X_{\text{ARDC 59}}} \]
Figure 2. Range of Density Deviation from ARDC 1959 Model Atmosphere for Northern Hemisphere

1. Low Lat. (Annual) from Nordberg’s “typical”
2. Hi Lat. Summer Profiles ref. 22
3. Hi Lat. Winter Profiles ref. 22
4. Estimated Limit of Density Range from ref. 1 and 32
5. From ref. 1 and 32
6. From ref. 27 and 29
7. From ref. 8
8. From ref. 17
\[ \Delta e_{\text{rel}} = \pm \sqrt{a} \left( e^b y \right) \]

- \( b = 0.01842 \text{ km}^{-1} \)
- \( y = \text{Altitude km} \)

Valid for altitude range: \( 0 = y \leq 125 \text{ km} \)

**Figure 3.** Idealized Range of Density Deviations from ARDC 1959 Model Atmosphere

**FIGURE 3**
Figure 4. Maximum and Minimum Density Variability Inferred from Thermoelectric Properties
Sources of Information

1. Jacchia ref. 9
2. Greenhow ref. 3
3. Nordberg ref. 22
4. Inferred from Temperature ref. 34
5. Wan-Cheng Chiu Ref. 33

Figure 5. Amplitude of Diurnal Density Variation, Day Positive Night Negative
BIBLIOGRAPHY


BIBLIOGRAPHY


BIBLIOGRAPHY


