

ANALYSIS OF PHOTOTHERMAL CHARACTERIZATION OF LAYERED MATERIALS— DESIGN OF OPTIMAL EXPERIMENTS ¹

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Abstract

In this paper numerical calculations are presented for the steady-periodic temperature in layered materials and functionally-graded materials to simulate photothermal methods for the measurement of thermal properties. No laboratory experiments were performed. The temperature is found from a new Green's function formulation which is particularly well-suited to machine calculation. The simulation method is verified by comparison with literature data for a layered material. The method is applied to a class of two-component functionally-graded materials and results for temperature and sensitivity coefficients are presented. An optimality criterion, based on the sensitivity coefficients, is used for choosing what experimental conditions will be needed for photothermal measurements to determine the spatial distribution of thermal properties. This method for optimal experiment design is completely general and may be applied to any photothermal technique and to any functionally-graded material.

KEY WORDS: functionally graded material; Green's functions; optimal experiment; photothermal; thermal properties

1 Introduction

Functionally-graded (FG) materials are being studied as possible components of aerospace thermal protection systems. These materials include composites with epoxy and metal matrices, metal foams, or any structure with properties designed to vary with position. In the future when FG materials are specified as part of a vehicle program, part of the procurement process will involve certification that the material meets the specifications.

To date there has been little research on accurate thermal characterization of FG materials. The present research is intended to close this gap in the procurement cycle by investigating photothermal methods for non-destructive and accurate measurement of thermal properties in FG materials. In this paper only numerical simulations are presented and no laboratory experiments were performed.

¹Paper presented at the Fifteenth Symposium on Thermophysical Properties, June 22-27, 2003, Boulder, Colorado, U.S.A.

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A review of the pertinent literature is given next in three areas: computer simulation of FG materials; heat transfer theory for photothermal methods; and, optimal experiment design.

Several researchers have found analytical solutions for the thermal response by representing a FG material as composed of multiple layers each with different, spatially uniform, thermal properties [1,2,3]. Other studies investigated an exponential-function variation of thermal properties along one spatial direction [4,5]. Another used Galerkin's method to find temperature in materials with arbitrary property distributions [6]. The primary motivation for these studies has been to determine temperature for the purpose of finding thermal stresses, or to find the distribution of thermal properties that optimizes the thermal stresses [7,8].

One research group has reported transient-heating experiments to measure thermal properties in a FG material [9,10]. This group studied a FG material containing an exponentially-varying spatial distribution of thermal properties. Their data analysis combines a single temperature datum with their transient theory to provide a single value for the parameter describing the spatial distribution of thermal properties. Although simple in concept, this approach is sensitive to measurement noise.

In the area of photothermal measurements, there are several pertinent publications. A diverse collection of thermal-wave Green's functions and temperature solutions has been published recently in book form [11]. Primarily homogeneous materials are treated, and layered materials are included by defining a global Green's function that embodies the effects of several layers in the material. Because the complexity of the layered-body Green's function increases rapidly as layers are added, no more than 3 layers are discussed.

Theory for many-layered bodies to laser heating has been studied previously by the author [12,13]. The volumetric heating is treated exactly from the optical absorption properties of all layers. The multi-layer body is treated efficiently by the use of local Green's functions which are found first in the time domain and are then transformed into the frequency domain. Each layer is linked to adjacent layers with appropriate interface conditions.

Theory for the photoacoustic response of a layered solid has been recently studied for which the temperature in each layer is linked with adjacent layers by interface conditions [14]. The optical absorption in each solid layer is described by an exponential distribution and an absorption coefficient. The photoacoustic response is found from both thermal effects and mechanical effects in the gas, however thermal effects predominate for solid materials. The method is used for analysis of experimental data in materials with 2 and 3 layers.

In the area of optimal experiment design, parameter estimation has been used for obtaining thermal properties from transient experiments for many years [15]. In these methods the desired parameters are found by non-linear regression between the experimental data (temperatures in this case) and a computational model of the experiment. Parameter estimation concepts have recently been applied to optimal experiment design for thermal characterization of uniform materials [16] and for materials with

temperature-varying properties [17]. The author has previously studied optimal experiment design for low-conductivity FG materials [18]. The simulated experiments involved time-series data collected from one or more temperature sensors, and the data analysis is carried out in the time domain. An optimality criterion was used to find the best experimental conditions for simultaneous estimation of several thermal properties. Results of simulations show that for FG materials with spatially-varying conductivity, it is better to heat the sample from the low-conductivity side.

In the present paper, FG materials are simulated with a large number of interconnected layers. The heat transfer theory draws upon the author's previous work with Green's functions, but here the Green's functions are given directly in the frequency domain in the form of algebraic expressions, not infinite-series expressions, that are numerically well behaved under all conditions. Likewise the temperature expressions found from these Green's functions are numerically well behaved. The Green's functions are given for a variety of boundary conditions; previously only specified-flux boundaries were treated. To the author's knowledge this paper describes the first application of optimal experiment design methods to frequency-domain analysis appropriate for photothermal experiments. This paper is divided into several sections, as follows: the temperature in one layer; the Green's functions; the temperature in a multi-layer material; the design of optimal experiments; results for a layered material; results for a FG material; and, a brief summary.

2 Temperature in one layer

Since the photothermal applications of interest involve periodic heating by a laser, the solution is sought in Fourier-transform space, and the solution is interpreted as the steady-periodic response at a single frequency ω . For a discussion of this point see [19]. Consider the heat conduction equation in Fourier transform space in one layer:

$$\frac{\partial^2 T}{\partial x^2} - \frac{j\omega}{\alpha} T = -\frac{1}{k} g(x, t); \quad 0 < x < L \quad (1)$$

$$k_i \frac{\partial T}{\partial n_i} + h_i T = f_i(\omega); \quad \text{at boundaries } i = 1, 2 \quad (2)$$

Here T is Fourier-space temperature (K s), α is thermal diffusivity (m^2/s), k is thermal conductivity ($\text{W}/\text{m}/\text{K}$), g is volume heating ($\text{W s}/\text{m}^3$) deposited by a laser, and f_i is a specified boundary condition. Index $i = 1, 2$ represents the boundaries at the limiting values of coordinate x . The boundary condition may be one of three types at each boundary: for type 1 f_i is a specified temperature ($k_i = 0$ and $h_i = 1$); for type 2 f_i is a specified heat flux ($h_i = 0$); and, type 3 represents a convection condition where h_i is a constant-with-time heat transfer coefficient (or contact conductance).

The temperature will be found with the Fourier-space Green's function, defined by the following equations:

$$\frac{\partial^2 G}{\partial x^2} - \sigma^2 G = -\frac{1}{\alpha} \delta(x - x') \quad (3)$$

$$k_i \frac{\partial G}{\partial n_i} + h_i G = 0; \quad i = 1, 2 \quad (4)$$

Here $\sigma^2 = j\omega/\alpha$ and $\delta(x-x')$ is the Dirac delta function. The coefficient $1/\alpha$ preceding the delta function in Eq. (3) provides the frequency-domain Green's function with units of seconds/meters. This is consistent with our earlier work with time-domain Green's functions.

Assume for the moment that the Green's function G is known, then the steady-periodic temperature is given by the following integral equation (see [20], p. 40-43):

$$T(x, \omega) = \frac{\alpha}{k} \int g(x', \omega) G(x, x', \omega) dx' \quad (\text{for volume heating}) \\ + \alpha f_i(\omega) \times \begin{cases} \frac{\partial G}{\partial n'}(x, x_i, \omega) & (\text{type 1 only}) \\ \frac{1}{k} G(x, x_i, \omega) & (\text{type 2 or 3}) \end{cases} \quad i = 1, 2 \quad (5)$$

3 Green's Function

The Green's function (GF) that satisfies Eqs. (3) and (4) is given by

$$G(x, x', \omega) = \frac{S_2^- (S_1^- e^{-\sigma(2L-|x-x'|)} + S_1^+ e^{-\sigma(2L-x-x')})}{2\alpha\sigma(S_1^+ S_2^+ - S_1^- S_2^- e^{-2\sigma L})} \\ + \frac{S_2^+ (S_1^+ e^{-\sigma(|x-x'|)} + S_1^- e^{-\sigma(x+x')})}{2\alpha\sigma(S_1^+ S_2^+ - S_1^- S_2^- e^{-2\sigma L})} \quad (6)$$

where the subscripts 1 and 2 represent the two boundaries at the smallest and largest x -values, respectively. Coefficients S_M^+ and S_M^- depend on the boundary conditions on side M and are given by

$$S_M^+ = \begin{cases} 1 & \text{if side } M \text{ is type 0, type 1, or type 2} \\ k\sigma + h_M & \text{if side } M \text{ is type 3} \end{cases} \\ S_M^- = \begin{cases} 0 & \text{if side } M \text{ is type 0} \\ -1 & \text{if side } M \text{ is type 1} \\ 1 & \text{if side } M \text{ is type 2} \\ k\sigma - h_M & \text{if side } M \text{ is type 3} \end{cases}$$

A boundary of type 0 designates a far-away boundary, as in a semi-infinite body. The derivation of the Fourier-space GF in Eq. (6) parallels that for steady-state GF given elsewhere [21]; however in the present work σ is complex.

This form of the GF is particularly well-behaved for machine computation, and most importantly, the temperature expressions based on these GF are similarly well-behaved for any layer thickness and for any frequency. This is a key contribution of this paper, in sharp contrast with previously reported difficulties in evaluating numerical values from exact solutions. For example, numerical overflow can occur with other formulations in thermally thick layers [14]. In a time-domain study, only short-time results were included due to numerical difficulties associated with longer times [3].

The GF expression given in Eq. (6) covers a number of boundary condition combinations, and a numbering system is used to distinguish among them. Designation XIJ is used to identify the GF for heat transfer in a layer with boundary condition of type $I = 1, 2, \text{ or } 3$ at $x = 0$ and with boundary condition of type $J = 1, 2, \text{ or } 3$ at $x = L$. For example, designation X12 represents the GF with type 1 boundary at $x = 0$ and type 2 boundary at $x = L$. Designation XI0 is used to identify the GF for a semi-infinite region with a boundary of type $I = 1, 2, \text{ or } 3$ at $x = 0$.

4 Temperature in Layered Materials

In this section the temperature caused by absorption of laser energy will be found in a domain consisting of non-absorbing air, N solid layers, and a substrate. At the interfaces, let q_{nm} represent the heat flux leaving layer n and entering layer m . Applying Eq. (5), the interface temperature in the air is:

$$T_0(0, \omega) = \frac{\alpha_0}{k_0} G_0(0, 0, \omega) q_{10} \quad (7)$$

In layer i ; $i = 1, 2, \dots, N$: the interface temperatures are:

$$T_i(0, \omega) = \frac{\alpha_i}{k_i} G_i(0, 0, \omega) q_{i-1,i} + \frac{\alpha_i}{k_i} G_i(0, L_i, \omega) q_{i+1,i} + B_i(0) \quad (8)$$

$$T_i(L_i, \omega) = \frac{\alpha_i}{k_i} G_i(L_i, 0, \omega) q_{i-1,i} + \frac{\alpha_i}{k_i} G_i(L_i, L_i, \omega) q_{i+1,i} + B_i(L_i) \quad (9)$$

In the substrate the temperature at the interface is:

$$T_{N+1}(0, \omega) = \frac{\alpha_{N+1}}{k_{N+1}} G_{N+1}(0, 0, \omega) q_{N,N+1} + B_{N+1}(0) \quad (10)$$

In the above expression, symbol B_i has been used for the volume-heating integral term from Eq. (5), specifically,

$$B_i(x) = \frac{\alpha_i}{k_i} \int_{x'} g(x', \omega) G_i(x, x', \omega) dx' \quad (11)$$

Here g is the laser energy absorbed in the layer per unit volume; this can be determined without approximation from the optical properties of the layers [12].

In the above temperature expressions, all of the interface heat fluxes are initially unknown. The heat flux leaving one layer enters the adjacent layer, $q_{i-1,i} = -q_{i,i-1}$ and the temperature difference between adjacent layers is related to heat flux through a contact resistance at each interface:

$$q_{i-1,i} R_i = T_i(0, \omega) - T_{i-1}(L_{i-1}, \omega); \quad i = 1, 2, \dots, N + 1 \quad (12)$$

The contact resistance R_i describes the size of the temperature jump across the interface. Next Eqs. (7-10) are combined with Eq. (12) to eliminate temperature. The

result is a set of $N + 1$ linear algebraic equations for the unknown heat fluxes, which may be stated in matrix form:

$$\begin{bmatrix} W_0 + U_1 + R_1 & -V_1 & 0 & \dots & 0 \\ -V_1 & U_1 + U_2 + R_2 & -V_2 & \dots & 0 \\ 0 & -V_2 & U_2 + U_3 + R_3 & \dots & 0 \\ \dots & \dots & \dots & \ddots & -V_N \\ 0 & 0 & \dots & -V_N & U_N + W_{N+1} + R_{N+1} \end{bmatrix} \times \begin{bmatrix} q_{10} \\ q_{21} \\ q_{32} \\ \dots \\ q_{N+1,N} \end{bmatrix} = \begin{bmatrix} B_1(0) \\ B_2(0) - B_1(L_1) \\ B_3(0) - B_2(L_2) \\ \dots \\ B_{N+1}(0) - B_N(L_N) \end{bmatrix} \quad (13)$$

Symbols W_i , U_i , and V_i used in the above expression are given below:

$$\begin{aligned} W_i &= \frac{\alpha_i}{k_i} G_i(0, 0, \omega) \\ U_i &= \frac{\alpha_i}{k_i} G_i(0, 0, \omega) = \frac{\alpha_i}{k_i} G_i(L_i, L_i, \omega) \\ V_i &= \frac{\alpha_i}{k_i} G_i(0, L_i, \omega) = \frac{\alpha_i}{k_i} G_i(L_i, 0, \omega) \end{aligned} \quad (14)$$

For any multilayered system, it is now possible to calculate the heat fluxes (q_{ij}) through all interfaces in the system. The above result is *exact*, and Cramer's rule may be used to solve for the q 's for a sample composed of one or two layers. For a sample with two or more layers, a numerical matrix solution is best. Once the heat fluxes are found, the temperature at any interface is given by Eq. (8 - 10), or the temperature within any layer may be found with Eq. (5).

Several different GF may be used in the above matrix equation. The non-absorbing gas (region 0) is a semi-infinite region so the GF needed is number X20. For layers $i = 1, 2, \dots, N$ the GF needed are type X22 (specified heat flux). The GF for the substrate depends on the heat transfer environment there. For example, a thick substrate could be described by GF number X20, or, a substrate in imperfect contact with a cold plate could be described by GF number X23.

5 Optimal Experiment Design

Sensitivity coefficients are central to the design of optimal experiments for thermal property evaluation. The sensitivity coefficients are defined by

$$X_{jk(i)} = b_k \frac{\partial T_j}{\partial b_k} \quad (15)$$

which is the sensitivity for the k th parameter, the j th temperature sensor, and the i th frequency. Parameters b_k may include conductivity, specific heat, density, etc. In this

research the sensitivity matrix has been computed from the real-valued amplitude and real-valued phase of the complex temperature at r frequencies, for which the distinct amplitude and phase values are treated as $2r$ measurements from each sensor.

The sensitivity coefficients were computed with a finite-difference procedure to approximate the derivative, as follows:

$$X_{jk}(i) \approx b_k \frac{[T_{ij}((1 + \epsilon)b_k) - T_{ij}(b_k)]}{\epsilon b_k} \quad (16)$$

Here T_{ij} is the temperature at the i th frequency for the j th sensor. The value of $\epsilon = 0.001$ was found to give well-behaved values for X .

There are two specific requirements that the sensitivity coefficients must satisfy. First, the sensitivity coefficients should be as large as possible. Second, when two or more parameters are to be measured in the same experiment, the sensitivity coefficients must be linearly independent. A formal procedure to quantify these two requirements may be constructed if the sensitivity coefficients are assembled into a sensitivity matrix \mathbf{X} , which is then multiplied by its transpose, given formally by $\mathbf{X}^T \mathbf{X}$, of size $[p \times p]$. The optimality criterion is the (normalized) determinant of matrix $\mathbf{X}^T \mathbf{X}$, given by [16]

$$D = \frac{1}{s r (T_{max})^2} \det(\mathbf{X}^T \mathbf{X}) \quad (17)$$

Note that optimality criterion D is normalized by the maximum temperature rise (squared), the number of sensors s , and the number of frequencies r . This is important so that D may be used to compare different experiments. When D is large, the sensitivity coefficients will be large and linearly independent [17].

6 Results for a Layered Material

In this section the techniques of simulation and optimal experiment design are applied to a SiO_2 film on a Si substrate. This material has been studied elsewhere by a photoacoustic technique over a frequency range from 2 to 20 kHz [14]. An opaque coating of Ni of 20 nm thickness is added to the sample to improve the optical absorption. Published results for the thermal conductivity of the SiO_2 film and the contact resistance between the film and the Si substrate are: $k = 1.52 \text{ W/m/K}$ and $R < 10^{-9} \text{ K m}^2/\text{W}$.

Using the present methods, the computed phase of the temperature agreed with published values, thus verifying the present approach. The sensitivity coefficients for the phase of the surface temperature to variations in the conductivity of the the SiO_2 layer, k , and the contact resistance with the Si substrate, R , are plotted in Fig. 1. The property values used are given in Table 1. The sensitivity to conductivity k has a maximum (negative) value at about 6000 Hz. The shape of the R -sensitivity curves and the location of the largest value depends strongly on the R value. For $R = 10^{-8}$ and $R = 10^{-7}$, the sensitivity curves are relatively small, with a broad peak near 3

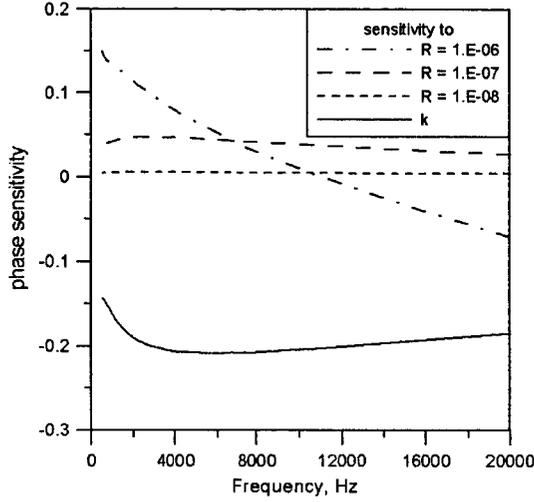


Figure 1: Phase sensitivity to thermal conductivity and to layer-substrate contact resistance for a layer of SiO_2 on silicon at various frequencies.

kHz. For $R = 10^{-6}$ however, the sensitivity values are large and positive at small frequencies and the curve slopes down to negative values as frequency increases. Some conclusions for $R < 10^{-7}$ are that the sensitivity coefficient is small, but the frequency range considered (0.5 - 20 kHz) captures the peak sensitivity. For $R = 10^{-6}$ the largest sensitivity may lie outside this frequency range.

Table 1. Properties used in calculations of a three-layer solid which is heated at the Ni surface and exposed to a layer of air on either side.

layer	i	d (μm)	k (W/m/K)	α (m^2/s)	R ($\text{K m}^2/\text{W}$)
air	0	–	0.0263	2.25E-05	–
Ni	1	0.07	80	1.98E-05	0.
SiO_2	2	0.485	1.52	9.09E-07	0.
Si	3	382	151	9.09E-04	varies
air	4	–	0.0263	2.25E-05	–

Next the sensitivities for k and R will be examined together. Optimality criterion D provides a numerical measure of the extent to which the sensitivity coefficients are both large and linearly independent, and at which frequencies this occurs. Figure 2 shows the optimality criterion for the layered material for both conductivity and contact resistance considered together. The highest curve is for k and $R = 10^{-6}$, and the peak for this curve is about 4 kHz. For the $R = 10^{-7}$ and $R = 10^{-8}$ curves the peak occurs around 8 kHz. Figure 2 indicates that the best experiment to measure both k and R when $R = 10^{-6}$ includes data at 4 kHz. For smaller R -values the most important frequency is 8 kHz, however because D is smaller the analysis of the data

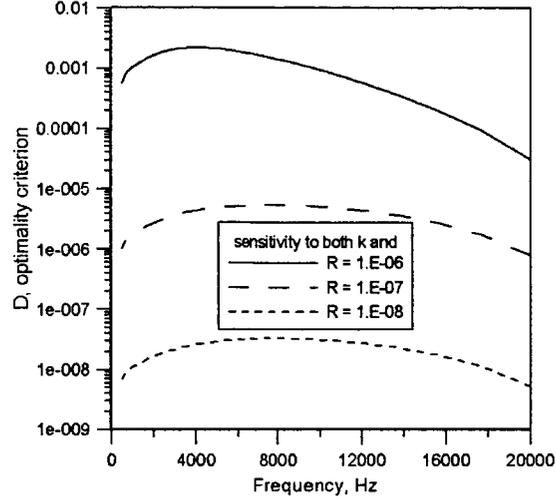


Figure 2: Optimality criterion D for simultaneous estimation of thermal conductivity and contact resistance, versus frequency, for a layer of SiO_2 on silicon.

may be more difficult. It is important to note that each point on Fig. 2 represents a value for D computed from a range of data extending from 20 kHz down to that point; data is added from high frequency to low frequency. To repeat, the D -values indicate what frequency range is needed for optimal estimation of both k and R from experimental data.

7 Results for a Functionally Graded Material

In this section the methods for experimental design are applied to a two-phase ceramic/ceramic material with graded volume fraction of the components. The volume fraction profile is assumed to have the form:

$$V_1(x) = 1 - (x/L)^p; \quad V_2(x) = (x/L)^p \quad (18)$$

where V_1 and V_2 are the volume fraction of the components. At location $x = 0$ the material is pure component 1 and at $x = L$ the material is pure component 2. The particular material considered is composed of TiC and SiC and the thermal properties are given in Table 2.

Table 2. Material properties of TiC and SiC.

	material	k (W/m/K)	ρ (kg/m ³)	c (J/kg/K)
1	TiC	20	4900	700
2	SiC	60	3200	1000

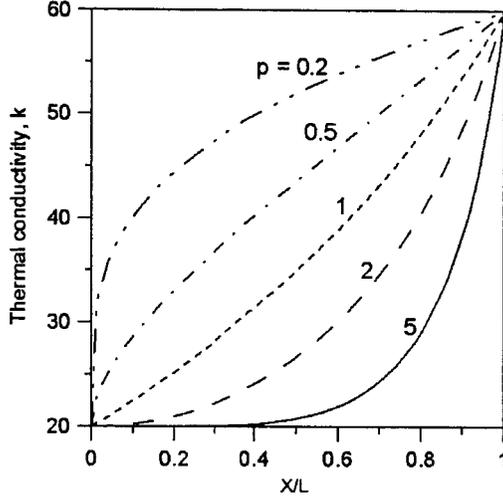


Figure 3: Thermal conductivity distribution in a functionally-graded material for several values of distribution parameter p .

The thermal conductivity of the material is given by

$$k(x) = k_1 \left[1 + \frac{V_2(x)(k_2 - k_1)}{k_1 + (k_2 - k_1)V_1(x)/3} \right] \quad (19)$$

where subscripts 1 and 2 stand for the properties of components 1 and 2, respectively. Figure 3 shows the spatial distribution of conductivity for the TiC/SiC material for several values of exponent p . The mass density and specific heat are determined by the rule of mixtures:

$$\rho(x) = V_1(x)\rho_1 + V_2(x)\rho_2 \quad (20)$$

$$c(x) = V_1(x)c_1 + V_2(x)c_2 \quad (21)$$

The thermal-stress behavior of this material has previously been studied [3].

In photothermal methods, the sample is heated by a periodically modulated laser beam, and the surface temperature (or a subsequent acoustic signal) is measured at the modulation frequency. The temperatures reported below are computed with the layered GF method with 50 layers used to simulate the spatial variation in the sample. Both surfaces of the sample are exposed to air. The surface temperature is shown in Fig. 4a (amplitude) and Fig. 4b (phase) versus dimensionless frequency. The frequency is normalized as $f^* = fL^2/\alpha_{av}$ where L is the material thickness and $\alpha_{av} = (\alpha_1 + \alpha_2)/2$, the average of the component values. In Fig. 4 the sample is heated at $x = 0$ and the temperature is reported at $x = 0$, the low- k side. This heating condition provides slightly higher temperature response than for heating on the high- k side (at $x = L$). Note that in Fig. 4a the amplitude curves are monotonic with few distinguishing features. In contrast the phase has a distinctive maximum

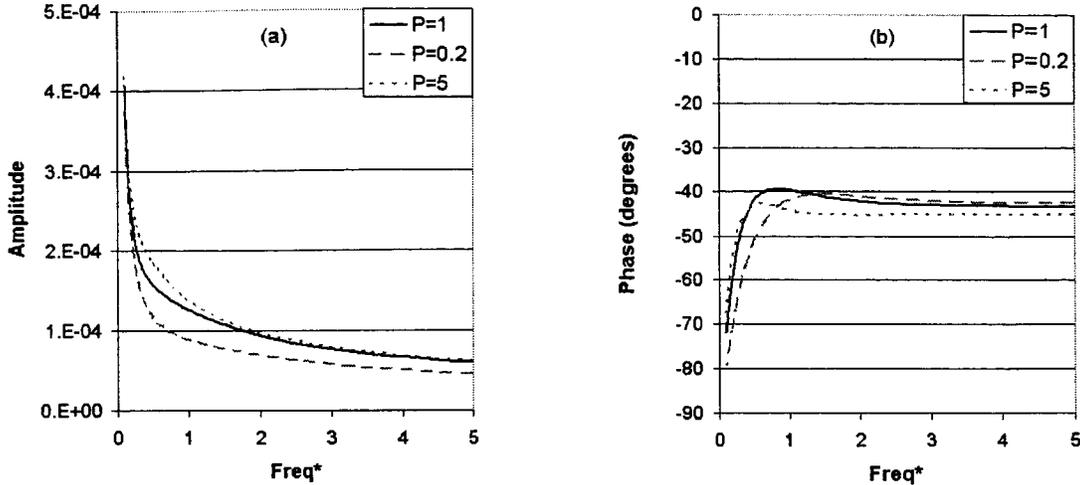


Figure 4: Surface temperature (a) amplitude, and (b) phase, versus dimensionless frequency for a functionally-graded material heated periodically at the surface.

for each p value, which supports the experimental observation that phase is more important than amplitude for photothermal measurement of thermal properties.

Figure 5a shows the sensitivity of the phase of temperature to k_2 , the thermal conductivity of component 2, for three values of spatial distribution parameter p . Note that the sensitivity is largest for $p = 1$, the linear k -distribution, and is small for other distributions. In Fig. 5a the peak sensitivities in a range of dimensionless frequencies below $f^* = 1$. This range of frequencies represents thermal waves (generated by the periodic heating) that penetrate all the way through the sample thickness. The sensitivity to k_1 , the component 1 thermal conductivity, is not shown because it is similar in size and shape to the k_2 sensitivities (but values are negative).

Figure 5b shows the sensitivity of the phase of the temperature to spatial distribution parameter p . The curve for $p = 1$ has a large positive peak at about $f^* = 0.2$ and a negative peak at about $f^* = 1.5$. The largest sensitivities are again located in the range $f^* < 1$.

The sensitivity to a single parameter is useful for determining a single parameter from an experiment. When two or more parameters are to be determined, optimality criterion D is instructive. Figure 6a shows optimality criterion D for both conductivity k_2 and exponent p . Both amplitude and phase information was used to compute these values. The largest curve is for $p = 1$ which indicates that this spatial distribution will provide better estimates of thermal properties k_2 and p from an experiment than for other p -values. The location of the peak for each curve indicates which frequencies should be included in an experiment. Since the D -values shown were computed from high-to-low frequencies, the peak indicates the lowest frequency of data that is needed for optimal data analysis.

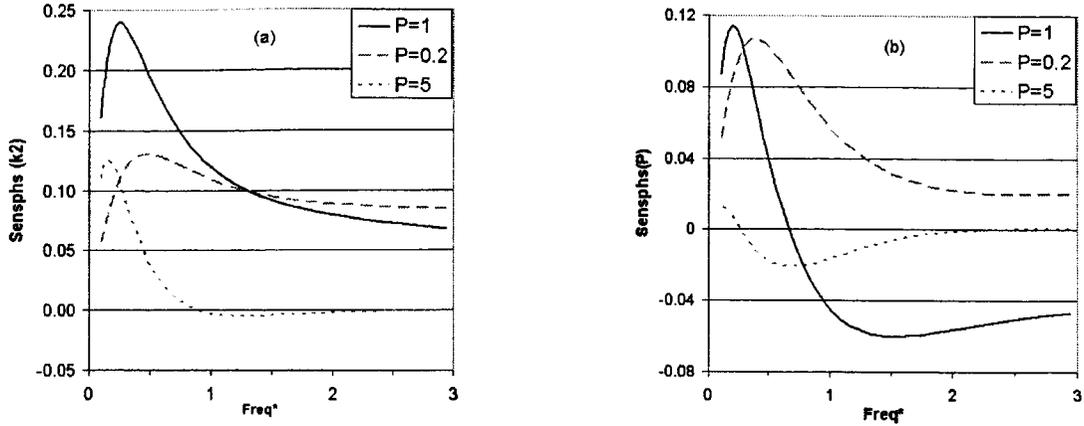


Figure 5: Sensitivity of the phase of temperature to (a) conductivity k_2 , and (b) exponent p , for the functionally-graded material heated periodically at the surface.

Optimality criterion D was also investigated for other combinations of parameters. The D -values computed for p and k_1 together, not shown, were identical to Fig. 6a, which is expected from the similarity in the shape of the sensitivities to k_1 and k_2 mentioned earlier. The D -values computed for k_1 and k_2 together are shown in Fig. 6b. The important feature of this figure is the vanishingly small values, five orders of magnitude smaller than Fig. 6a values, which indicates that k_1 and k_2 should not be sought simultaneously. As mentioned before the sensitivity coefficients for k_1 and k_2 have a similar shape, so the fact that their combined D -value is near zero reinforces the idea that the sensitivities for k_1 and k_2 are not linearly independent. Finally, D -values were also computed for p , k_1 , and k_2 considered simultaneously (not shown), and these values were predictably near zero because of the dependence of k_1 and k_2 .

8 Summary

This paper investigates photothermal methods for thermal characterization of functionally graded materials through numerical modeling and experiment design. No laboratory experiments are reported. A new formulation is given for the temperature response of layered materials to periodic heating, based on the method of Green's functions, which is numerically better behaved than previous work. The method has been applied to a SiO_2 layer on silicon and compared to literature values to validate the method. Optimality criterion D indicates which frequency range of experimental data should provide the best possible estimates of the layer conductivity and contact conductance.

The new methods have also been applied to a two-component functionally-graded material with a power-law distribution of thermal properties. The largest temperature response is found by heating the sample on the low- k side, and the phase of the tem-

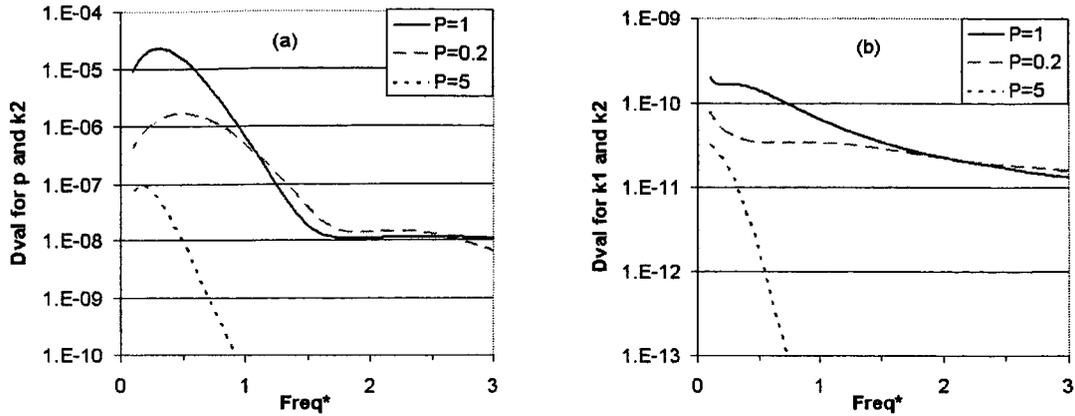


Figure 6: Optimality criterion D for simultaneous estimation of (a) conductivity k_2 distribution parameter p , and (b) conductivities k_1 and k_2 , for the functionally-graded material.

perature is particularly important for estimation of thermal properties. Component conductivities k_1 and k_2 have similar-shaped sensitivity coefficients, and consequently both cannot be estimated simultaneously from experimental data. The most important parameter is p which describes the spatial distribution of thermal properties in the functionally-graded material. Values for optimality criterion D indicate that values for p may be found simultaneously with one of the conductivities, but not both. Dimensionless frequencies in the range $f^* < 1$ are important for measurement of spatial-distribution parameter p . The magnitude of the optimality criterion D also suggests that it will be easier to estimate parameters for $p \approx 1$ (near-linear spatial variation) compared to other values of p .

Acknowledgement

This work was supported by NASA Langley grant NAG-1-01087 under the supervision of Max Blosser. The author would like extend thanks to Xinwei Wang for helpful discussions of photoacoustic methods and to Ryan Sneed for carrying out many numerical simulations. Mr. Sneed's salary was provided by the Undergraduate Creative Activity and Research Experiences (UCARE) program at the University of Nebraska.

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