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# TECHNICAL NOTE

D-803

# EFFECTS OF MASS-LOADING VARIATIONS AND APPLIED MOMENTS ON MOTION AND CONTROL OF A MANNED ROTATING SPACE VEHICLE

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON May 1961

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

TECHNICAL NOTE D-803

# EFFECTS OF MASS-LOADING VARIATIONS AND APPLIED MOMENTS ON MOTION AND CONTROL OF A MANNED ROTATING SPACE VEHICLE

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## SUMMARY

An analytical study has been made to determine the effects of mass-loading variations and onboard rotating machinery on a hypothetical earth-satellite space station, rotating to provide an artificial gravity equal to one-fourth of that at the earth's surface. Attempts were also made to damp out or minimize undesirable motions by using mass shifts, constant-rate inertia wheels, or jet-reaction moments.

Results obtained indicate that the shifting of masses within the rotating space station could bring about large roll oscillations ($\pm 100^{\circ}$) or even continuous rolling motions if the craft is rotating about the axis of intermediate moment of inertia. The pitch angles obtained were generally small ($< \pm 1^{\circ}$). The amplitudes of the roll and pitch oscillations are dependent upon the angle of displacement of the greatest principal axis of inertia from the initial spin axis.

In attempting to damp out or minimize undesirable motions, it was found that a constant-rate inertia wheel located on and rotating about the axis about which the craft is rotating (Z-axis) was beneficial in keeping the roll angles relatively small, provided it had a sufficient amount of angular momentum. It was also found that the use of jet-reaction moments was very satisfactory for damping undesirable motions in that the roll oscillations could be damped.


## INTRODUCTION

The movement of personnel or equipment on a rotating orbital space station may lead to dynamic cross-coupling moments due to simultaneous angular velocities about more than one principal axis of the space station. These moments in turn may cause significant divergencies from the intended single-axis rotary motion of the craft. Accordingly, a study was undertaken to determine the effects of such moments brought into play by mass-loading variations on a hypothetical earth-satellite space station, rotating to provide for its occupants an artificial gravity equal to one-fourth of that at the earth's surface. The space station is assumed to be of cylindrical shape with a weight of 7,000 pounds, a length of 50 feet (X-axis), and a diameter of 10 feet (Y- and Z-axes). Figure 1 shows a sketch of the configuration.

In addition to determining the effects of mass-loading variations simulating those due to a movement of personnel or equipment, the effects of onboard rotating machinery were investigated. Attempts were also made to damp out or minimize undesirable motions by using mass shifts, constant-rate inertia wheels, or jet-reaction moments.

## SYMBOLS

The angular motions of the space station are about the body system of axes. This system of axes and related Eulerian angles are illustrated in figure 2.

| | |
|---|---|
| $c$ | angle between a body axis and its respective principal axis, deg |
| $I$ | mass moment of inertia, slug-ft$^2$ |
| $I_X,I_Y,I_Z$ | moment of inertia about X-, Y-, and Z-axis, respectively, slug-ft$^2$ |
| $I_{XY}$ | product of inertia about X- and Y-axes, positive when a point on X'-axis has positive components along both X- and Y-axes, slug-ft$^2$ |
| $I_{XZ}$ | product of inertia about X- and Z-axes, positive when a point on X'-axis has positive components along both X- and Z-axes, slug-ft$^2$ |
| $I_{YZ}$ | product of inertia about Y- and Z-axes, positive when a point on Y'-axis has positive components along both Y- and Z-axes, slug-ft$^2$ |
| $I_{X,m}\omega_m$ | angular momentum of masses rotating about X-axis within craft, positive when masses rotate clockwise when viewed from negative end of X-axis, ft-lb-sec |
| $I_{Y,m}\omega_m$ | angular momentum of masses rotating about Y-axis within craft, positive when masses rotate clockwise when viewed from negative end of Y-axis, ft-lb-sec |

| | |
|---|---|
| $I_{Z,m}\omega_m$ | angular momentum of masses rotating about Z-axis within craft, positive when masses rotate clockwise when viewed from negative end of Z-axis, ft-lb-sec |
| $k$ | radius of gyration, ft |
| $k_X, k_Y, k_Z$ | radius of gyration about X-, Y-, and Z-axis, respectively, ft |
| $M_X, M_Y, M_Z$ | moment (produced by rocket) about X-, Y-, and Z-axis, respectively, ft-lb |
| $m$ | mass of a movable object in craft, slugs |
| $m_t$ | total mass of craft, 217.594 slugs |
| $p,q,r$ | angular velocity about X-, Y-, and Z-axis, respectively, radians/sec |
| $X,Y,Z$ | body axes |
| $X',Y',Z'$ | principal axes |
| $x_1, y_1, z_1$ | coordinate along X-, Y-, and Z-axis, respectively, indicating location of mass before it is moved, positive when on positive end of respective body axis, ft |
| $x_2, y_2, z_2$ | coordinate along X-, Y-, and Z-axis, respectively, indicating location of mass after it is moved, positive when on positive end of respective body axis, ft |
| $x_3, y_3, z_3$ | coordinate along X-, Y-, and Z-axis, respectively, indicating location of center of gravity of craft before mass is moved, positive when on positive end of respective body axis, ft |
| $x_4, y_4, z_4$ | coordinate along X-, Y-, and Z-axis, respectively, indicating location of center of gravity of craft after mass is moved, positive when on positive end of respective body axis, ft |
| $\theta_e$ | total angular movement of X-axis from horizontal plane measured in vertical plane, positive when front of vehicle is above horizontal plane, radians or deg |

$\phi_e$ total angular movement of Y-axis from horizontal plane measured in YZ-plane, positive when clockwise as viewed from rear of vehicle (if X-axis is vertical, $\phi_e$ is measured from reference position in horizontal plane), radians or deg

$\psi_e$ horizontal component of total angular deflection of X-axis from reference position in horizontal plane, positive when clockwise as viewed vertically from above vehicle, deg or radians

Subscript:

O value present at geometric center of craft (reference point) before mass is moved

A dot over a symbol represents a derivative with respect to time.

## METHODS AND CALCULATIONS

A digital computer was utilized for the calculations of this investigation.

Three degrees of freedom in rotation were used and the moment equations of motion used to correspond with these degrees of freedom are as follows:

$$I_X\dot{p} = I_{XY}\dot{q} + I_{XZ}\dot{r} - q\left(I_Z r - I_{YZ}q - I_{XZ}p\right) + r\left(I_Y q - I_{YZ}r - I_{XY}p\right) + I_{Y,m}\omega_m r - I_{Z,m}\omega_m q + M_X \tag{1}$$

$$I_Y\dot{q} = I_{YZ}\dot{r} + I_{XY}\dot{p} - r\left(I_X p - I_{XY}q - I_{XZ}r\right) + p\left(I_Z r - I_{YZ}q - I_{XZ}p\right) - I_{X,m}\omega_m r + I_{Z,m}\omega_m p + M_Y \tag{2}$$

$$I_Z\dot{r} = I_{YZ}\dot{q} + I_{XZ}\dot{p} - p\left(I_Y q - I_{XY}p - I_{YZ}r\right) + q\left(I_X p - I_{XY}q - I_{XZ}r\right) + I_{X,m}\omega_m q - I_{Y,m}\omega_m p + M_Z \tag{3}$$

In addition to these equations of motion, the following associated formulas were used:

$$\dot{\theta}_e = q \cos \phi_e - r \sin \phi_e \tag{4}$$

$$\dot{\phi}_e = p + r \tan \theta_e \cos \phi_e + q \tan \theta_e \sin \phi_e \tag{5}$$

$$\dot{\psi}_e = \frac{r \cos \phi_e + q \sin \phi_e}{\cos \theta_e} \tag{6}$$

It should be noted that, when an attempt is made to visualize the motions obtained, it may be helpful to consider the Euler angles in the order $\psi_e$, $\theta_e$, and $\phi_e$. The angular motions of the space station about its center of gravity were considered to be independent of the linear velocity in orbit; hence, no force equations of motion were included.

If the configuration is assumed to be initially symmetrical, the original moments of inertias $I_X$, $I_Y$, and $I_Z$ can be estimated from $I = m_t k^2$. If the radii of gyration $k_X$ and $k_Y$ $\left(k_Y = k_Z\right)$ are taken to be 2 feet and 10 feet, respectively, the moments of inertia and products of inertia for the 7,000-pound space station are

$$I_X = 870 \text{ slug-ft}^2$$

$$I_Y = I_Z = 21,760 \text{ slug-ft}^2$$

$$I_{XY} = I_{XZ} = I_{YZ} = 0$$

To compute new moments and products of inertia when a weight was relocated in the space station, the following formulas were used:

$$I_X = I_{X,0} - m\left(y_1^2 + z_1^2\right) + m\left(y_2^2 + z_2^2\right) - m_t\left(y_4^2 + z_4^2\right) \tag{7}$$

$$I_Y = I_{Y,o} - m\left(x_1^2 + z_1^2\right) + m\left(x_2^2 + z_2^2\right) - m_t\left(x_4^2 + z_4^2\right) \quad (8)$$

$$I_Z = I_{Z,o} - m\left(x_1^2 + y_1^2\right) + m\left(x_2^2 + y_2^2\right) - m_t\left(x_4^2 + y_4^2\right) \quad (9)$$

$$I_{XY} = I_{XY,o} - m\left(x_1 y_1\right) + m\left(x_2 y_2\right) - m_t\left(x_4 y_4\right) \quad (10)$$

$$I_{XZ} = I_{XZ,o} - m\left(x_1 z_1\right) + m\left(x_2 z_2\right) - m_t\left(x_4 z_4\right) \quad (11)$$

$$I_{YZ} = I_{YZ,o} - m\left(y_1 z_1\right) + m\left(y_2 z_2\right) - m_t\left(y_4 z_4\right) \quad (12)$$

The location of the center of gravity of the craft after the mass has been moved $\left(x_4,\ y_4,\ \text{and}\ z_4\right)$ was computed from

$$x_4 = \frac{mx_2 - mx_1 + m_t x_3}{m_t} \quad (13)$$

$$y_4 = \frac{my_2 - my_1 + m_t y_3}{m_t} \quad (14)$$

and

$$z_4 = \frac{mz_2 - mz_1 + m_t z_3}{m_t} \quad (15)$$

As a result of utilizing figure 2, where the body system of axes is shown, and equations 7 to 12, the following two basic observations were made as regards the effects of relocating weights inside the space station on the moments of inertia and products of inertia:

(1) Moving weights located on one body axis along the axis toward the center of gravity of the craft reduces the magnitude of the moments

of inertia about the other two axes. Conversely, of course, moving weights along this axis away from the center of gravity increases the moments of inertia about the other two axes. For example, if a 200-pound object is moved from $z_1 = 4$ feet along the Z-axis to $z_2 = 1$ foot, $I_X$ and $I_Y$ would be decreased by 92 slug-ft$^2$.

(2) Moving a weight in the craft within a given plane causes the principal axes to shift so that the principal axes and their respective body axes no longer coincide; this weight shift produces a product of inertia about the respective body axes. For example, if a 200-pound object is moved from $(x_1 = 1,\ z_1 = 1)$ in the XZ-plane to $(x_2 = 10,\ z_2 = 3)$, a product of inertia $I_{XZ}$ of 177 slug-ft$^2$ would be produced. Similarly, moving this weight in the XY-plane produces a product of inertia $I_{XY}$, and moving it in the YZ-plane produces $I_{YZ}$. Also, if moving this weight involves more than one body plane, more than one product of inertia will result.

To determine the effects of mass-loading variations, an initial attitude and motion of the rotating craft were assumed for an initial rotation about the Z-axis (referred to as yawing). (See table I.) The input data for each calculation are listed in table II. When the vehicle motions were calculated, the abrupt insertion of one or more products of inertia and/or a term representative of the gyroscopic moments due to rotating machinery as a computer input provided the disturbance moment or moments which affected the basic yawing motion of the craft. When a product of inertia was used as the disturbance moment, the basic yawing motion of the craft was disturbed by using either $I_{XZ}$ or $I_{YZ}$ or $I_{XZ}$, $I_{YZ}$, and $I_{XY}$ simultaneously; no calculations were made with $I_{XY}$ alone because, as can be seen by examining the equations of motion, the initial pure yawing motion $(p = q = 0)$ would not be affected by $I_{XY}$. However, once the yawing motion is disturbed ($p$ and $q \neq 0$), the product of inertia $I_{XY}$ will have an effect on the motion and must be considered when weights are moved in the craft. No calculations were made with only two products of inertia inasmuch as it was considered that such an arrangement was less likely to occur. No transient or gradually changing disturbance moments were used.

When the basic motion of the vehicle was disturbed by one of the aforementioned factors, this disturbance was kept in throughout the calculation, unless otherwise stated. Also, in discussing the results of the present calculations, small oscillations ($\pm 2^\circ$) are considered good relative to large oscillations ($\pm 100^\circ$) or divergencies but are not necessarily considered adequate.

When jet-reaction moments were used to damp out or minimize the undesirable motions, predetermined moments were put into the system (for example, $M_X = \pm 2$ ft-lb) and the jets were considered to be automatically activated when assigned limits of the angular velocities of the craft were obtained. The limits of the angular velocities used in this approach were

$$p = \pm 0.001 \frac{\text{radian}}{\text{sec}}$$

$$q = \pm 0.001 \frac{\text{radian}}{\text{sec}}$$

$$r = 0.60 \text{ to } 0.65 \frac{\text{radian}}{\text{sec}}$$

The procedure worked as follows: If it is assumed that a jet was placed at some point on the Y-axis and was able to produce a moment $M_X$ of $\pm 2$ ft-lb and that the craft, for some reason, began to roll to the left (negative $p$), then once the angular velocity $p$ reached a magnitude of -0.001 the jet would fire positively and produce a moment $M_X$ of 2 ft-lb. The jet would continue to fire until $p$ became closer to zero than -0.001. The same technique would apply if the craft had originally rolled to the right, except the jet would have automatically fired negatively and produced a moment $M_X$ of -2 ft-lb when $p$ became as large as 0.001. The moments $M_Y$ and $M_Z$ were handled in the same manner.

## RESULTS AND DISCUSSION

The results of the calculations are presented as time histories of roll angle $\phi_e$, pitch angle $\theta_e$, and yaw rate $r$ in figures 3 to 20. In addition, for the time histories of the motions in which jet-reaction moments were used in an attempt to minimize the undesirable motion, the magnitudes and timing sequences of these jet-reaction moments are indicated (figs. 18 to 20).

### Effects of Relative Magnitudes of $I_Y$ and $I_Z$

In the first two calculations, the basic yawing motion of the craft was disturbed by inserting as a computer input a product of inertia $I_{XZ}$

of -162 slug-ft$^2$ for a case in which $I_Y < I_Z$ and for a case in which $I_Y > I_Z$; the results are shown in figures 3 and 4, respectively. When the axis of the basic yawing rotation of the craft was the axis of greatest moment of inertia $\left(I_Y < I_Z\right)$, the results (fig. 3) indicated that the oscillations in roll were no greater than approximately ±2° and the pitch angle $\theta_e$ was, in general, less than -1°.

A qualitative explanation of these results is presented by use of a method equivalent to the analytical treatment known as Poinsot's construction (presented in a number of text books on mechanics) and is as follows: From an examination of the inputs used in calculating the results shown in figure 3, it is found from the equation (ref. 1)

$$I_{X'Z'} = 0 = \frac{I_X - I_Z}{2} \sin 2c + I_{XZ} \cos 2c$$

that the $I_{XZ}$ term indicates that the principal axis of greatest moment of inertia $Z'$ is displaced from the spin axis (Z-axis) by 1/2° ($c = 1/2°$). It is known that a body prefers to rotate about its principal axes of inertia; in this case, since the body is initially spinning about an axis displaced from the preferred axis of rotation by 1/2° and since there is no damping in the system, an oscillation with an amplitude of 1° (twice the displacement angle) is set up in the XZ-plane. The indicated oscillations in roll $\phi_e$ are due to the cross-couple angular velocities brought about by the disturbance in pitch.

When the axis of basic yawing rotation was the axis of intermediate moment of inertia $\left(I_Y > I_Z\right)$, the results (fig. 4) indicated that a continuously rolling motion was obtained. A qualitative explanation of the results shown in figure 4 is similar to that for the results in figure 3 and is as follows: The inputs used to calculate the time history shown in figure 4 were examined; these inputs indicated that the principal axis of greatest moment of inertia was $Y'$ (which coincides with the Y-axis in this case) and that, again, the principal axes of inertia in the XZ-plane were slightly displaced ($c = 1/2°$) from their respective body axes due to the $I_{XZ}$ term. It is known that a body prefers to rotate about its principal axis of maximum or minimum moment of inertia. Since the body was initially spinning about the Z-axis which was displaced 90° from the principal axis of maximum moment of inertia $Y'$, an oscillation in roll with an amplitude of 180° might have been expected. However, instead of an oscillation, a steady divergence was achieved. Thus, the rolling motion would be pictured as follows: The rotation vector of the body is initially directed downward. Also, initially, the Z-axis is directed downward. However, the preferred situation is to have the Y'-axis alined with the rotation vector. Therefore, the body rolls $-90^\circ$ about the X-axis until the -Y-axis is pointing downward, and, since there is no damping in the system, the body continues to roll through $180^\circ$ until the -Z-axis is pointing downward. After a brief pause, instead of reversing the direction of roll about the X-axis in order to bring the -Y-axis back in coincidence with the spin vector, the body continues to roll in the same direction about the X-axis in order to bring the +Y-axis downward. The motion continues until the +Z-axis is directed downward and, after another pause, continues in order to bring the -Y-axis downward again. During all this time, and throughout the remainder of the calculation, the X-axis continues to trace a nearly horizontal plane, as is indicated by the fact that the variation of $\theta_e$ is small.

These results agree qualitatively with the statement made in reference 2 to the effect that, if a body is set spinning about its axis of greatest moment of inertia, a slight displacement will not make the axis of rotation deviate very far from its initial position and the motion may be regarded as stable; if the initial axis of rotation were that of intermediate moment of inertia, a small displacement would cause the axis of rotation to change and the rotation about the initial axis would be said to be unstable.

A calculation was made to determine whether applying a very large product of inertia $I_{XZ}$ of -1,095 slug-ft$^2$ would cause an appreciable disturbance when the craft is rotating about its axis of greatest moment of inertia. (See table II for inputs used and fig. 5 for the resulting motion.) The very large product of inertia $I_{XZ}$ led to a slightly more oscillatory motion (fig. 5) than did the smaller value of $I_{XZ}$ (fig. 3). This result was expected, of course, since the method used in explaining the results of figure 3 indicated that the principal axis of maximum moment of inertia was displaced from the spin axis (Z-axis) approximately $3^\circ$ (fig. 5) as compared with $1/2^\circ$ for the inputs used in computing the results of figure 3.

Calculations were also made for cases in which a product of inertia $I_{YZ}$ was used to disturb the motion. For the case in which $I_Y < I_Z$, an oscillation in roll from $0^\circ$ to approximately $52^\circ$ was obtained (fig. 6); whereas, when $I_Y > I_Z$, an oscillation in roll from $0^\circ$ to approximately $108^\circ$ resulted (fig. 7). The oscillations in pitch were less than $\pm 1^\circ$ for both cases.

The explanation of the differences in the amplitudes of the oscillations in roll shown in figures 6 and 7 is similar to the explanation of the results of figure 3. By utilizing the following equation (ref. 1)

$$I_{Y'Z'} = 0 = \frac{I_Y - I_Z}{2} \sin 2c + I_{YZ} \cos 2c$$

and the inputs used in computing the results of figure 6, it is found that the Y'- and Z'-axes are displaced from their respective body axes approximately $26^\circ$ ($c = 26^\circ$); therefore, as shown in figure 6, the amplitude of the roll oscillation is $52^\circ$ (twice the displacement angle). However, the inputs used for calculating the case in which the craft was initially rotating about its axis of intermediate moment of inertia (fig. 7) indicate that the principal axes of inertia Y' and Z' are displaced approximately $36^\circ$ ($c = 36^\circ$) from their respective body axes. Therefore, since the principal axis of greatest moment of inertia is Y', which is now displaced $54^\circ$ from the initial spin axis, an oscillation in roll with an amplitude of $108^\circ$ is obtained.

A brief calculation was made with the absolute value of the $I_{YZ}$ disturbance increased from -53 to -745 slug-ft$^2$ $\left(I_Y\right.$ still larger than $\left.I_Z\right)$ and the results (fig. 8) showed oscillations larger in amplitude. In addition, when the value of $I_{YZ}$ of -745 slug-ft$^2$ was used but with the difference between $I_Y$ and $I_Z$ decreased from 1,500 to 391 slug-ft$^2$ $\left(I_Y\right.$ still larger than $\left.I_Z\right)$, the results (fig. 9) indicated that the magnitude of the oscillation in roll decreased. (Compare figs. 8 and 9.) Although not specifically computed, analysis indicates that when $I_Y < I_Z$ an increase in the difference between the values of $I_Y$ and $I_Z$ would lead to less oscillatory motions.

A further study was made to evaluate the effect of the relative magnitudes of $I_Y$ and $I_Z$ in which the motion was disturbed by using, simultaneously, three products of inertia $I_{XY}$, $I_{XZ}$, and $I_{YZ}$. The time history in figure 10 shows that an oscillation in roll from $0^\circ$ to $-68^\circ$ was obtained when the craft was rotating about its axis of greatest moment of inertia, and the time history in figure 11 shows an oscillation in roll from $0^\circ$ to $-135^\circ$ when the craft was rotating about its axis of intermediate moment of inertia. Therefore, as would be expected from previous results, the oscillation in roll was larger when the craft was initially rotating about its axis of intermediate moment of inertia. Also, it is interesting to note that this latter result indicated an oscillation in roll even though the product of inertia $I_{XZ}$ was present; whereas, the corresponding motion discussed previously and presented in figure 4 had shown that when the basic yawing motion was disturbed by $I_{XZ}$ alone, when $I_Y > I_Z$, the craft rolled continuously.

A comparison of figures 4 and 11 indicates that $I_{XY}$ and/or $I_{YZ}$ apparently had a damping effect on the motion presented in figure 11. It can be seen from the equation

$$I_{Y'Z'} = 0 = \frac{I_Y - I_Z}{2} \sin 2c + I_{YZ} \cos 2c$$

that the reason the time history presented in figure 11 did not indicate a continuous rolling motion, even though $I_Y > I_Z$ and $I_{XZ}$ was present, was that the $I_{YZ}$ term shifted the principal axis of greatest moment of inertia Y' toward the initial spin axis (Z-axis) which meant that the configuration could not roll over since the principal axis of greatest moment of inertia was located less than $90^{\circ}$ from the initial spin axis. Therefore, effectively, the $I_{YZ}$ term had a so-called damping effect for the calculation presented in figure 11.

## Effects of Rotating Machinery in the Space Station

For this study, it was assumed that an auxiliary power unit to be used for such things as the life cycle, radio, hydraulic system, etc., would have rotating parts in the boiler feed pump, primary sodium pump, turbine, alternator, etc. It was assumed that, collectively, these rotating parts were equivalent to an inertia wheel with a weight of 1 pound and a diameter of 0.456 foot and that, when the wheel rotated at a rate of 36,000 rpm, it would produce an angular momentum of approximately 6 ft-lb-sec.

The effect of the machinery rotating about the X-axis on the motion of the craft was calculated for the condition of the craft rotating about its axis of intermediate moment of inertia $(I_Y > I_Z)$. The results indicate that this small disturbance made the craft roll continuously (fig. 12). A similar calculation for the case in which the craft was initially rotating about its axis of greatest moment of inertia $(I_Y < I_Z)$ indicated stability in that very small oscillations $(\phi_e < \pm 1^{\circ})$ occurred (fig. 13). These results indicate that it is important that the craft be rotated about its axis of greatest moment of inertia even if no products of inertia are present when machinery is rotating about the X-axis of the craft.

## Attempts to Damp Out or Minimize Undesirable Motions

Some calculations were made in attempts to control, minimize, or damp out some undesirable motions that have been discussed. Essentially, the following three methods were used:

(1) Removal of product of inertia which acted as the disturbance factor

(2) Use of constant-rate inertia wheels to counteract the effects of products of inertia

(3) Use of jet-reaction moments, when needed, to keep angular velocities within a given range

In general, the removal of the disturbance - that is, relocating the weights in such a manner that no products of inertia were present - proved to be unsatisfactory. For example, the results shown in figure 14 indicate that, when the craft was initially rotating about its axis of greatest moment of inertia $\left(I_Y < I_Z\right)$ and a product of inertia was present, the craft oscillated in roll $\pm 2^{\circ}$ and that, when the weights that had orginally been moved to produce a product of inertia were moved back to their original positions where no product of inertia was present, the oscillatory motion was essentially unchanged. Furthermore, the results in figure 15, where the craft was originally set rotating about its axis of intermediate moment of inertia $\left(I_Y > I_Z\right)$, show that the craft rolled continuously to the left when the product of inertia was present and that, when the product of inertia was taken out (weights put back to their initial positions), the craft continued to roll to the left. These results might have been expected since there was nothing in the system to damp the acquired motions.

Calculations were made in an attempt to minimize undesirable motions by use of a constant-rate inertia wheel. The wheel was assumed to be rotating about the Y-axis of a configuration which had a continuously rolling motion when the wheel was not operating. For this configuration, $I_Y$ was greater than $I_Z$ and a product of inertia $I_{XZ}$ of -373 slug-ft$^2$ was present. The assumed inertia wheel produced an angular momentum successively of +10, +50, and +500 ft-lb-sec (positive angular momentum indicates that the wheel is rotating clockwise when viewed from negative end of Y-axis) and was assumed to be in operation from time zero. A typical time history with the inertia wheel in operation is presented as figure 16 and indicates an oscillation in roll from $0^{\circ}$ to $180^{\circ}$ and a pitch oscillation of approximately $\pm 2^{\circ}$. A comparison of the results for the wheel operating and not operating indicates that the inertia wheel caused the craft to oscillate in roll instead of allowing it to roll continuously. Although it is not shown in the figures, the frequency of the roll oscillation decreased as the angular momentum produced by the wheel was decreased.

The inertia wheel was also assumed to be rotating about the Z-axis and a calculation was made for which the wheel was assumed to be producing an angular momentum of +50 ft-lb-sec. The results are presented in figure 17. As can be seen from this time history, the roll oscillation was approximately $\pm 6^{\circ}$; this appeared to be an appreciable improvement over the case in which the wheel was not operating and in which the wheel was operating about the Y-axis (fig. 16).

A qualitative explanation of the results shown in figure 17 is as follows: Consider the following terms in the roll equation of motion (eq. (1)):

$$\left(I_Y - I_Z\right)qr - I_{Z,m}\omega_m q$$

Assume that $r = 0.6$ and constant, then the first term is (see table II for numbers)

$$(21,760 - 21,710)(0.6)q = +30q$$

This term is positive solely because $I_Y > I_Z$, and it is known from the results shown in figure 4 that the craft will diverge in roll for these conditions. However, when the effect of the inertia wheel is added, the result is

$$(21,760 - 21,710)(0.6)q - 50q = -20q$$

which number is similar in sign to the cross-couple inertia term when the time history in figure 3 was computed. Therefore, it can be reasoned that adding the inertia wheel to the system makes the configuration equivalent to a system in which $I_Z > I_Y$. A comparison of the results of figure 17 with those of figure 3, where $I_Z$ was larger than $I_Y$, shows that the results are similar. (The differences in the results are due to the larger value of $I_{XZ}$ used for the calculation shown in fig. 17.) It could be concluded from these results that rotating an inertia wheel about the Z-axis (the spin axis), which has a sufficient amount of angular momentum, is equivalent to making the moment of inertia about the Z-axis greater than that about the Y-axis which, as stated previously, is preferable.

The last approach toward minimizing undesirable motions was the use of jet-reaction moments, when needed, to keep angular velocities of the craft within a given range. Since it has been pointed out in the text that the craft should always be rotated about its axis of greatest moment of inertia, all attempts made to minimize undesirable motions by use of jet-reaction moments were for a configuration where $I_Y$ was less than $I_Z$.

The first calculation, in which jet-reaction moments were used, was made to minimize the motion that was presented in figure 6. The time history shown in figure 6 was calculated for the condition when the craft was initially set rotating about its axis of greatest moment of inertia and was disturbed by a product of inertia $I_{YZ}$, and it indicated an oscillation in roll from $0^{\circ}$ to $52^{\circ}$. (The pitch angle was disturbed less than $\pm 1^{\circ}$ and the yaw rate $r$ remained between 0.40 and 0.63 radian/sec.) Essentially, this calculation was made again with the jet-reaction-moment terms included in the equations of motion and the craft's angular velocities used to activate the jets, as described in the section entitled "Methods and Calculations." Jets were assumed to be located on the craft in such a manner that the following moments could be produced:

$$M_X = \pm 2 \text{ ft-lb}$$

$$M_Y = \pm 45 \text{ ft-lb}$$

$$M_Z = \pm 45 \text{ ft-lb}$$

The resulting motion is shown in figure 18 and indicates that the roll oscillation was damped to less than $1^{\circ}$, $\theta_e$ remained at less than $\pm 1^{\circ}$, and $r$ remained about 0.63 radian/sec, but the firing of all three jets continuously was required in order to keep the craft at this attitude.

Another calculation was made with the same inputs but with only one jet that produced a moment $M_X$ of $\pm 2$ ft-lb. The resulting motion is presented in figure 19 and shows that the roll oscillation was again damped to less than $1^{\circ}$, $\theta_e$ remained even closer to zero than it did when $M_X$, $M_Y$, and $M_Z$ were used simultaneously, and the jet was no longer required once the motion reached its new equilibrium. From a comparison of figures 18 and 19 it can be concluded that $M_Y$ and $M_Z$ (fig. 18) were producing undesirable cross-couple effects and that using $M_X$ alone (fig. 19) gave much more effective as well as efficient results. This is true, however, only because the original motion was disturbed by the product of inertia $I_{YZ}$ which is in the roll plane.

These two calculations were for the case in which one or more jet-reaction moments were used from zero time to try to keep the motion damped. A third calculation was made to determine whether, after the craft had already achieved its oscillatory motion, it could be damped to a more desirable motion. For this calculation, the oscillatory motion in figure 6 was again used as the motion to be damped. This time history (fig. 6) was picked up at approximately $t = 94$ seconds with all conditions and inputs remaining the same except that a value of $M_X$ of $\pm 2.0$ ft-lb was used. The same limit on $p$ ($\pm 0.001$) was used here as had been used in the previous calculations. The resulting motion is presented in figure 20 and shows that the roll oscillation was damped to less than $1^\circ$, $\theta_e$ was less than $\pm 1^\circ$, and the rate of yaw was steady at about 0.56 radian/sec. (The time history in fig. 6 is repeated in fig. 20 for comparison.) This result indicated that, even though unfavorable products of inertia may result when weights are shifted in the orbiting vehicle, the use of a jet-reaction moment $M_X$ will enable the craft to achieve again a relatively steady-state motion. It should be mentioned, however, that, although these results (figs. 18, 19, and 20) indicated that the roll oscillation was damped to less than $1^\circ$, the roll angle about which the craft was damped was not zero. It is evident why the roll angle was not damped about $0^\circ$ in each of these computations and the explanation for this is similar to the explanation for the results of figure 6. The roll oscillation was damped about an angle which was one-half of the original amplitude of the oscillation when no jets were used (fig. 6). This is due to the fact that, as explained previously concerning figure 6, the principal axis of greatest moment of inertia is displaced from the spin axis (Z-axis) by approximately $26^\circ$. Therefore, from an examination of figures 18, 19, and 20, it can be seen that the roll angle was damped about the $26^\circ$ roll attitude.

## CONCLUSIONS

An analytical study has been made to determine the effects of mass-loading variations on the angular motions of a hypothetical cylindrical earth satellite space station, rotating about its Z-axis to provide an artificial gravity equal to one-fourth of that at the earth's surface. The results of this study are as follows:

1. When the moment of inertia about the Y-axis was less than the moment of inertia about the Z-axis, a product of inertia $I_{XZ}$ did not disturb the craft in pitch more than twice the angle of displacement of the principal axis of greatest moment of inertia Z' from the initial spin axis Z. For these conditions, the amplitude of the oscillations in roll was greater than those in pitch and depended upon the cross-couple inertias of the configuration. A product of inertia $I_{YZ}$ did not disturb the craft in roll more than twice the angle of displacement of the greatest principal axis Z'. For this condition, the amplitude of the oscillations in pitch depended upon the cross-couple inertias of the configuration.

2. When the moment of inertia about the Y-axis was greater than the moment of inertia about the Z-axis, a product of inertia $I_{XZ}$ caused the craft to experience a continuous rolling motion because the principal axis of greatest moment of inertia Y' was displaced $90^{\circ}$ from the initial spin axis Z and because there was no damping in the system. The pitch angle $\theta_e$ remained small ($\pm 1^{\circ}$). A product of inertia $I_{YZ}$ did not disturb the craft in roll more than twice the angle between the Y'-axis and the initial spin axis.

3. When all three products of inertia $I_{XY}$, $I_{XZ}$, and $I_{YZ}$ were used simultaneously as disturbance factors, large oscillations in roll resulted; pitch angles were generally small ($< \pm 1^{\circ}$). Again, the amplitudes of the oscillations of these angles depended upon the location of the principal axes with respect to the initial spin axis.

4. The rotating parts of the machinery in the craft could cause the craft to experience a continuously rolling motion if the basic rotation of the craft was about its axis of intermediate moment of inertia; however, small effects were experienced if the rotation was about its axis of greatest moment of inertia.

5. In general, the removal of a disturbance - that is, relocating the weights in such a manner that no products of inertia were present - was not satisfactory as regards returning the motion of the craft to its original steady-state yawing condition once a disturbed motion was set up, because there was no damping in the system.

6. The use of constant-rate inertia wheels, particularly one located on and rotating about the Z-axis (spin axis), was quite successful in minimizing undesirable motions if the angular momentum of the wheel was large enough and in the right direction.

7. In attempting to damp an undesirable roll oscillation, jet-reaction moments were used successfully; the ensuing roll angle to which the craft damped was equal to the angle of displacement of the principal axis of greatest moment of inertia from the initial spin axis.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Field, Va., February 8, 1961.

## REFERENCES

1. Baumeister, Theodore, ed.: Mechanical Engineers' Handbook. Sixth ed., McGraw-Hill Book Co., Inc., 1958, pp. 3-19 - 3-22.

2. Smart, E. Howard: Advanced Dynamics. Vol. II - Dynamics of a Solid Body. Macmillan and Co., Ltd., 1951.

TABLE I.- INITIAL CONDITIONS USED FOR CALCULATIONS

| Variables | Initial values |
| --- | --- |
| $\theta_e$ | 0 |
| $\phi_e$ | 0 |
| $\psi_e$ | 0 |
| p | 0 |
| q | 0 |
| r | +0.628 |
| Time | 0 |

TABLE II.- INPUTS USED IN CALCULATIONS

[Basic configuration: $I_X$ = 870 slug-ft$^2$; $I_Y$ = $I_Z$ = 21,760 slug-ft$^2$; all other terms zero]

| Figure | Inertia terms, slug-ft$^2$ | | | | | | Angular-momentum terms, ft-lb-sec | | | Jet-reaction-moment terms, ft-lb | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $I_X$ | $I_Y$ | $I_Z$ | $I_{XY}$ | $I_{XZ}$ | $I_{YZ}$ | $I_{X,m}\omega_m$ | $I_{Y,m}\omega_m$ | $I_{Z,m}\omega_m$ | $M_X$ | $M_Y$ | $M_Z$ |
| 3 | 811 | 19,675 | 19,734 | 0 | -162 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 811 | 19,769 | 19,640 | 0 | -162 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 776 | 21,322 | 21,416 | 0 | -1,095 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 866 | 19,369 | 19,482 | 0 | 0 | -71 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 824 | 21,754 | 21,720 | 0 | 0 | -53 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 2,000 | 19,500 | 18,000 | 0 | 0 | -745 | 0 | 0 | 0 | 0 | 0 | 0 |
| 9 | 479 | 21,760 | 21,369 | 0 | 0 | -745 | 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | 781 | 20,417 | 20,507 | -467 | -267 | +112 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11 | 716 | 19,769 | 19,544 | +576 | -161 | +104 | 0 | 0 | 0 | 0 | 0 | 0 |
| 12 | 776 | 21,760 | 21,666 | 0 | 0 | 0 | -6 | 0 | 0 | 0 | 0 | 0 |
| 13 | 776 | 21,666 | 21,760 | 0 | 0 | 0 | -6 | 0 | 0 | 0 | 0 | 0 |
| 14 | 811 | 19,675 | 19,734 | 0 | -162 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 776 | 21,666 | 21,760 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 15 | 811 | 19,769 | 19,640 | 0 | -162 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 776 | 21,760 | 21,666 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 16 | 820 | 21,760 | 21,710 | 0 | -373 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 820 | 21,760 | 21,710 | 0 | -373 | 0 | 0 | +50 | 0 | 0 | 0 | 0 |
| 17 | 820 | 21,760 | 21,710 | 0 | -373 | 0 | 0 | 0 | +50 | 0 | 0 | 0 |
| 18 | 866 | 19,369 | 19,482 | 0 | 0 | -71 | 0 | 0 | 0 | ±2 | ±45 | ±45 |
| 19 | 866 | 19,369 | 19,482 | 0 | 0 | -71 | 0 | 0 | 0 | ±2 | 0 | 0 |
| 20 | 866 | 19,369 | 19,482 | 0 | 0 | -71 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 866 | 19,369 | 19,482 | 0 | 0 | -71 | 0 | 0 | 0 | ±2 | 0 | 0 |

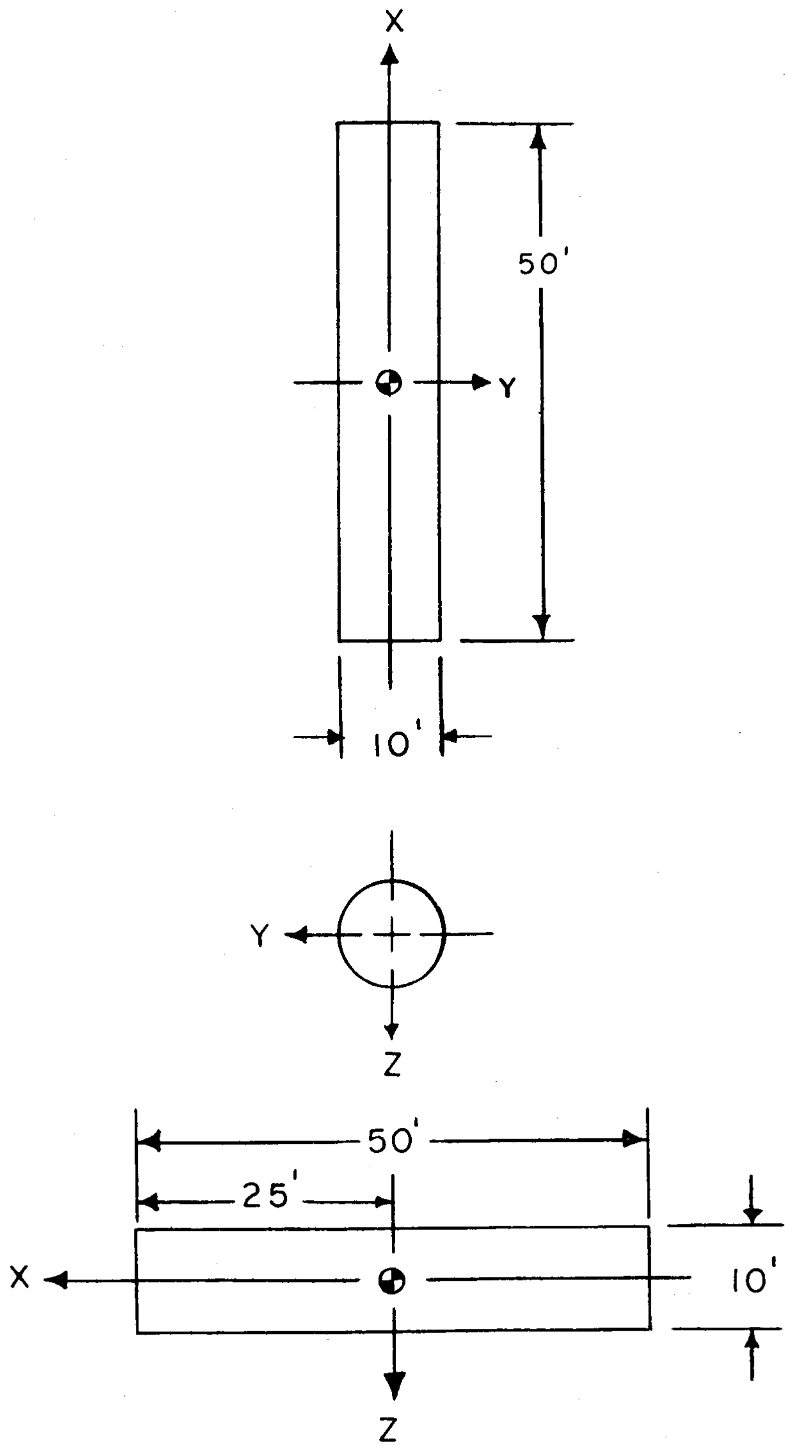


Figure 1.- Three-view sketch of configuration showing nominal location of reference axes.

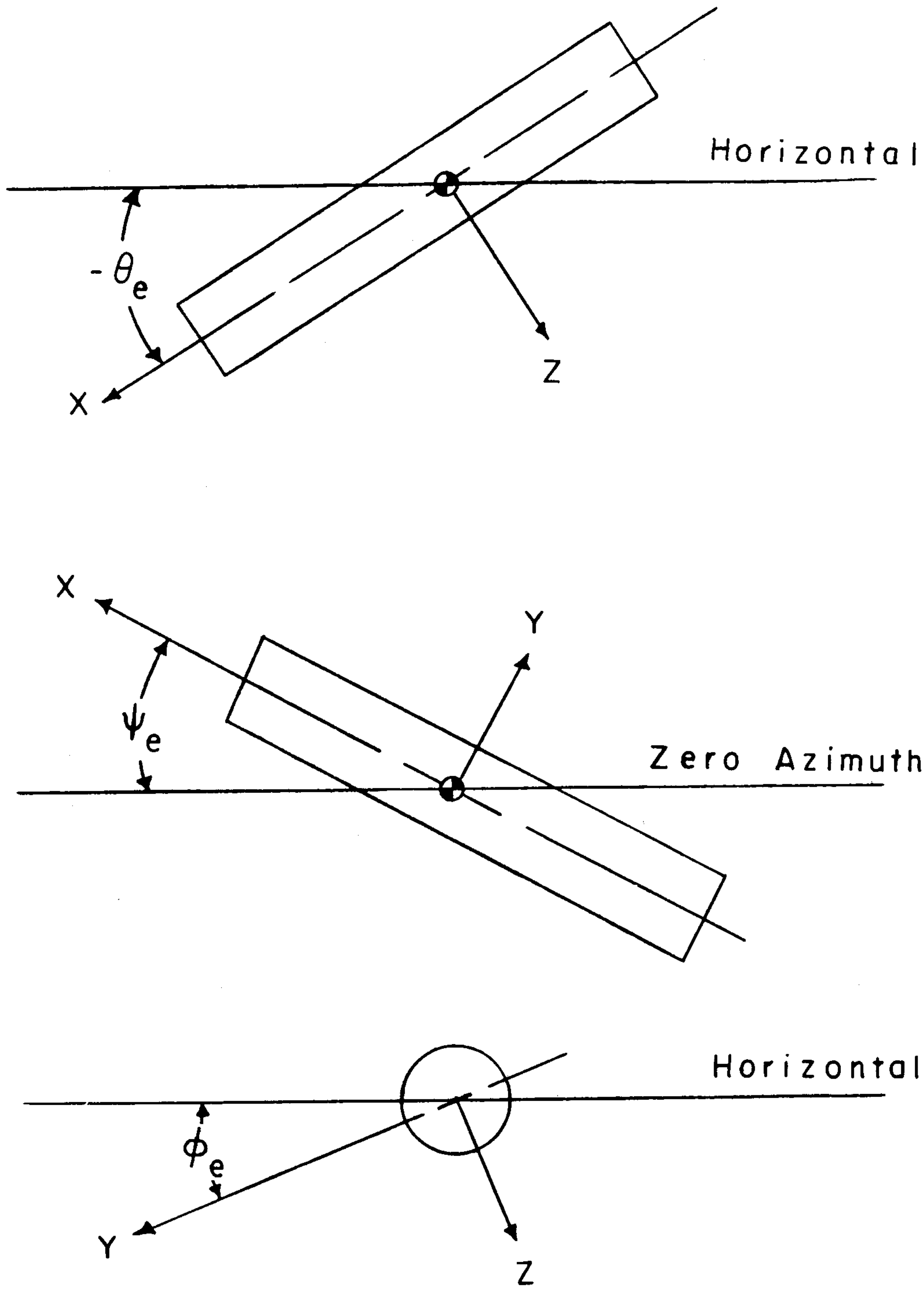


Figure 2.- Body system of axes and related angles.

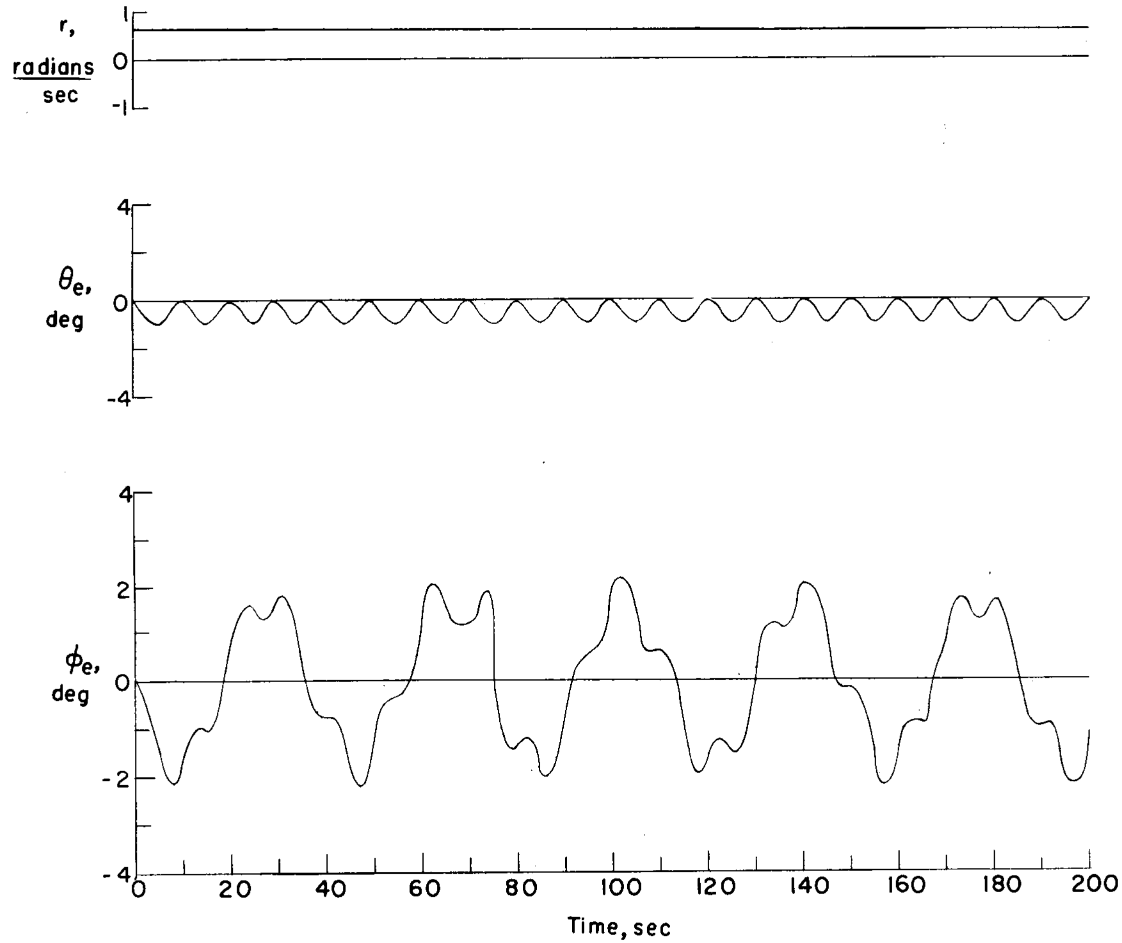


Figure 3.- Calculated motion indicating effects of a product-of-inertia disturbance $I_{XZ}$ of -162 slug-ft$^2$ when the craft is initially rotating about its axis of greatest moment of inertia $\left(I_Y < I_Z\right)$.

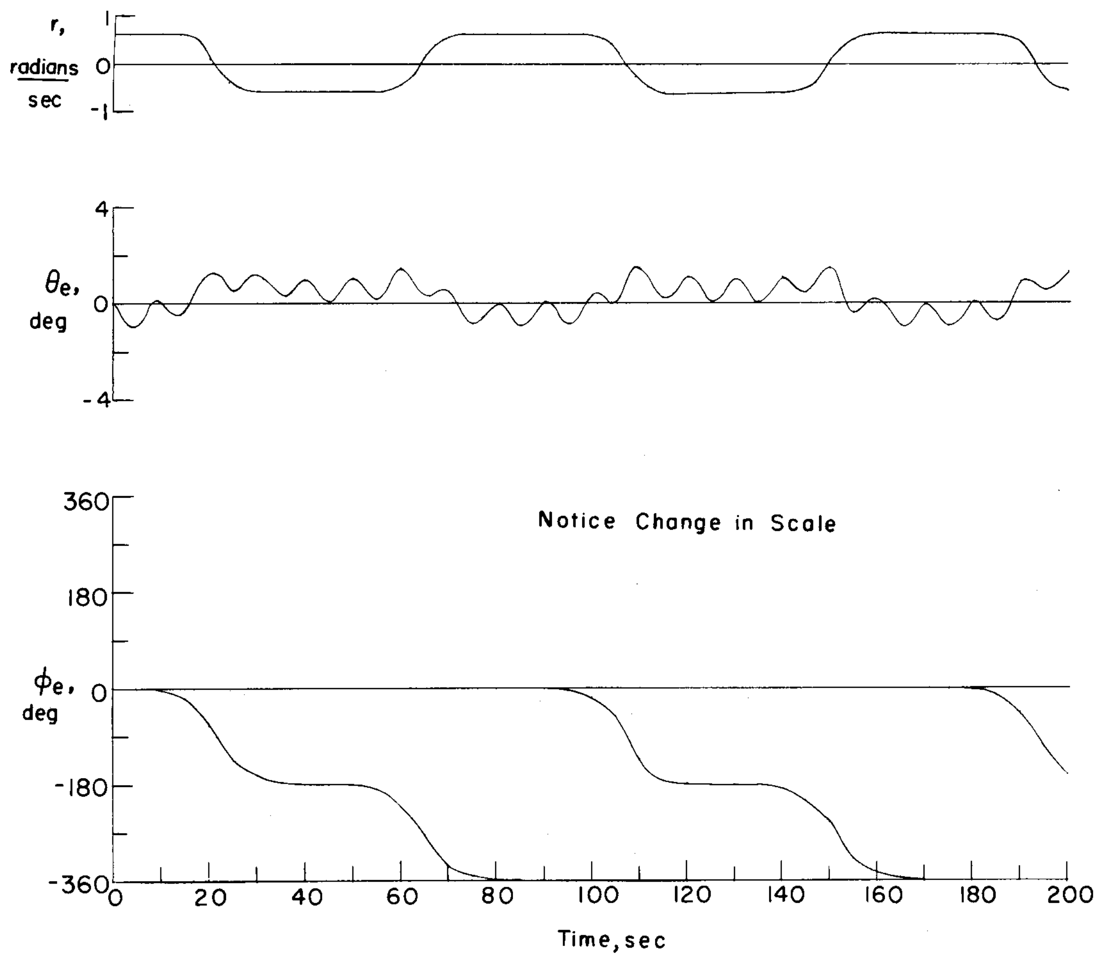


Figure 4.- Calculated motion indicating effects of a product-of-inertia disturbance $I_{XZ}$ of -162 slug-ft$^2$ when the craft is initially rotating about its axis of intermediate moment of inertia $\left(I_Y > I_Z\right)$.

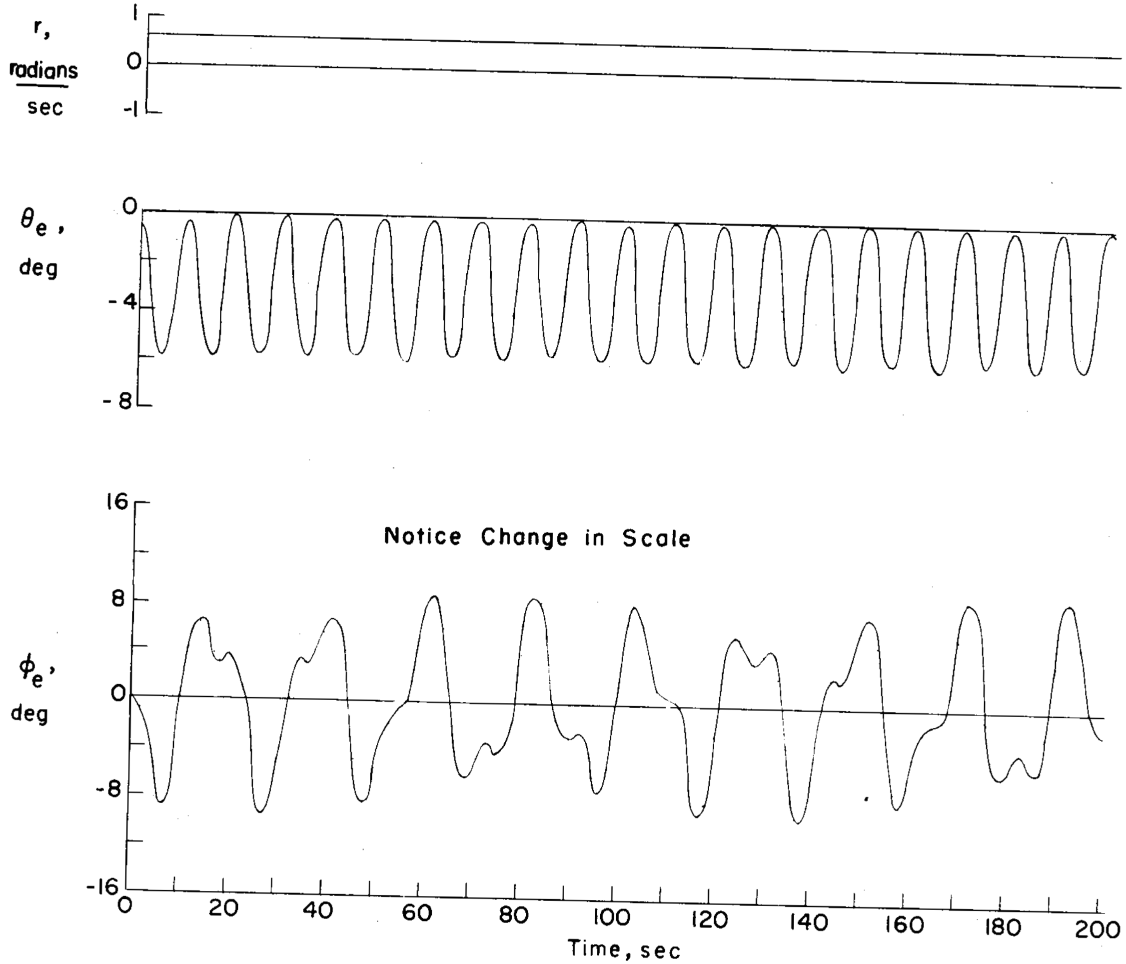


Figure 5.- Calculated motion indicating effects of a very large product-of-inertia disturbance $I_{XZ}$ of -1,095 slug-ft$^2$ when the craft is initially rotating about its axis of greatest moment of inertia $\left(I_Y < I_Z\right)$.

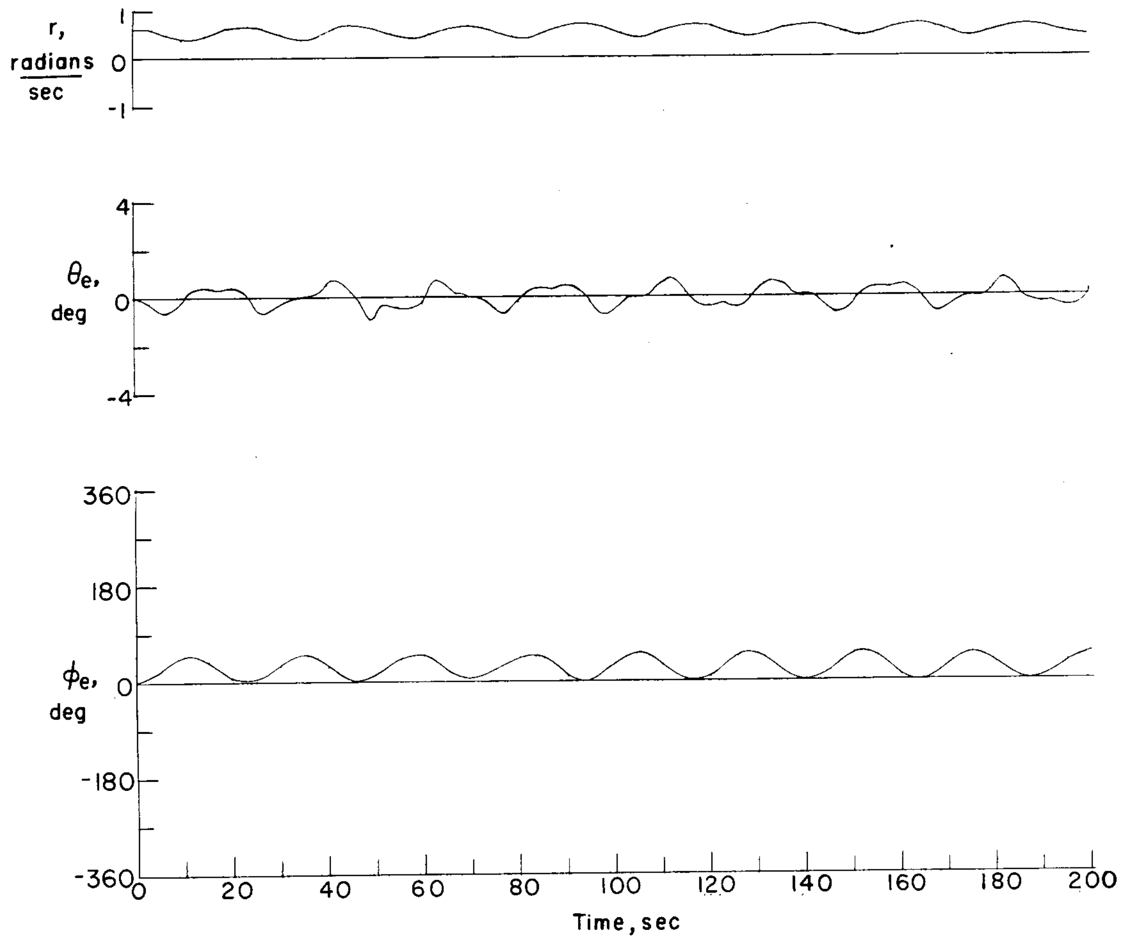


Figure 6.- Calculated motion indicating effects of a product-of-inertia disturbance $I_{YZ}$ of $-71$ slug-ft$^2$ when the craft is initially rotating about its axis of greatest moment of inertia $\left(I_Y < I_Z\right)$.

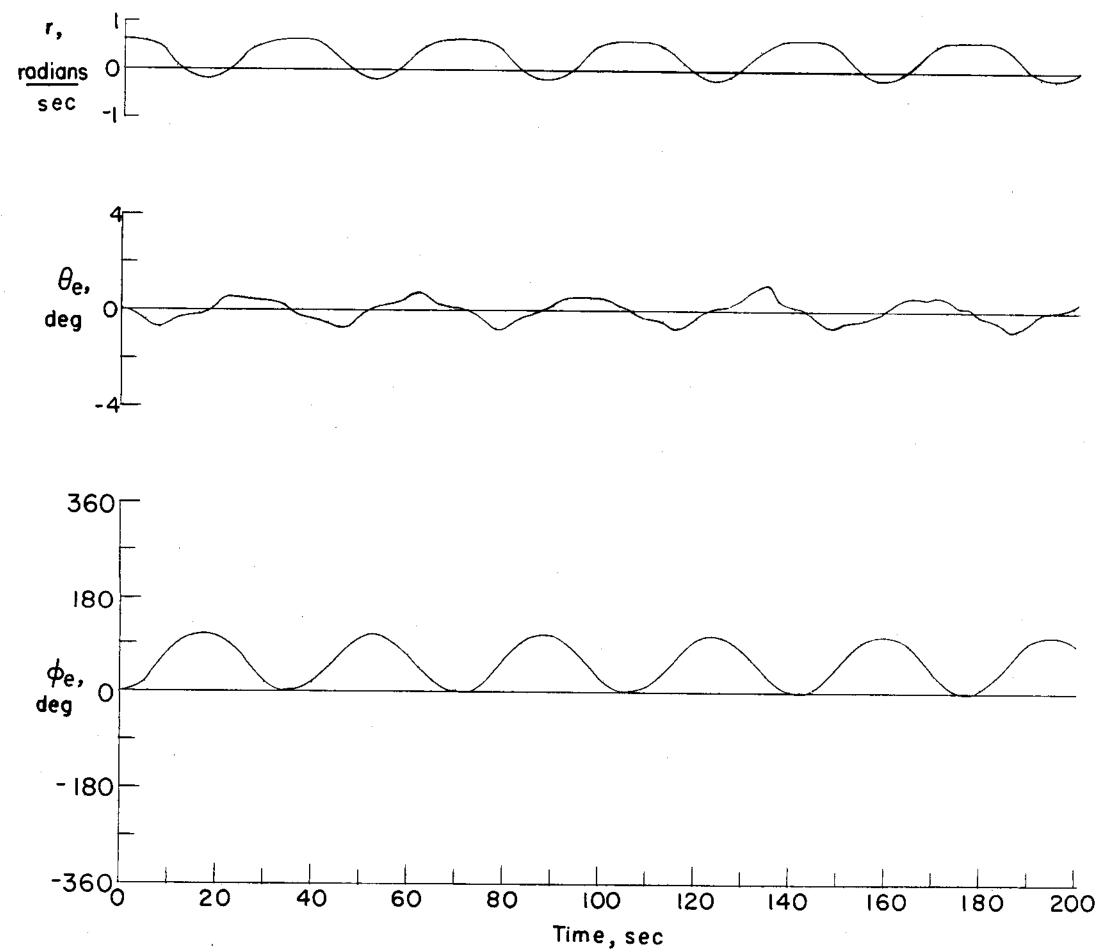


Figure 7.- Calculated motion indicating effects of a product-of-inertia disturbance $I_{YZ}$ of $-53$ slug-ft$^2$ when the craft is initially rotating about its axis of intermediate moment of inertia $\left(I_Y > I_Z\right)$.

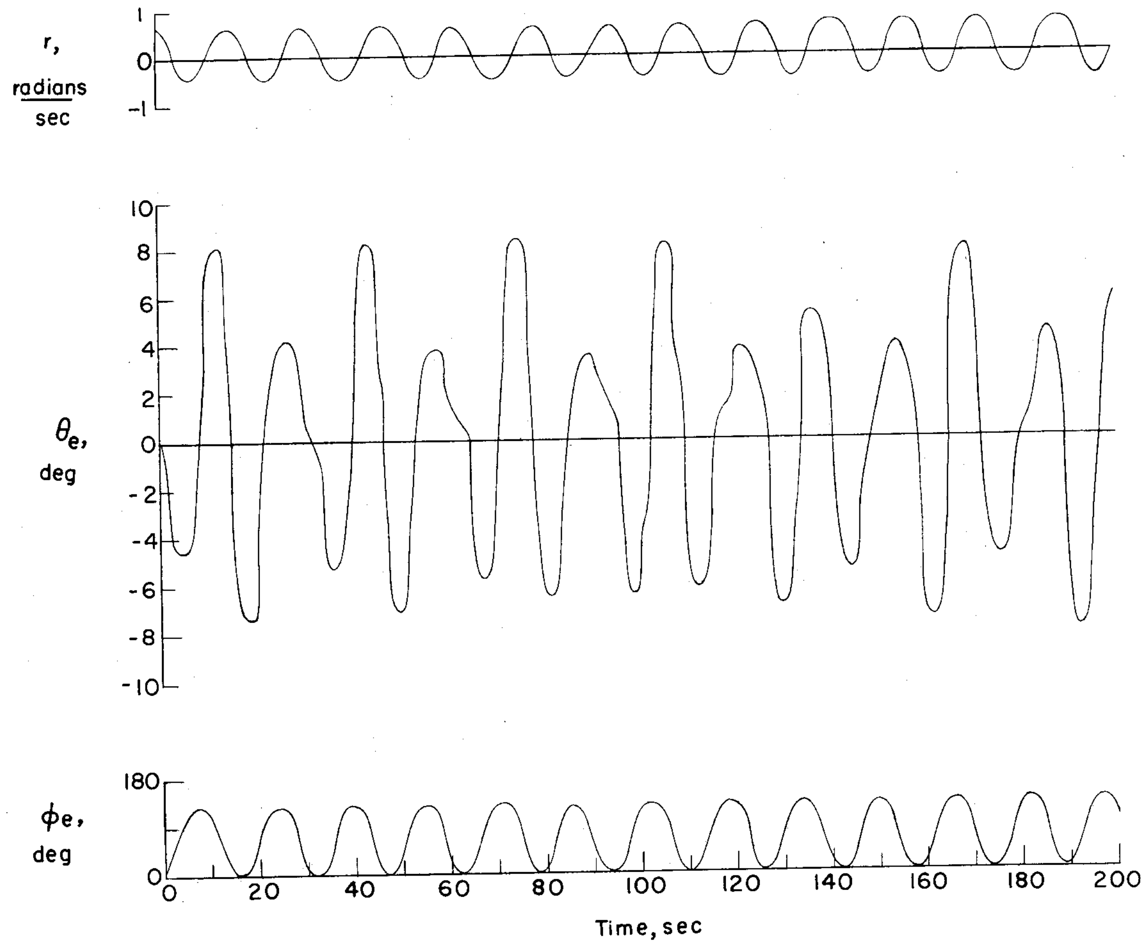


Figure 8.- Calculated motion indicating effects of a very large product-of-inertia disturbance $I_{YZ}$ of -745 slug-ft$^2$ when the craft is initially rotating about its axis of intermediate moment of inertia $\left(I_Y > I_Z\right)$.

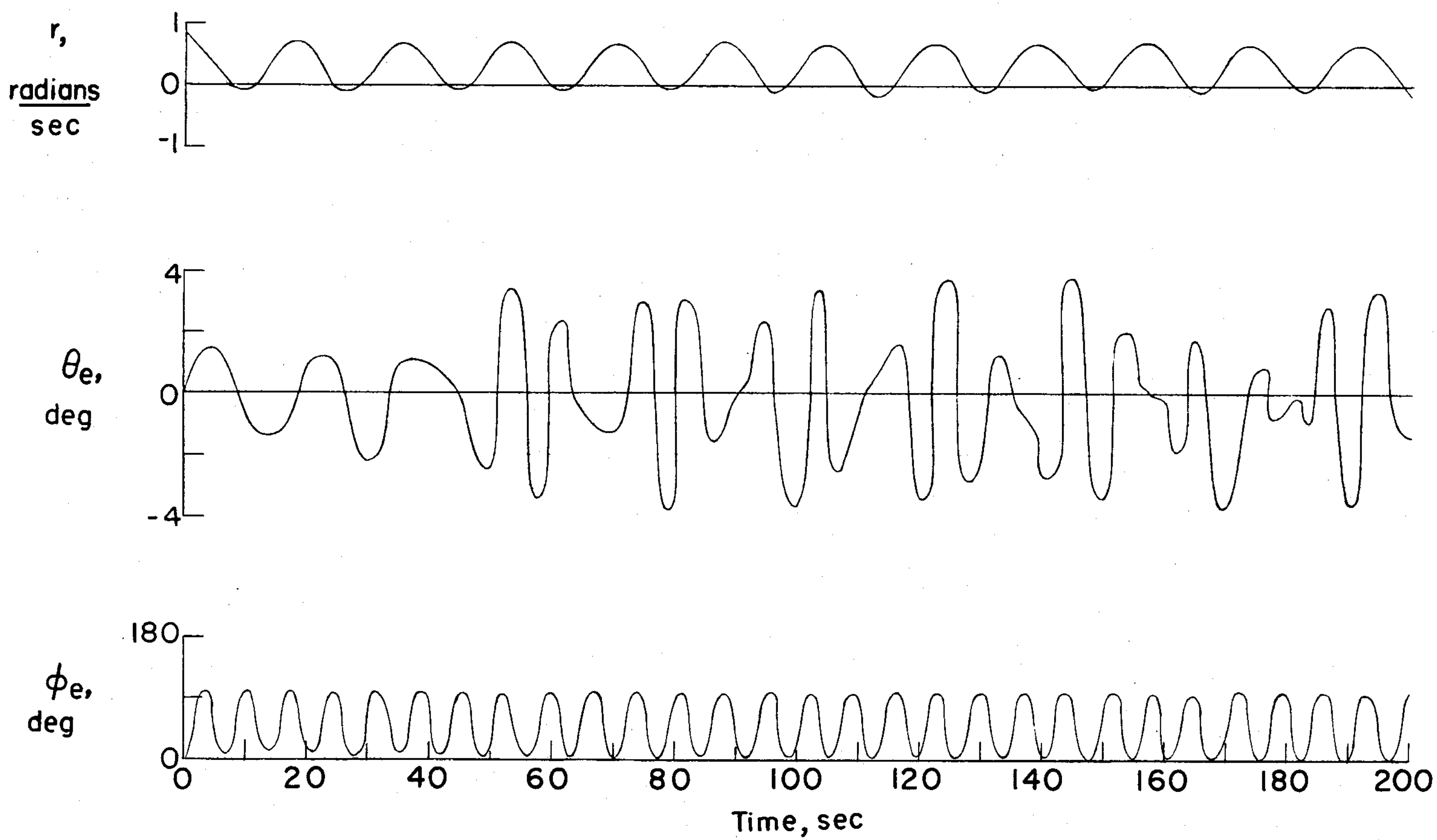


Figure 9.- Calculated motion indicating effects of relative magnitude of the difference between $I_Y$ and $I_Z$ when the craft is disturbed by a product of inertia $I_{YZ}$ of -745 slug-ft$^2$ and initially rotating about its axis of intermediate moment of inertia $\left(I_Y > I_Z\right)$.

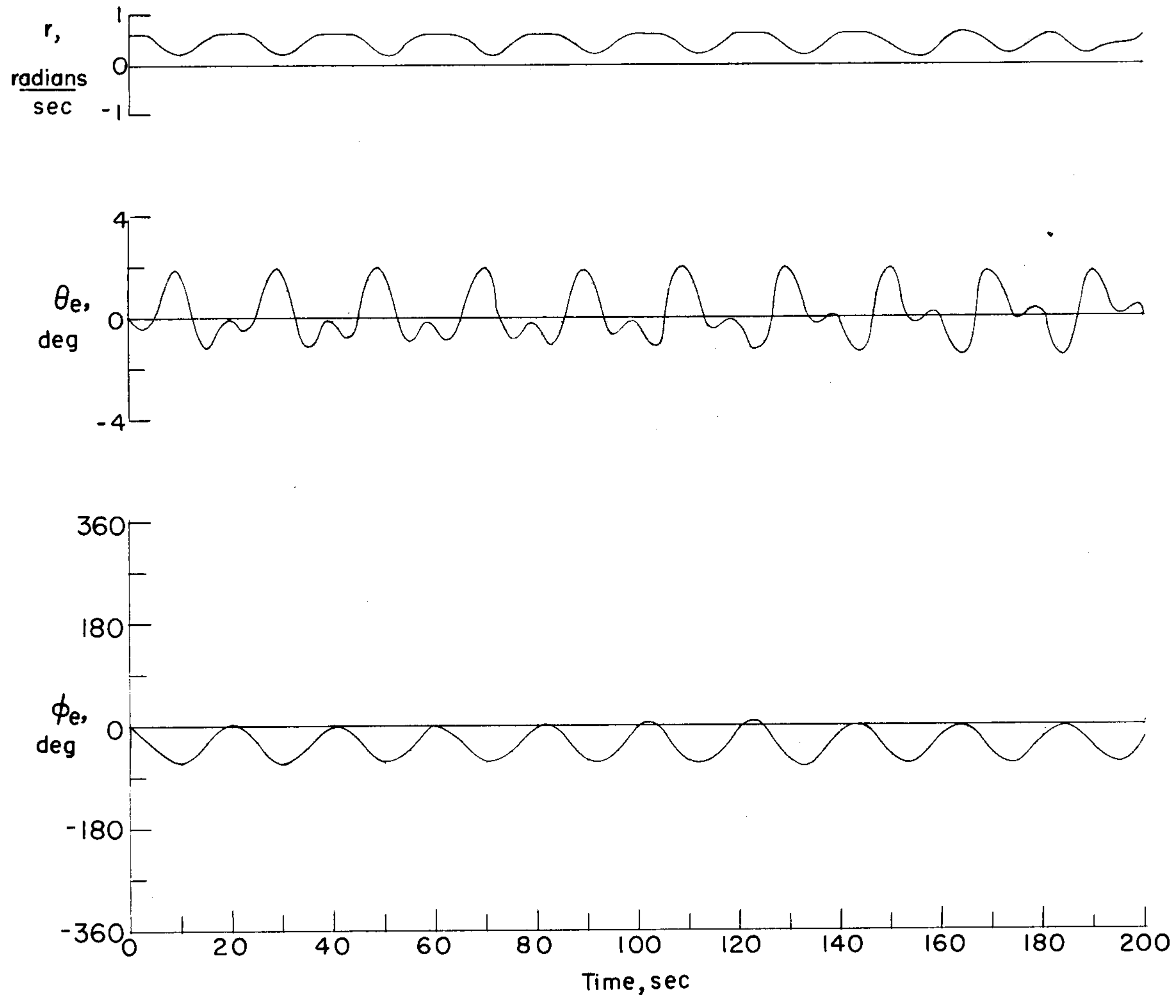


Figure 10.- Calculated motion indicating effects of disturbing the pure yawing motion of the craft by using, simultaneously, three products of inertia $I_{XY}$, $I_{XZ}$, and $I_{YZ}$ when the craft is initially rotating about its axis of greatest moment of inertia $\left(I_Y < I_Z\right)$.

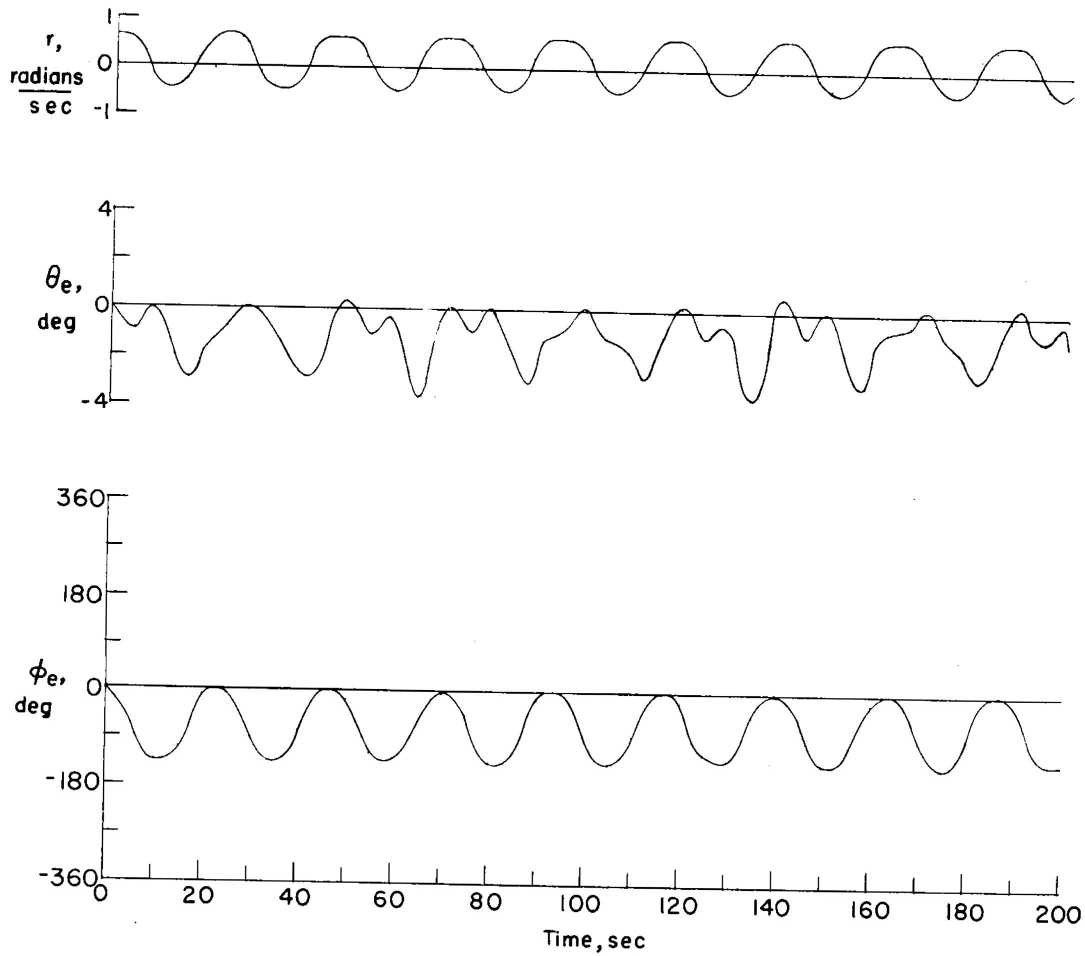


Figure 11.- Calculated motion indicating effects of disturbing the pure yawing motion of the craft by using, simultaneously, three products of inertia $I_{XY}$, $I_{XZ}$, and $I_{YZ}$ when the craft is initially rotating about its axis of intermediate moment of inertia $\left(I_Y > I_Z\right)$.

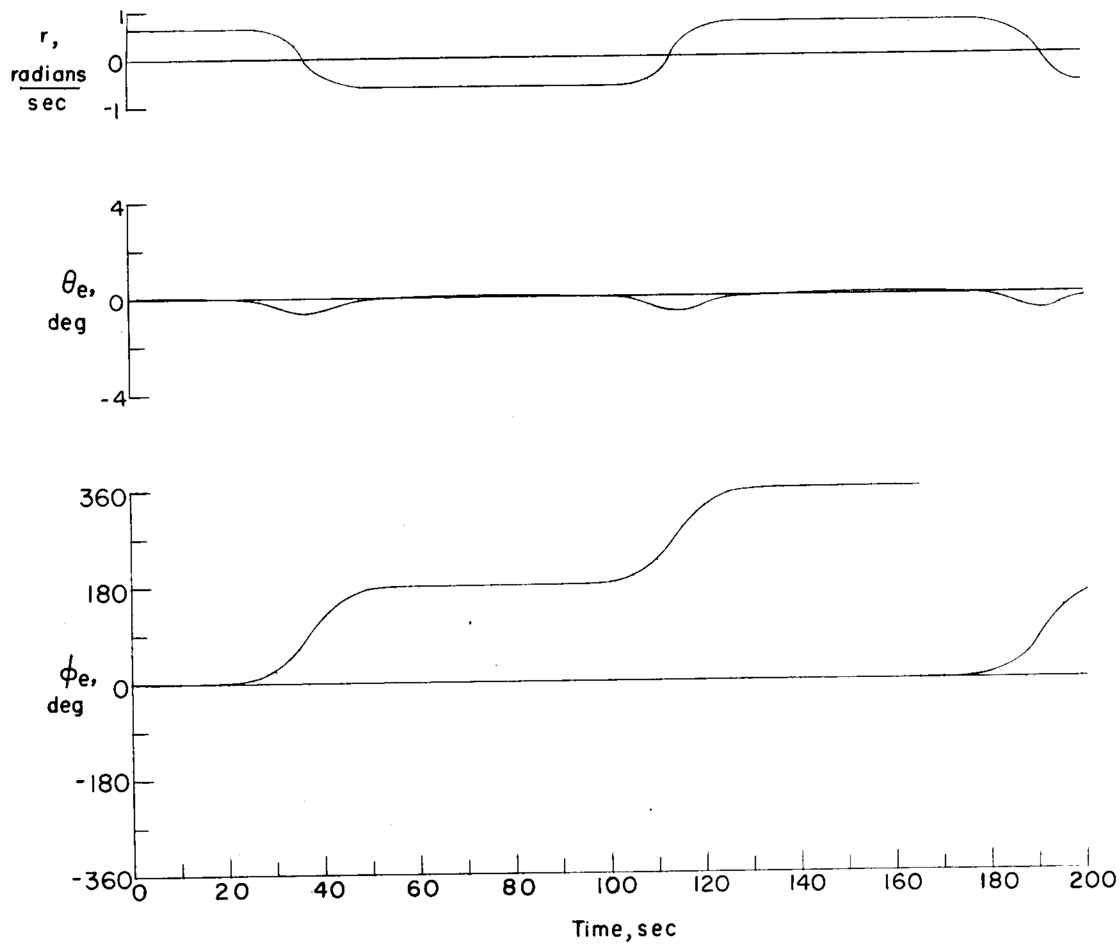


Figure 12.- Calculated effects of rotating machinery on the motion of the craft where the craft was initially rotating about its axis of intermediate moment of inertia $\left(I_Y > I_Z\right)$.

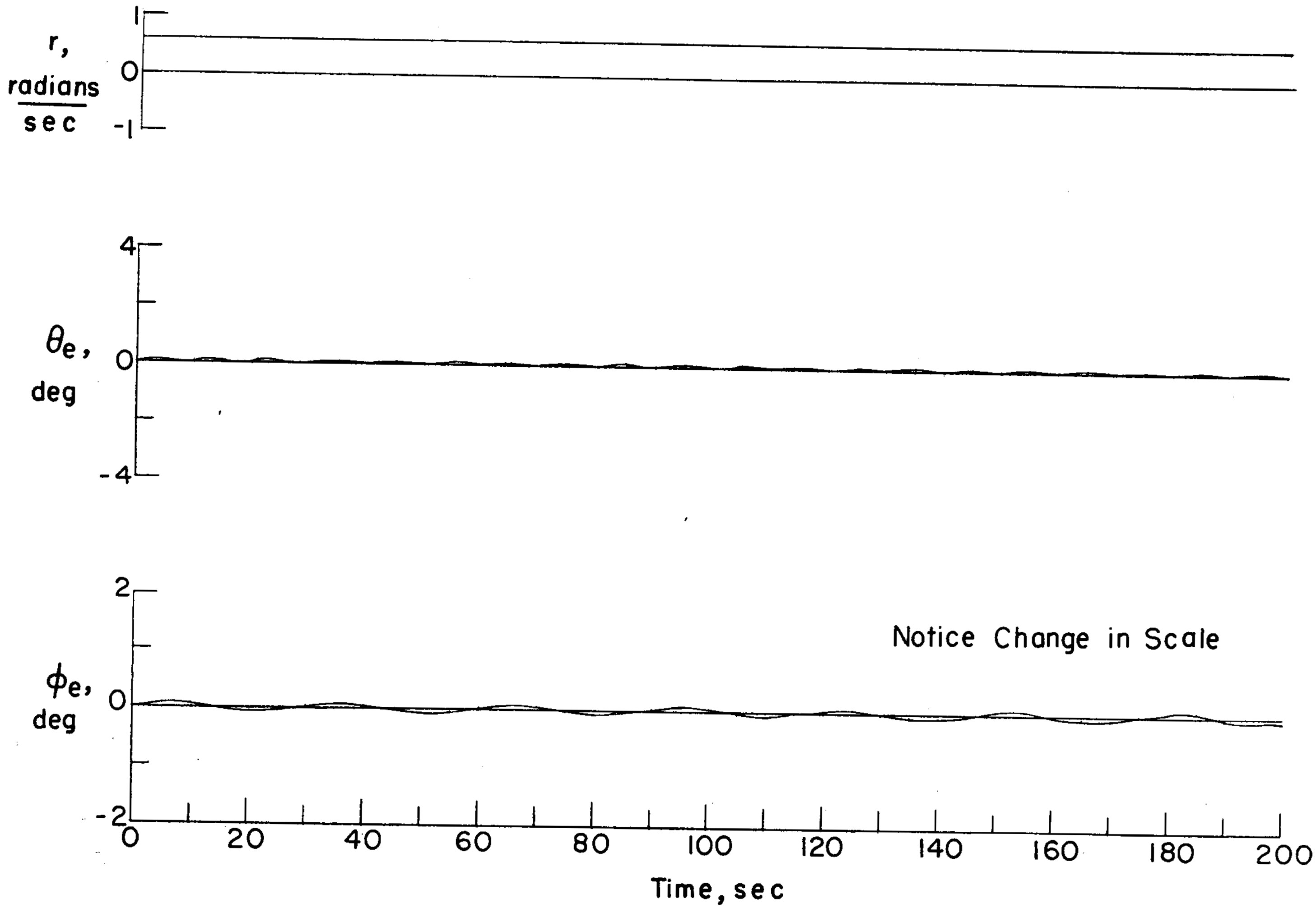


Figure 13.- Calculated effects of rotating machinery on the motion of the craft where the craft was initially rotating about its axis of greatest moment of inertia $\left(I_Y < I_Z\right)$.

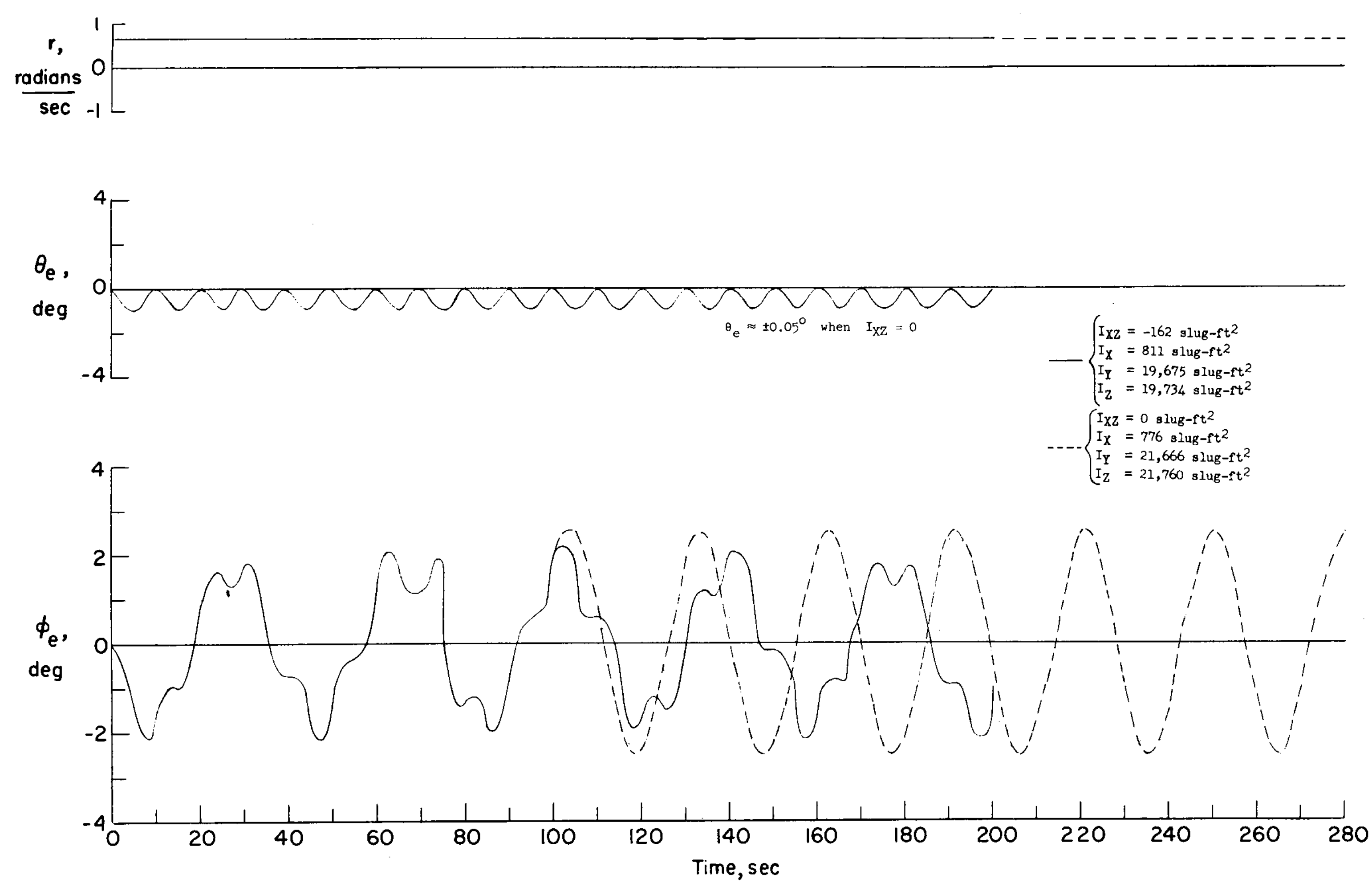


Figure 14.- Calculated effects of removing $I_{XZ}$ which disturbed the motion when the craft was rotating about its axis of greatest moment of inertia $\left(I_Y < I_Z\right)$.

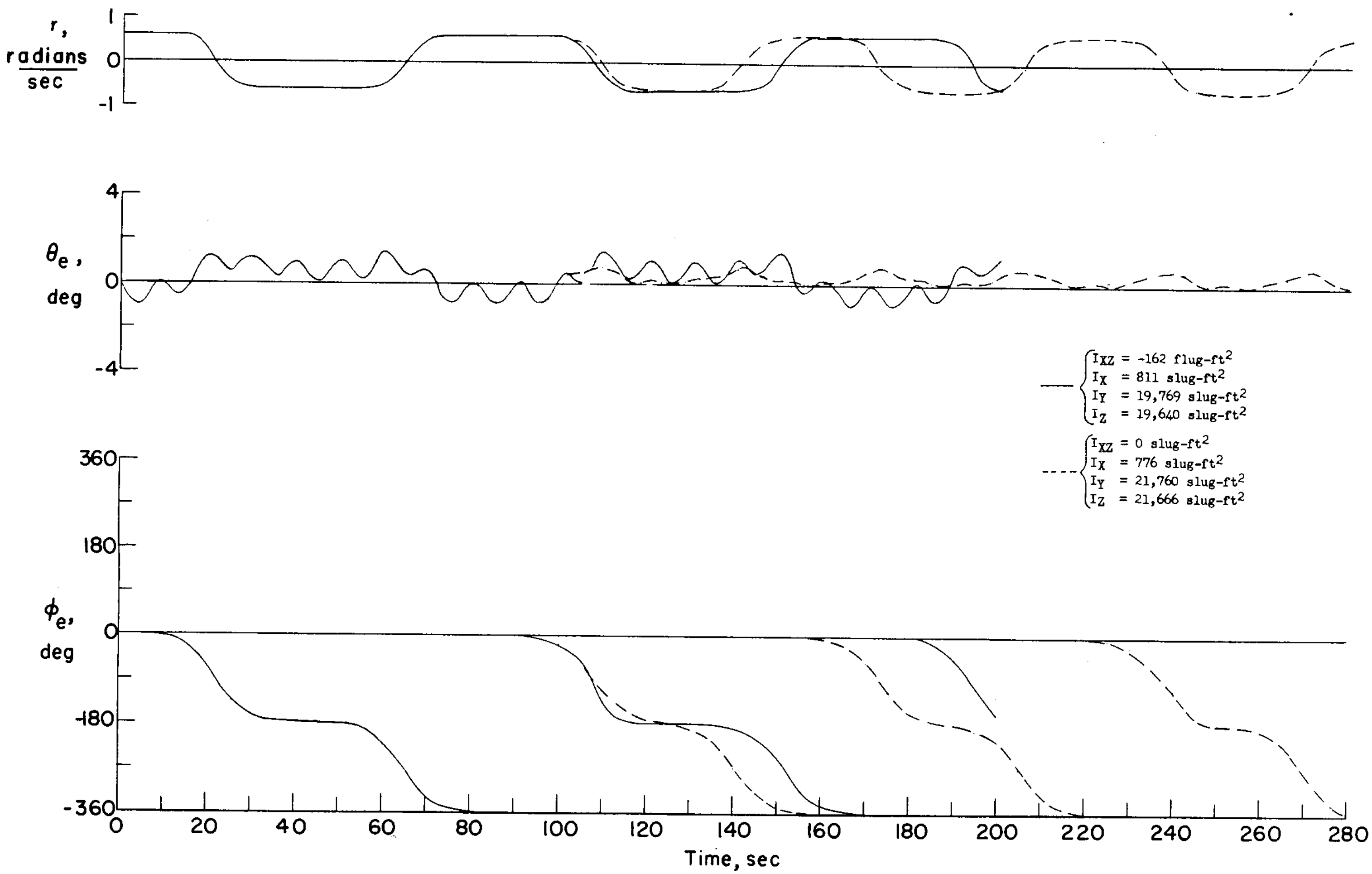


Figure 15.- Calculated effects of removing $I_{XZ}$ which disturbed the motion when the craft was rotating about its axis of intermediate moment of inertia $\left(I_Y > I_Z\right)$.

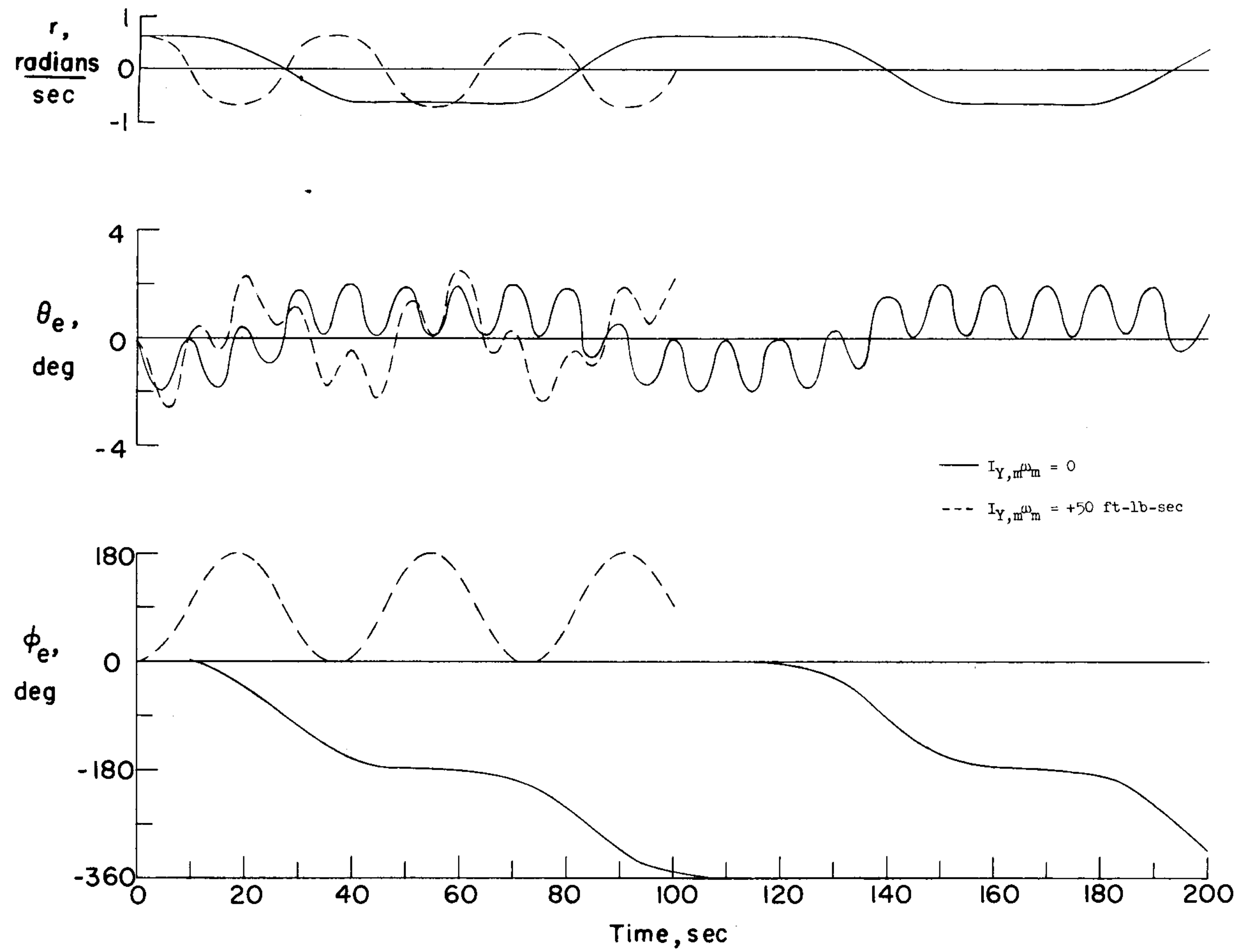


Figure 16.- Calculated motions indicating the effects of an inertia wheel, rotating about the Y-axis and producing +50 ft-lb-sec angular momentum, on the motion of the craft which is initially rotating about its axis of intermediate moment of inertia $\left(I_Y > I_Z\right)$ and has a product of inertia $I_{XZ}$ of -373 slug-ft$^2$ present.

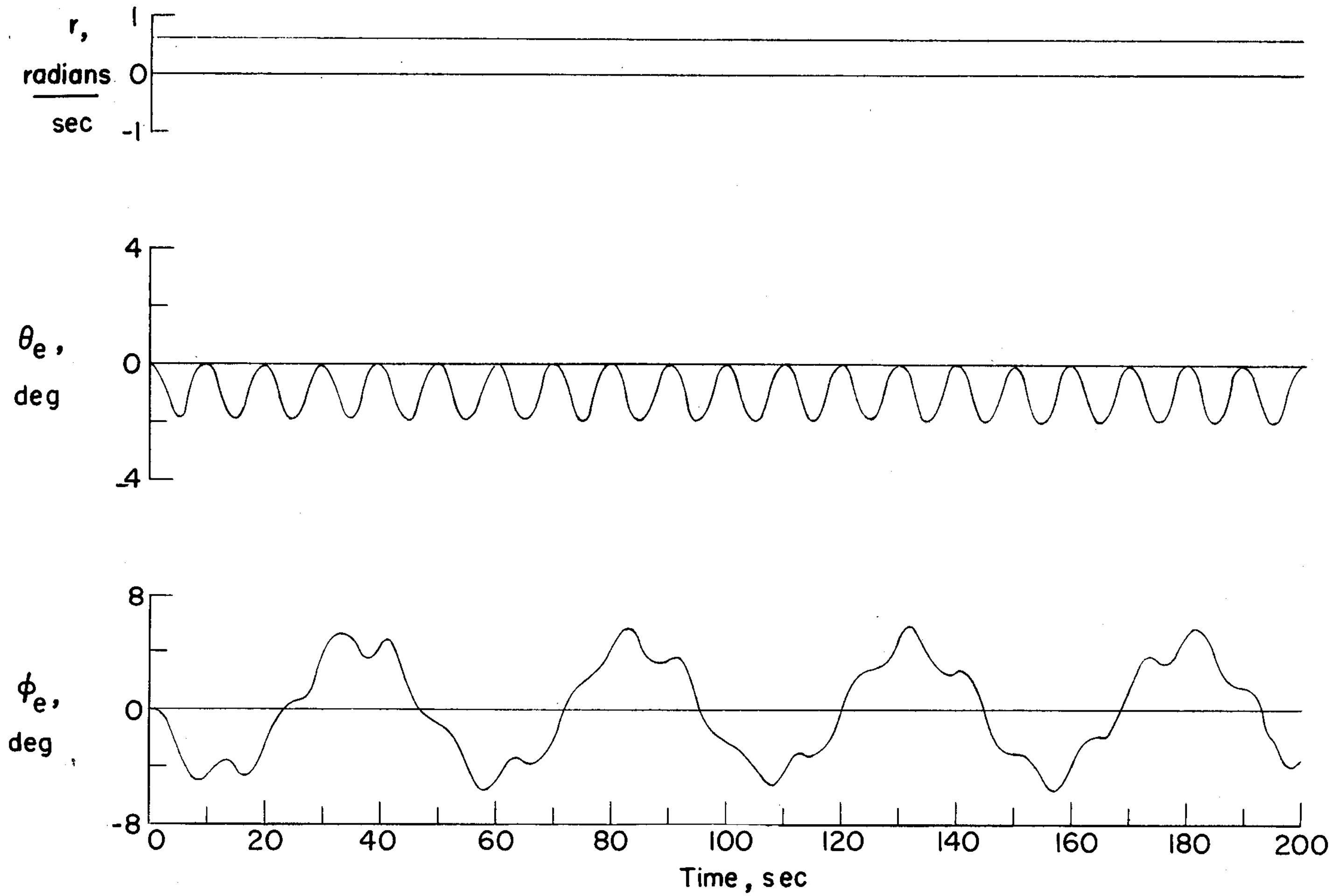


Figure 17.- Calculated motion indicating the effects of an inertia wheel, rotating about the Z-axis and producing +50 ft-lb-sec angular momentum, on the motion of the craft which is initially rotating about its axis of intermediate moment of inertia $\left(I_Y > I_Z\right)$ and has a product of inertia $I_{XZ}$ of -373 slug-ft$^2$ present.

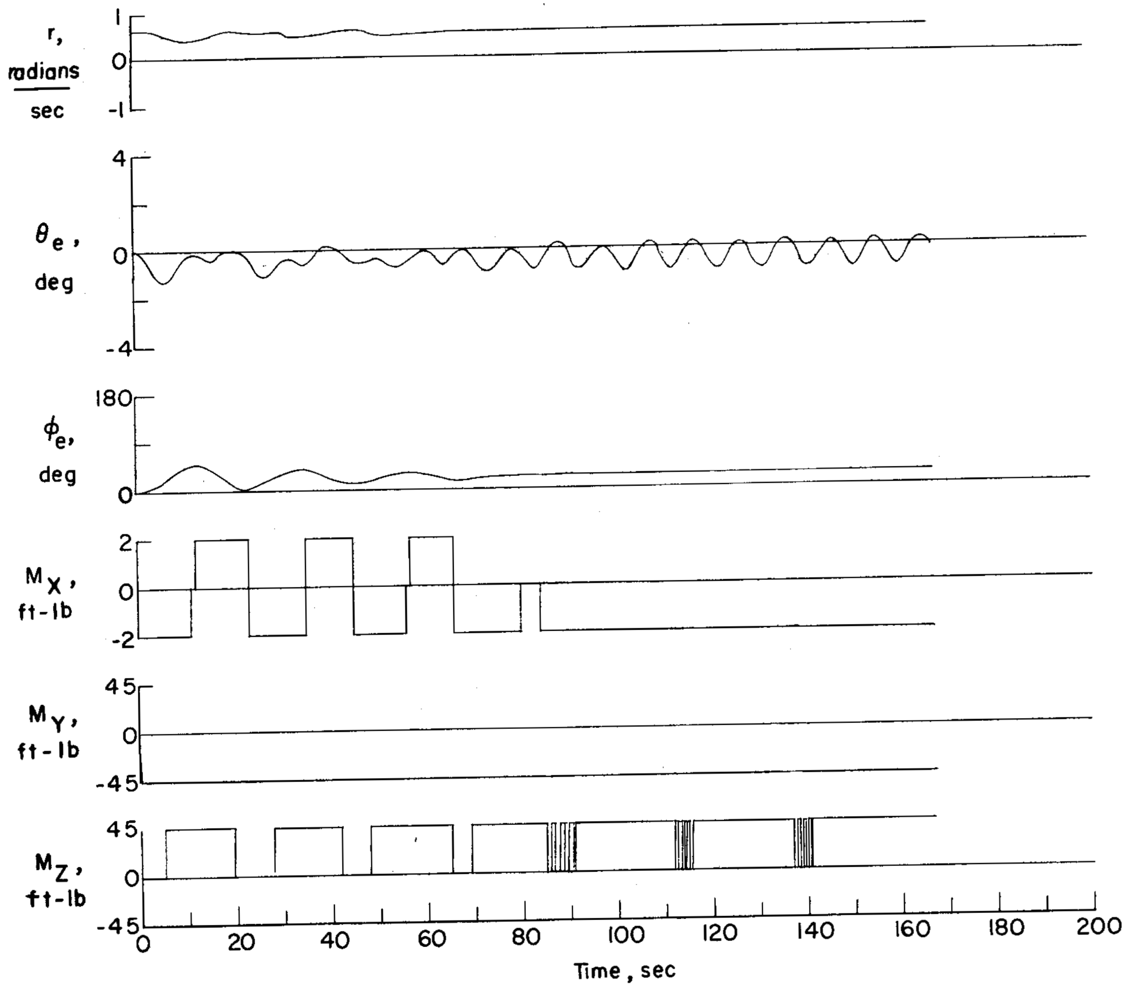


Figure 18.- Calculated motion indicating the effects of using three jets, when needed, to minimize the oscillatory motion presented in figure 6 and assuming that jets are available from time zero.

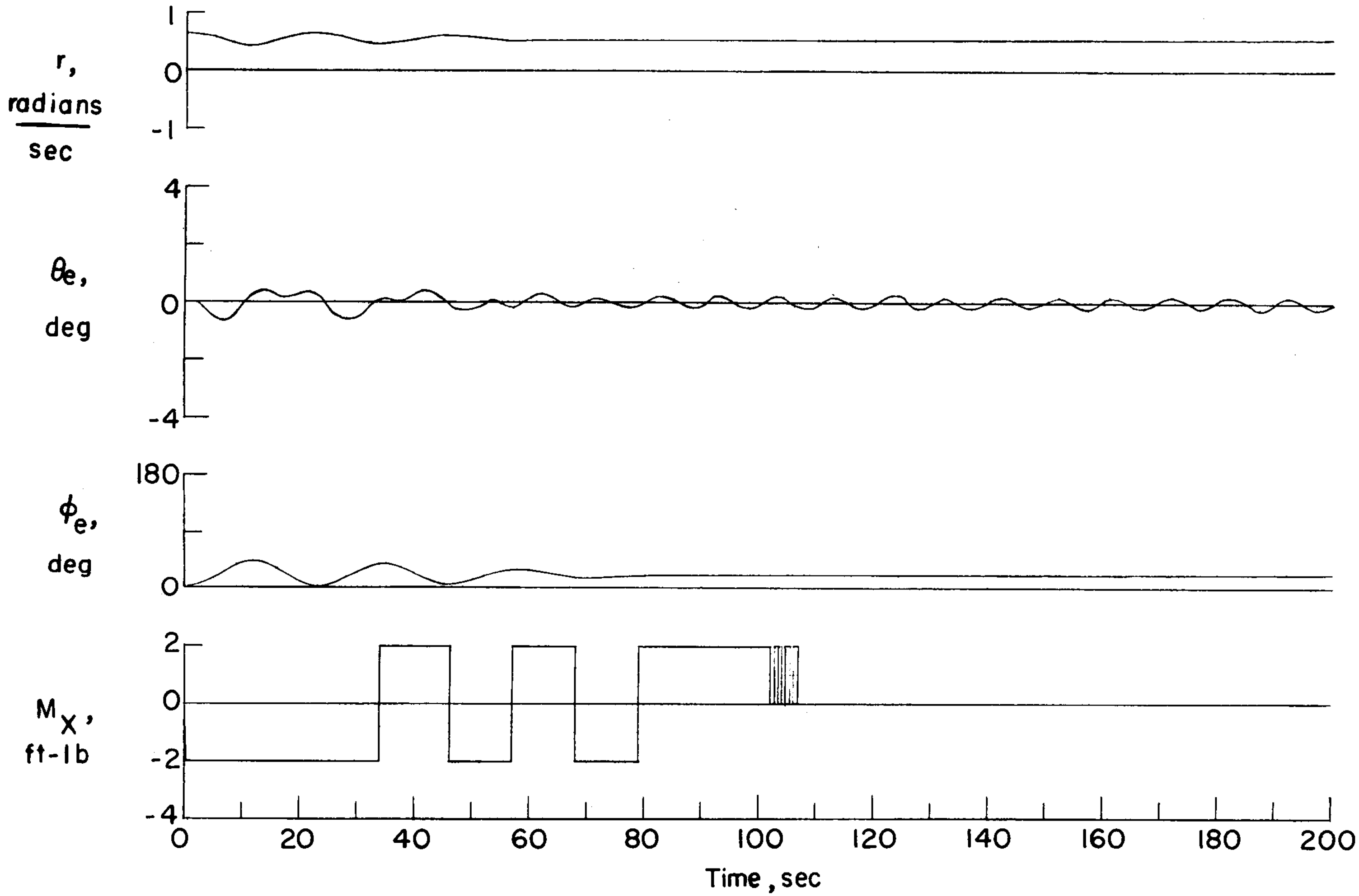


Figure 19.- Calculated motion indicating the effects of using one jet, when needed, to minimize the oscillatory motion presented in figure 6 and assuming that jet is available from time zero.

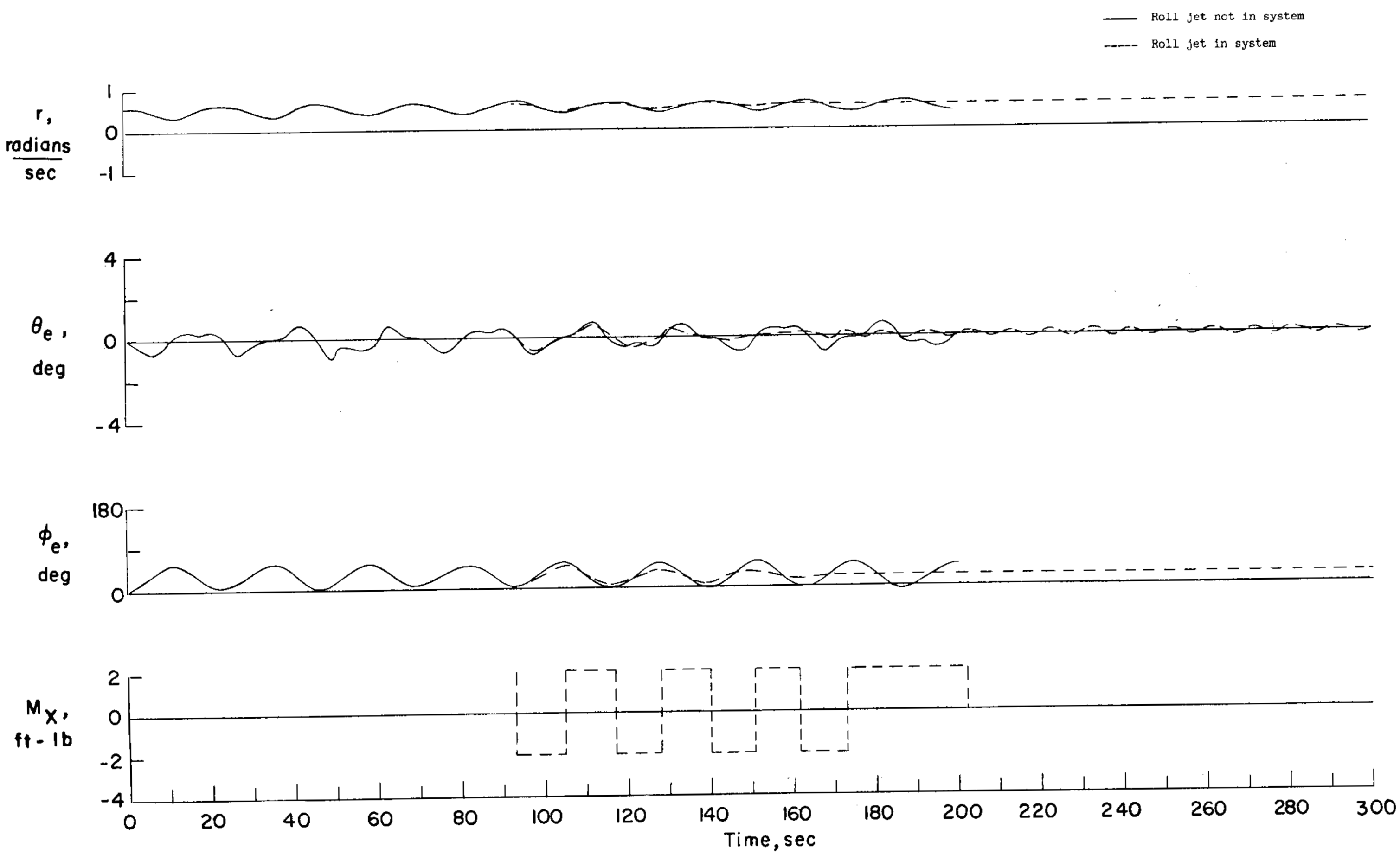


Figure 20.- Calculated motion indicating the effects of using one jet, when needed, to minimize the oscillatory motion presented in figure 6 and assuming that jet is available after 94 seconds.