Validation of the Poisson stochastic radiative transfer model

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ABSTRACT

A new approach to validation of the Poisson stochastic radiative transfer method is proposed. In contrast to other validations of stochastic models, the main parameter of the Poisson model responsible for cloud geometrical structure – cloud aspect ratio – is determined entirely by matching measurements and calculations of the direct solar radiation. If the measurements of the direct solar radiation is unavailable, it was shown that there is a range of the aspect ratios that allows the stochastic model to accurately approximate the average measurements of surface downward and cloud top upward fluxes. Realizations of the fractionally integrated cascade model are taken as a prototype of real measurements.

1. Background

Clouds cover significant part of the globe and are the key factor in the Earth radiation budget. Unfortunately, available information on cloud macrostructure and cloud microphysical properties is not sufficient for a unique determination of the radiation regime in a particular point and a given direction. However, it was long recognized (Stephens and Platt 1987) that some important conclusions on the dynamics of the radiation regime could be drawn from the averaged over the ensemble of realizations statistical characteristics of cloud and radiation fields. This gave an impulse to intense development of many stochastic approaches for the description of radiative transfer in cloudy atmosphere. Starting from a simple cloud model by Mullamaa et al. (1972), several different stochastic models for single-layer broken clouds have been developed (Kargin 1984; Titov 1990; Malvagi and Pomraning 1992; Barker et al. 1992; Malvagi et al. 1993; Kargin and Prigarin 1994; Prigarin and Titov 1996; Marshak et al 1998; Prigarin et al. 1998, 2001; Evans et al. 1999, 2000; etc.)

Though the main criterion of applicability for any stochastic cloud model is its agreement with measured statistical characteristics for both cloud and radiation fields, there are yet very few examples of model validations. Recently Lane et al. (2002) compared experimental and theoretical downwelling shortwave radiation (DWSR) at the surface. They used two models: (*i*) a horizontally homogeneous one-layer cloud model and (*ii*) the stochastic model developed by Byrne, Malvagi, Pomraning and Somerville (Malvagi et al. 1993; Malvagi and Pomraning 1993; Byrne et al. 1996). Observational data from the Atmospheric Radiation Measurements (ARM) Program Southern Great Plains (SGP) site were used to derive the necessary input. The main focus of the comparison was on broken cloud conditions characterized by fair-weather cumulus and cumulus fractus clouds. The discrepancies between observations and predicted by both models radiation fields to a great extent came from the uncertainties in input cloud microphysics. The paper showed that in most cases the calculations of DWSR at the surface by the one-layer homegeneous model fit observations better (by 30-35 Wm^{-2}) than the stochastic model. It was suggested that the main reason for the discrepancies between models is a limited ability of the stochastic model to simulate cloud vertical inhomogeneity. Indeed, the stochastic model splits a cloud field between consecutive atmospheric layers and since the distribution of clouds between adjacent layers is not correlated, all calculations are made assuming "cloud random overlap"; as a result, DWSR is substantially underestimated.

The goal of the present study is to check the validity and applicability of the *statistically homogeneous* Poisson stochastic cloud model proposed by Titov (1990). The earlier attempts to apply the Poisson model to experimental data (Titov 1990) showed that, in general, the radiative transfer processes in real broken clouds were well approximated by the model. Unfortunately, initially there was little data available for validation and even available measurements were incomplete. One of the main problems for validation of the Poisson model was the ambiguity in accurate estimation of the aspect ratio, one of the most important parameters of the model. Here we propose a new validation approach that determines the aspect ratio from measurements of direct solar radiation by matching the mean direct radiative fluxes with those calculated from the stochastic model. Instead of real data we use realizations of a modified version of the fractionally integrated cascade model (Schertzer and Lovejoy 1987) with the modifications suggested by Marshak et al. (1998) to simulate broken cloudiness. We assume that realizations of the fractionally integrated cascade model represent real measurements.

The outline of this paper is as follows. Poisson stochastic cloud model and the fractionally integrated cascade model are briefly described in Section 2. Section 3 discusses our approach to validation and shows some testing results. The conclusions are provided in Section 4.

2. Stochastic cloud models

a. Poisson stochastic cloud model

The detailed description of the one-layer Poisson stochastic model is given by Titov (1990). Here we briefly summarize only those points of the model that are relevant to the present research.

The *statistically homogeneous* Poisson model (called here the "Poisson model") is completely determined by only three parameters: cloud fraction N; cloud optical depth τ (assumed to be constant for all cloud elements); and aspect ratio $\gamma = H/D$ where H is the geometrical thickness of a cloud layer and D is the horizontal size of clouds. For statistically homogeneous cloud fields, a closed system of equations for mean intensity was obtained, and efficient algorithms of its solution by the Monte Carlo method were developed. The accuracy and the range of applicability of these equations were estimated by comparison with direct numerical simulations. The results of comparison showed that the equations for the mean intensity have an acceptable accuracy and, hence, can be used to study the influence of effects of random cloud geometry on radiative properties of broken clouds. The main advantage of the method of closed equations is that once the radiative transfer equation is analytically averaged (using some assumptions concerning probabilistic properties of the cloud field), the calculations of the average radiative characteristics of clouds are computationally fast and straightforward.

At the initial stage of the Poisson model validation (Titov 1990), there was little data and the available measurements were not sufficiently integrated, i.e., the radiative properties and optical and geometrical parameters of clouds were not simultaneously determined and tied with atmospheric and surface characteristics required in calculations. However, since the method of closed equations for calculations of statistical characteristics of radiation is computationally efficient, the question of validation became critical.

Validation is especially important since recent efforts to generalize the Poisson model to the case of *multilayer* broken clouds (Titov and Zhuravleva 1999; Prigarin et al. 2002; Kassianov 2003). First, Titov and Zhuravleva (1999) assumed statistical independence of cloud fields in each layer determined by an independent set of parameters (cloud fraction N, cloud optical depth τ, and aspect ratio γ). Then Prigarin et al. (2002) generalized the Poisson model to statistically dependent fields with a given correlation function for cloud fields at different vertical layers. They also studied the effect of vertical correlation on radiative transfer. Recently, Kassianov (2003) using a Marcovian approach derived approximate equations for direct and diffuse solar radiation in broken cloudiness with an arbitrary horizontal and vertical inhomogeneity. Kassianov et al. (2003) estimated the accuracy and robustness of these equations. In contrast to the Titov and Zhuravleva's (1999) model, the last two models require a lot of information on correlation between cloud fields at different vertical levels. So far this information is very limited but the recent increase in interest to complex cloud systems (Wang and Rossow 1995; Bergman and Hendon 1998; Wang et al 1999) gives us a hope that this information will be available in the near future. Our approach to test the statistically homogeneous Poisson models described in section 3 can be also useful for validation of the stochastic models of multilayer broken cloudiness.

b. Fractionally integrated cascade model

Each realization of the fractionally integrated cascade model (called here the "cascade model") has four well defined and easily estimated from real data parameters (Schertzer and Lovejoy 1987): two of them come from a single-point statistics (mean optical depth, τ_{mean}, and standard deviation, or rather a direct function of it, p), one comes from a two-point statistics (scaling exponent, β), and one is a cloud fraction, N (Marshak et al. 1998). The fractionally integrated model transforms singular cascades with spectral exponent $\beta < 1$ into a more realistic one with $\beta > 1$ using a power-law filtering in Fourier space. In physical space, this operation is known as "fractional integration"; thus the name of the model. For simplicity, through the whole paper $\beta = 5/3, p = 0.35$ (Cahalan 1994) were selected.

1) STATISTICAL CHARACTERISTICS OF CLOUDS

Because for given β and p, cloud fraction varies from one realization to another, it is reasonable to use cloud fractions in statistical sense with its mean, $\langle N \rangle$, standard deviation, σ_N, minimum, N_{min}, and maximum, N_{max}, as well as the probability density function $f(N)$. (Symbol $\langle \rangle$ is used for the ensemble-averaged statistics.)

Cloud fraction statistics can be estimated from a sample of M cloud realizations. Figure 1 shows that $M = 10^4$ cloud realizations are sufficient to represent adequately the cloud fraction statistics. Further increase of the number of realizations does not lead to significant changes in N statistics, though substantially increases computer time. Note that while going from one realization to another, not only the cloud fraction varies but there are also changes in τ_{min} and τ_{max} for a fixed average value τ_{mean}.

In addition to Fig. 1, Fig. 2 shows examples of statistical characteristics of the cloud fraction. As $<N>$ increases the distribution of cloud fraction becomes broader. Indeed, while at $<N>$ = 0.32 the cloud fraction variability range is $N_{max} - N_{min}$=0.07, at $<N>$ = 0.70 it is almost twice larger and is equal to 0.12.

2) STATISTICAL CHARACTERISTICS OF RADIATION

Let us first discuss how can we effectively calculate the radiative characteristics $<R>$ averaged over a number of realizations of the cascade model. Here by R we mean either direct transmitted radiation S, diffuse transmitted radiation Q_s, or albedo A.

The simplest way to calculate $<R>$ would be a sequential generating of realizations of a cloud field, "accurate" solution of the three-dimensional (3D) radiative transfer equation for each realization and finally statistical averaging over the ensemble of radiation fields. However, this way is at best inefficient; it requires tremendous amount of computer time since every cloud realization represents a complex inhomogeneous 3D medium.

To get efficiently the radiative characteristics averaged over a number of realizations we use the randomization procedure (Mikhailov 1986) that is based on introduction of an additional randomness. According to this approach, statistical characteristics of radiation can be obtained by estimating randomly chosen m independent trajectories of photons. The optimum number of photon trajectories for each realization is usually selected from special numerical tests. Our preliminary calculations showed that for estimation of average fluxes the optimal m =1 while for standard deviation we need m = 10000. To calculate the radiative characteristics for each cloud realization, the Monte Carlo "maximal cross section" method (Marchuk et al. 1981) has been used.

Let a unit solar flux be incident at the top of the cloud layer in direction $\Omega_0 = (\theta_0, \varphi_0)$ where θ_0 and φ_0 are zenith and azimuth solar angles, respectively. For simplicity, we assume here an absorbing surface and conservative cloud droplet scattering with the C1 scattering phase function (Deirmendjian 1969). In this case, $S + Q_s + A = 1$. No aerosols are taken into account. Pixel sizes are chosen to be 0.1 km×0.1 km (with 6 cascades, a modeled cloud field is 6.4 km×6.4 km) and periodic boundary conditions are assumed.

The stochastic nature of a radiative field comes from a random choice of the cloud fraction N and differences in cloud realizations that correspond to equal cloud fraction values. As an example, Fig. 3 shows two realizations of the cloud optical depth distribution with $N_1 = N_2 = \langle N \rangle = 0.515$. Note that the realization in Fig. 3b contains a large fragment of a clear sky. The geometry of the scene is such that for an oblique solar zenith angle and $\varphi_0 = 0$, the fraction of the direct radiation S will be much larger than of its Fig. 3a counterpart. Indeed, going from realization 1 to realization 2, S almost doubles, while the diffuse radiation, Q_s, decreases by ≈20%. The least sensitive to a cloud field realization is the albedo A; its variations do not exceed ≈10% in this example. In general, the radiative properties of the cascade model are characterized by strong variations.

To obtain the statistical characteristics of solar radiation for the realizations of the cloud model, we calculate the mean values $\langle R \rangle$ and the root-mean-square deviations σ_R of radiative fluxes. (Here again R stands for S, Q_s or A). The probability density functions $f(R)$ and variability ranges $(R_{\min}, R_{\max})$ of radiative fluxes are also of interest; however, because it requires much higher accuracy and thus more labor-intensive calculations than mean and standard deviation, we calculated $f(R)$, $R_{\min}$, and $R_{\max}$ only for some selected cases. An example of these calculations is shown in Fig. 4.

3. Validation of the Poisson broken cloud model

The main methodological aspect here is the development of a reasonable approach to specify the Poisson model parameters for validation of the mean radiative fluxes against realizations of the cascade model, which are taken as a prototype of real measurements. In contrast to Lane at al. (2002) and as the first approximation, we limit ourselves to a monochromatic case because broadband fluxes require integration over wavelength spectrum and have more uncertainties in parameterization of cloud optical characteristics, water vapor profiles, etc. – the complications we wanted to escape. There are two main methodological points determining our approach.

First, the Titov's (1990) approach permits the efficient calculation of the ensemble-averaged radiative characteristics $\langle R(\tau) \rangle_{pois}$ assuming that the cloud optical depth τ is a constant and does not change from one realization to another. In reality, cloud optical depth substantially varies horizontally inside a cloud field. To combine the Titov's model efficiency with the horizontal variability of cloud optical depth, we follow Barker et al. (1996) and assume that the distribution of cloud optical depth is well approximated (at least for marine low-level cumulus) by the gamma distribution

$$p_\Gamma(\tau,\nu)=\frac{1}{\Gamma(\nu)}\left(\frac{\nu}{\tau_{mean}}\right)^{\nu}\tau^{\nu-1}\exp\left(-\frac{\nu}{\tau_{mean}}\tau\right). \qquad (1)$$

Here parameter $\nu=\left(\frac{\tau_{mean}}{\sigma_\tau}\right)^2$ where τ_{mean} and σ_τ are the average and the standard deviation of the cloud optical depth distribution, respectively. Averaging $\langle R(\tau)\rangle_{pois}$ over the set of optical depth values yields:

$$\rangle R(\gamma,N,\theta_0)\langle_{pois}=\int_0^\infty \langle R(\tau,\gamma,N,\theta_0)\rangle_{pois}\, p_\Gamma(\tau,\nu)d\tau\,,\; R=S,\,A,\,Q_s. \qquad (2)$$

Symbol $\rangle R\langle_{pois}$ indicates that the radiative characteristics are averaged both over a set of cloud realizations and over a set of cloud optical depths.

Second, the main geometrical input parameter in Poisson model is the aspect ratio γ; to define it the information on cloud vertical thickness H and its horizontal sizes D is required. The most accurate but perhaps the most difficult way to estimate *jointly* H (or rather cloud base height and cloud top height) and D is to have a set of complex measurements as it was done by Lane et al. (2002). In case of a lack of direct measurements, the distribution of geometrical characteristics of a certain type of clouds is usually taken from the literature. For example, Plank (1969) found that for Florida cumulus clouds, the number density increased exponentially with decreasing cloud size. Cahalan and Joseph (1989) argued that for California marine Sc cloud sizes follow a power-law distribution. Hozumi et al. (1982), Wielicki and Welch (1986), Benner and Curry (1998) among others characterized the dimensions and spatial distribution of cumulus clouds. A thorough statistical discussion on an exponential versa a power-law distribution of cloud sizes can be found in Astin and Latter (1998). However, because vertical and geometrical cloud sizes strongly depend on the region of observation, season, type of clouds, their altitude, etc., the climatological approach to the choice of aspect ratio is often a source of large uncertainties and in many cases is inappropriate for the validation of stochastic models.

Based on the above, in addition to the average cloud optical depth τ_{mean} and standard deviation σ_τ, we propose to determine the other two parameters of the Poisson model as follows: (*i*) cloud fraction N to be equal to $<N>$ and (*ii*) the aspect ratio γ to be chosen in such a way that for $N = <N>$,

$$\rangle S(\gamma, N, \theta_0) \langle_{pois} = \langle S(N, \theta_0) \rangle, \tag{3}$$

i.e., the average direct radiation for the Poisson model, $\rangle S(\gamma, N, \theta_0) \langle_{pois}$ coincides with the direct radiation averaged over all realizations of the cascade model, $\langle S(N, \theta_0) \rangle$.

Next we take the statistical approach first stating and then verifying the following two hypotheses.

Hypothesis 1. *If for a given oblique solar zenith angle* $\theta_0 > 0$ *and a cloud fraction* $N = \langle N \rangle$*, the aspect ratio* $\gamma(\theta_0)$ *is determined from Eq. (3) then the calculated average albedo* $\rangle A(\gamma(\theta_0), N, \theta_0) \langle_{pois}$ *and transmittance* $\rangle Q_s(\gamma(\theta_0), N, \theta_0) \langle_{pois}$ *will be within the confidence intervals defined by the standard deviations of the cascade model,*

$$\rangle R(\gamma(\theta_0), N, \theta_0) \langle_{pois} \in \left[\langle R(N, \theta_0) \rangle - \sigma_R(N, \theta_0), \langle R(N, \theta_0) \rangle + \sigma_R(N, \theta_0)\right], \quad R = A,\ Q_s. \tag{4}$$

Assume that θ_0 is fixed. Since the mean flux of the direct radiation in the Poisson model for $\theta_0 = 0$ does not depend on γ, we will compare the mean values of the albedo and diffuse transmittance only for the oblique solar angles $10^\circ \le \theta_0 \le 75^\circ$. Our intense numerical calculations summarized in Table 1 confirm that, when γ is specified by (3), Eq. (4) is valid. Note that here, for given $<N>$, τ_{mean}, and σ_τ, the γ value depends on θ_0, i.e., $\gamma = \gamma(\theta_0)$.

Hypothesis 2. *For a fixed cloud fraction $N=<N>$, there is a range of the aspect ratios $\gamma \in [\gamma_{\min}, \gamma_{\max}]$ that for any solar zenith angle in the range $0 \le \theta_0 \le 75^o$:*

$$\langle R(\gamma, N, \theta_0)\rangle_{pois} \in [\langle R(N,\theta_0)\rangle - \sigma_R(N,\theta_0), \langle R(N,\theta_0)\rangle + \sigma_R(N,\theta_0)],\quad R=S, A, Q_s. \tag{5}$$

To test the second hypothesis, we use the following approach. For given $<N>$, τ_{mean} and σ_τ, and fixed solar zenith angle $10^\circ \le \theta_0 \le 75^\circ$, we first calculate the mean direct radiation, $\langle S(N,\theta_0)\rangle$, and its root-mean-square deviation, $\sigma_S(N,\theta_0)$. Next, for the Poisson models, we select $\gamma_{\min}(\theta_0)$ and $\gamma_{\max}(\theta_0)$ in such a way that

$$\begin{aligned} \langle S(\gamma_{\min}(\theta_0), N, \theta_0)\rangle_{pois} &= \langle S(N,\theta_0)\rangle + \sigma_S(N,\theta_0), \\ \langle S(\gamma_{\max}(\theta_0), N, \theta_0)\rangle_{pois} &= \langle S(N,\theta_0)\rangle - \sigma_S(N,\theta_0). \end{aligned} \tag{6}$$

Finally, for chosen $\gamma_{\min}(\theta_0)$ and $\gamma_{\max}(\theta_0)$ we calculate the mean values of A and Q_s.

Figure 5 shows $\gamma_{\min}(\theta_0)$ and $\gamma_{\max}(\theta_0)$ for θ_0 varying in the range $10^\circ < \theta_0 \le 75^\circ$. Evidently, there is a common region of the aspect ratios $(\gamma_{\min}, \gamma_{\max})$ for the entire angular range $10^\circ < \theta_0 \le 75^\circ$. Moreover, because of weak dependence of the direct radiation in the Poisson model on parameter γ for $\theta_0 = 0$, it can be extended to $0^\circ \le \theta_0 \le 75^\circ$. This means that with respect to γ, Eq. (4) will be valid for all solar zenith angles $0^\circ \le \theta_0 \le 75^\circ$. For $<N>=0.515$ and $\tau_{mean}=13$, $\gamma_{\min}$ and $\gamma_{\max}$ correspond approximately to the aspect ratio for $\theta_0 = 30^\circ$ and are found to be 1.33 and 1.93, respectively.

Qualitatively, the above conclusions also apply to other mean cloud fractions and mean cloud optical depths (see Tables 2 and 3). It follows from these results that for a wide range of model input parameters $0^\circ \le \theta_0 \le 75^\circ$, $6 \le \tau_{mean} \le 26$, there is a set of the aspect ratios around $\gamma^* \approx 5/3$ which can be used to calculate mean radiative fluxes in the Poisson cloud model with an acceptable accuracy. These values of the aspect ratio are in good agreement with their climatological values as well as with the results of complex measurements of horizontal and optical sizes of cumulus clouds (Lane et al. 2002; Plank 1969; Cahalan and Joseph 1989; Hozumi et al. 1982; Wielicki and Welch 1986; Benner and Curry 1998).

4. Summary

The proposed approach allowed us to validate the stochastic Poisson model of broken clouds (Titov 1990) against realizations of the cascade cloud model that served as a prototype of real measurements. The results of the validation test suggest that the Poisson cloud model can be successfully used to calculate the mean radiative properties of broken clouds. As soon as we know the average cloud fraction and mean and variance of the in-cloud optical depth (assumed to be gamma distributed), we can estimate the average radiative transfer characteristic by setting the aspect ratio in the Poisson stochastic model to 5/3 for any reasonable solar zenith angles. If, in addition, we know the direct radiation, the aspect ratio can be determined more accurately from the condition of matching the mean direct radiative fluxes with those calculated from the Poisson cloud model.

In this study, the fractionally integrated cascade model determined the input parameters for the Poisson model. The next step will be the use of cloud properties retrieved from satellite (MODIS and MISR) and ground-based (ARM SGP site) observations: cloud-base and cloud top heights, cloud fraction and cloud optical depth. These data will be used to determine the input parameters for the Poisson model to validate it against the data from the ARM's shortwave radiometer archive.

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TABLES

TABLE 1. Mean solar radiation fluxes calculated by the cascade and Poisson models: τ_{mean} = 13, σ_τ =11.9, <*N*> = 0.515. Cloud geometrical thickness H = 1 km.

$\theta_0 = 60°$		$\theta_0 = 75°$	
Cascade model	**Poisson model** $\gamma = 1.56$, $D = 0.64\ km$	**Cascade model**	**Poisson model** $\gamma = 2.04$, $D = 0.49\ km$
$\langle S \rangle = 0.19, \sigma_S = 0.04$	$\rangle S \langle_{pois} = 0.191$	$\langle S \rangle = 0.047, \sigma_S = 0.05$	$\rangle S \langle_{pois} = 0.047$
$\langle A \rangle = 0.356, \sigma_A = 0.015$	$\rangle A \langle_{pois} = 0.348$	$\langle A \rangle = 0.517,\ \sigma_A = 0.03$	$\rangle A \langle_{pois} = 0.512$
$\langle Q_s \rangle = 0.454$	$\rangle Q_s \langle_{pois} = 0.461$	$\langle Q_s \rangle = 0.436$	$\rangle Q_s \langle_{pois} = 0.441$

TABLE 2. Parameters $\gamma_{\min}$ and $\gamma_{\max}$ for different mean cloud fractions $\langle N \rangle$. Here $\tau_{mean} = 13$ and $\sigma_\tau = 11.9$.

	$\langle N \rangle = 0.318$	$\langle N \rangle = 0.515$	$\langle N \rangle = 0.701$
$\gamma_{\min}$	1.52	1.35	1.25
$\gamma_{\max}$	1.93	1.93	1.93

TABLE 3. Parameters $\gamma_{\min}$ and $\gamma_{\max}$ for different values of mean optical depth τ_{mean} and σ_τ. Mean cloud fraction $\langle N \rangle = 0.515$.

	$\tau_{mean} = 6$, σ_τ=5.47	$\tau_{mean} = 13$, σ_τ=11.9	$\tau_{mean} = 26$, σ_τ=23.7
$\gamma_{\min}$	1.15	1.35	1.47
$\gamma_{\max}$	1.92	1.93	1.75

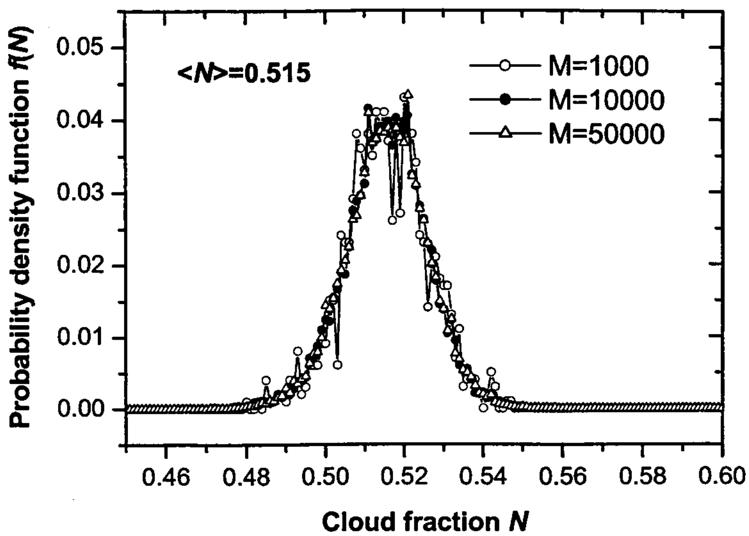

FIG. 1. The influence of the number of cloud realizations M on estimate of the probability density function $f(N)$ for a given $\langle N \rangle$.

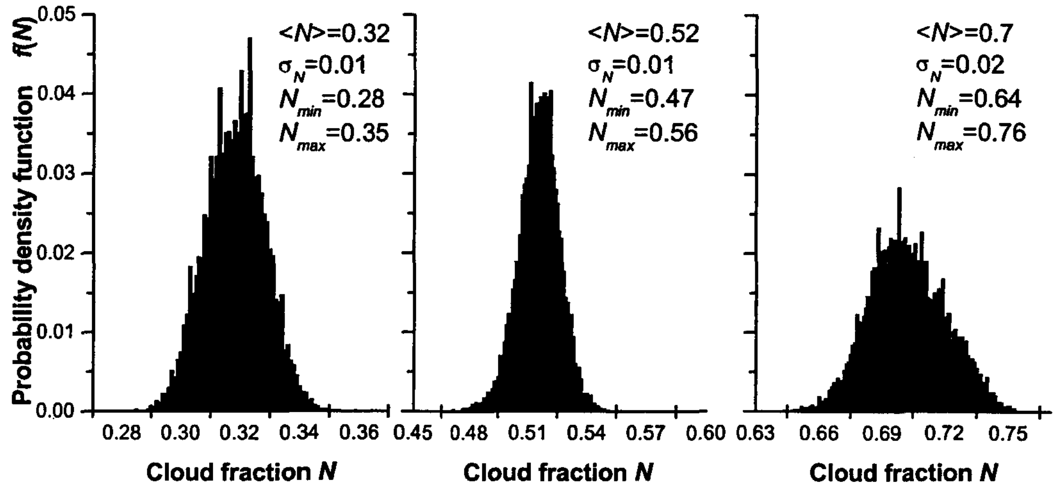

FIG. 2. Statistical characteristics of cloud fraction in the modified version of fractionally integrated cascade model for different values of the mean cloud fraction $<N>$ with number of cascades L=6, $\beta = 5/3$, p=0.35 and the number of cloud realizations M=10000.

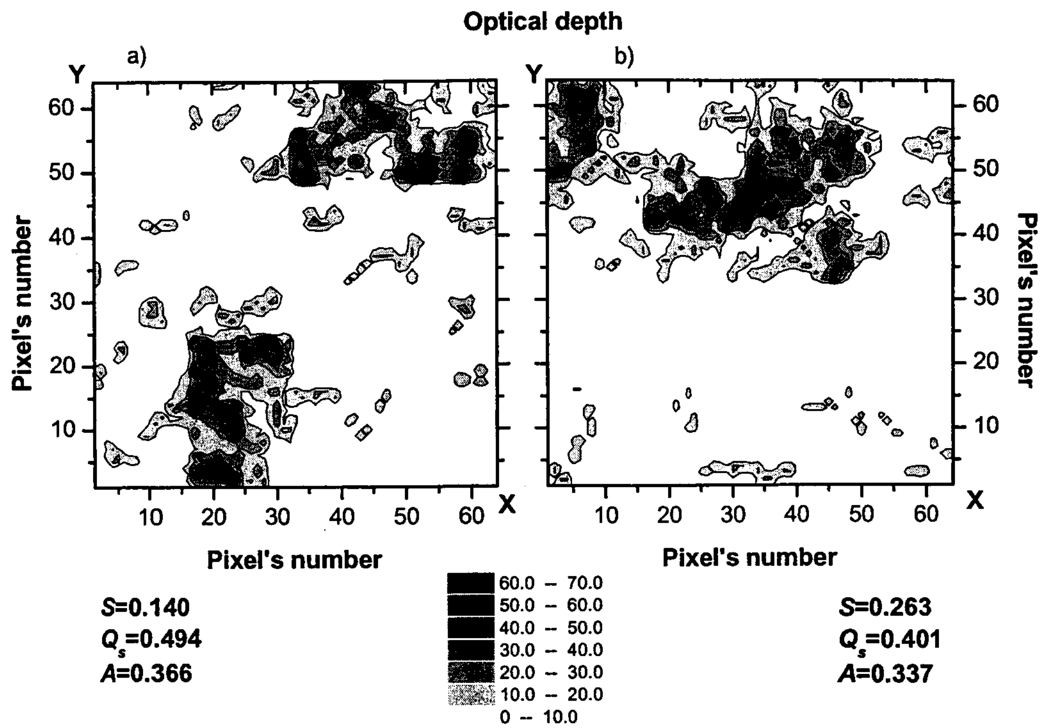

FIG. 3.. Two realizations of the cascade models that correspond to the same (average) cloud fraction $<N>$ = 0.515. Illumination (θ_0=60^0 and φ_0=0^0) is the same for both realizations. The calculated radiative characteristics for both realizations are also shown. Here S, Q_s and A are direct transmittance, diffuse transmittance and albedo, respectively.

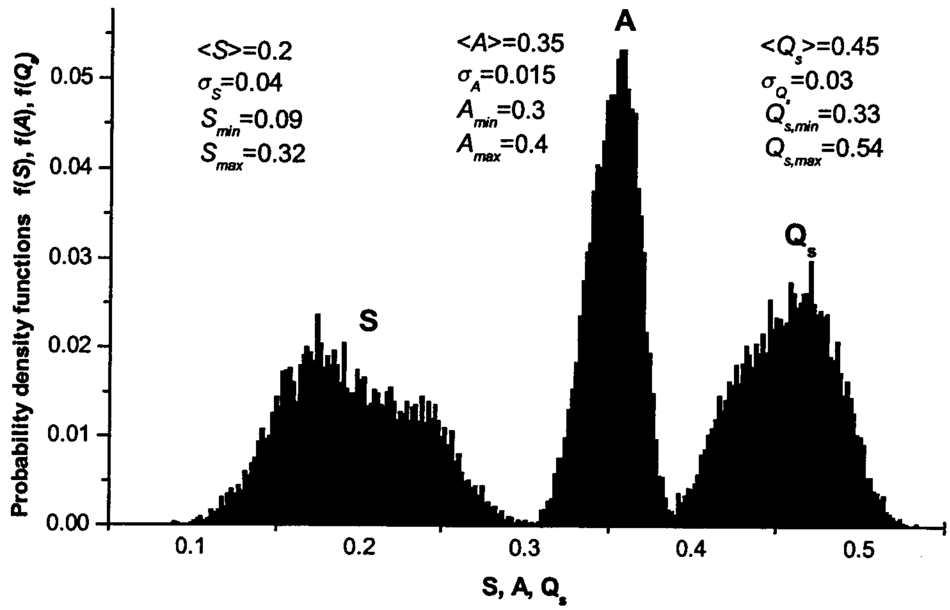

FIG. 4. Statistical characteristics of albedo A, direct radiation S and diffuse radiation Q_s for the cascade model with $\beta = 5/3$, $<N>$=0.515. Mean optical depth τ_{mean}=13, standard deviation σ_τ=11.9, pixel size 0.1 km×0.1 km, cloud thickness H=1 km, and solar zenith angle $\theta_0 = 60°$.

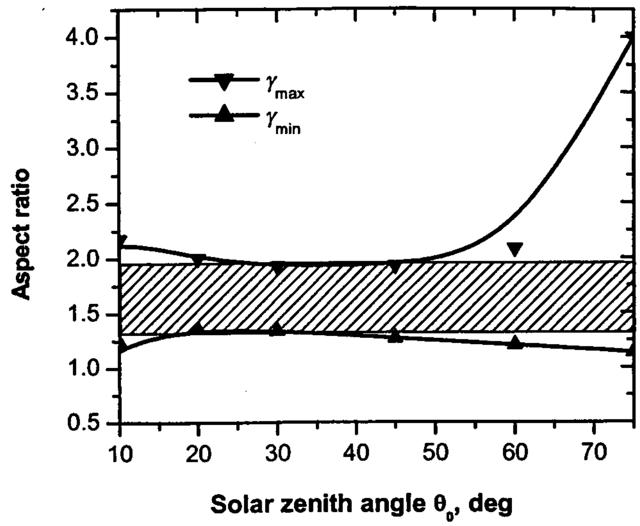

FIG. 5. Variability range of the aspect ratio γ in the Poisson cloud model for which Eq. (5) holds. The hatched region corresponds to the values of γ common for the entire range $0 \leq \theta_0 \leq 75°$. Mean optical depth τ_{mean}=13, standard deviation σ_τ=11.9, cloud fraction N=0.515.