PSEUDO LINEAR ATTITUDE DETERMINATION OF SPINNING SPACECRAFT

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This paper presents the overall mathematical model and results from pseudo linear recursive estimators of attitude and rate for a spinning spacecraft. The measurements considered are vector measurements obtained by sun-sensors, fixed head star trackers, horizon sensors, and three axis magnetometers. Two filters are proposed for estimating the attitude as well as the angular rate vector. One filter, called the \( q \)-Filter, yields the attitude estimate as a quaternion estimate, and the other filter, called the \( D \)-Filter, yields the estimated direction cosine matrix. Because the spacecraft is gyro-less, Euler's equation of angular motion of rigid bodies is used to enable the estimation of the angular velocity. A simpler Markov model is suggested as a replacement for Euler's equation in the case where the vector measurements are obtained at high rates relative to the spacecraft angular rate.

Extended Abstract

\( q \)-Filter Dynamics

The first dynamics equation we consider is the following Euler's equation for the angular motion of a spacecraft (SC). It is [1, pp. 522, 523]

\[
\dot{\omega} = I^{-1}[(I\omega + h) \times \omega] + I^{-1}(T - \dot{h})
\]

(1)

where \( I \) is the SC inertia matrix, \( \omega \) is the angular velocity vector, \( h \) is the angular momentum of the momentum wheels, and \( T \) is the external torque acting on the SC. The symbol \( [a \times] \) denotes the cross product matrix of the general vector \( a \). Attitude is represented by the attitude quaternion whose kinematic equation is [1, p. 512]

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\[ \dot{q} = \frac{1}{2} Q \omega \]  

(2)

where

\[
Q = \begin{bmatrix}
q_4 & -q_3 & q_2 \\
q_3 & q_4 & -q_1 \\
-q_2 & q_1 & q_4 \\
-q_1 & -q_2 & -q_3
\end{bmatrix}
\]  

(3)

In the q-filter we augment Eqs. (1) and (2) to form the following dynamics equation, which includes the noise terms

\[
\begin{bmatrix}
\dot{\omega} \\
\dot{q}
\end{bmatrix} = \begin{bmatrix}
\mathbf{I}^{-1}[(I\omega + h) \times] & 0 \\
\frac{1}{2} Q & 0
\end{bmatrix} \begin{bmatrix}
\omega \\
q
\end{bmatrix} + \begin{bmatrix}
\mathbf{I}^{-1}(T - \dot{h}) \\
0
\end{bmatrix} \begin{bmatrix}
w_\omega \\
w_q
\end{bmatrix}
\]  

(4.a)

The unbiased white-noise vector \( w_\omega \) accounts for the inaccuracies in the modeling of the SC angular dynamics, and \( w_q \) is an unbiased white-noise vector that accounts for modeling errors in the quaternion kinematics.

When the measurements come at a relatively high frequency we may be able to replace the SC angular dynamics model in Eq. (4.a) with a simpler Markov model [2]. Consequently, Eq. (4.a) is replaced by the model

\[
\begin{bmatrix}
\dot{\omega} \\
\dot{q}
\end{bmatrix} = \begin{bmatrix}
-\tau & 0 \\
\frac{1}{2} Q & 0
\end{bmatrix} \begin{bmatrix}
\omega \\
q
\end{bmatrix} + \begin{bmatrix}
w_\omega \\
w_q
\end{bmatrix}
\]  

(4.b)

where \( \tau \) is a diagonal matrix whose elements are the inverse of suitable time constants.

\textbf{q-Filter Measurement Model}

\[ b_{jm} = \begin{bmatrix} 0_3 & H_j(r, q) \end{bmatrix} \begin{bmatrix} \omega \\ q \end{bmatrix} + v_{jb} \]  

(5)

where
is the reference vector corresponding to vector sensor \( j \), and \( v_{j}\) is white noise.

**D-Filter Dynamics**

Using Euler’s equation and assuming the spacecraft attitude is represented as a direction cosine matrix, the dynamics take on the following form:

\[
\begin{bmatrix}
\dot{\mathbf{\omega}} \\
\dot{d}
\end{bmatrix} = \begin{bmatrix}
I^{-1}[(J\omega + h)\times] & 0 \\
D & 0
\end{bmatrix} \begin{bmatrix}
\mathbf{\omega} \\
d
\end{bmatrix} + \begin{bmatrix}
I^{-1}(T - \dot{h}) \\
0
\end{bmatrix} + \begin{bmatrix}
w_{\omega} \\
w_{d}
\end{bmatrix}
\]  \tag{6}

where \( d^T = [d_1^T \quad d_2^T \quad d_3^T] \), \( d_j^T \) is the transpose of the \( j \)th column of the direction cosine matrix, and \( D = \begin{bmatrix}
[d_1 \times] \\
[d_2 \times] \\
[d_3 \times]
\end{bmatrix} \) where \([d_j \times]\) is the skew symmetric matrix for \( j \)th column of the direction cosine matrix.

**The D-Filter Measurement Model**

For vector measurements, \( b_{jm} = [d_1r_j \mid d_2r_j \mid d_3r_j] + v_{j,b} \) where \( r \) is the corresponding reference vector for the observation and \( d_i \) is the \( i \)th column of the direction cosine matrix. This equation can be rearranged to form the measurement model:
Conclusion

Both the q-Filter and the D-Filter will be tested against simulated data and a comparison will be made of the relative performance of each.

References
