Magnetic Circuit Model of PM Motor-Generator to Predict Radial Forces

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Abstract

A magnetic circuit model is developed for a PM motor for flywheel applications. A sample motor is designed and modeled. Motor configuration and selection of materials is discussed, and the choice of winding configuration is described. A magnetic circuit model is described, which includes the stator back iron, rotor yoke, permanent magnets, air gaps and the stator teeth. Iterative solution of this model yields flux linkages, back EMF, torque, power, and radial force at the rotor caused by eccentricity. Calculated radial forces are then used to determine motor negative stiffness.

Introduction

High speed flywheel systems are currently under development at NASA Glenn Research Center (GRC) in Cleveland, Ohio. Flywheels are being considered as an alternative to batteries and reaction wheels in space vehicles. Strengths of this technology include high energy density, long life, deep depth of discharge, and pulse power capability. Also, flywheels can be deployed in an array which provides both energy storage and attitude control. A system level flywheel testbed is operational at GRC. The system uses active magnetic bearings to provide a long life, low-loss suspension of the rotor. Permanent magnet (PM) motor-generators are used to store and retrieve energy from the rotor.

This paper discusses a magnetic circuit model which will be used as a design tool for subsequent flywheel motor-generator designs. A sample motor-generator is developed to validate the design code. For simplicity, the motor-generator unit will be referred to simply as a motor for the remainder of the paper.

Nomenclature

“back iron” designates stator back iron
“yoke” designates rotor back iron
cross sectional area of stator tooth tip
flux density of stator tooth tip
slot depth
rotor inner diameter
stator outer diameter
MMF vector (A/turns)
mechanical air gap
harmonic number
unit vectors in x and y direction
winding distribution factor
pitch factor
reluctance (H⁻¹)
back emf
coil pitch
permanent magnet thickness
yoke width
number of coils per phase belt
arc length of coils in winding group
flux vector (Wb)
torque
pole pitch
flux linkage
mechanical speed (rad/sec)
**Design Approach**

A variety of methods are brought together to design the motor. The following is a quick summary of the design approach used:

1. Select motor configuration and materials
2. Optimize fractional pitch
3. Optimize winding distribution factor
4. Define motor magnetic circuit model
5. Solve model iteratively for flux (360°)
6. Calculate flux linkages, and back EMF
7. Calculate power and torque versus angle
8. Calculate radial force versus rotor displacement

Details describing implementation of each of the above steps are presented in the following sections.

**Motor Configuration**

The motor configuration chosen for the flywheel design is a PM, radial gap, outer stator, inner rotor configuration; the motor will be a three phase, four pole design with 24 slots. This configuration was selected based on previous experience with motors used in flywheel applications; size of the motor is also defined by the specific application. The layout for this motor is shown schematically in Figure 1.

All of the iron parts (the stator back iron, the stator teeth and the rotor yoke) are made of an iron cobalt alloy, which was selected for its ability to carry high flux densities (~2.2 Tesla), and for its high strength, which will allow high speed operation.

The two candidate permanent magnet materials were samarium cobalt and neodymium iron boron; samarium cobalt was selected due to its superior high temperature performance. For the analysis performed in this paper, the magnets are assumed to have constant radial magnetization.

**Motor Winding Configuration**

Next, the fractional pitch is selected, to optimize harmonic content of the MMF harmonics. The pitch factor for a given harmonic is given as [1]:

\[
k_{ph} = \sin \left( \frac{h W}{\tau_p} \frac{\pi}{2} \right) \sin \left( \frac{h \pi}{2} \right)
\]  

The amplitude of the MMF harmonics is proportional to \( k_{ph} \); we plot this versus the fractional pitch, which is defined as \( \frac{W}{\tau_p} \), and select fractional pitch to optimize desired harmonic content. The plot is shown in Figure 2. Note that the harmonic number, \( h \), has only odd values possible, since only odd harmonics exist in this motor.

For this design, we wish to minimize third harmonic content. One full pitch for this motor is 24 slots/4 poles = 6 slots; thus reasonable options are 3/6, 4/6, 5/6 fractional pitch or 6/6 full pitch. Since third harmonic...
content in Figure 2 is minimized with a pitch factor of 0.67, a coil pitch of 4 was selected.

The winding structure for the motor stator is double layer lap wound. The winding distribution factor is given as [1]:

\[
k_{dh} = \frac{1}{q} \sin \left( \frac{hz \pi}{2q} \right) \sin \left( \frac{hz \pi}{2q} \right)
\]

(2)

where \(z/\tau_p\) is the fraction of a pole pitch that is occupied by the phase belt. With a four pole, three phase, 24 slot, 4/6 fractional pitch machine, each 360 electrical degree phase belt has four coils, and the only design options available are a 60° or a 120° phase belt. Since considerably more voltage is generated by a 60° phase belt \((k_{dl} = 0.958\) for a 60° phase belt versus 0.837 for a 120° phase belt), the 60° phase belt was selected.

With all three phases, 360 electrical degrees is spanned by 12 coils, at 30° increments. The electrical phase relationship of these coils is shown in Figure 3.

Figure 4 shows the MMF’s of the coils when they are collected into 60° degree phase belts. Note that some of the coils are wound in with negative polarities.

Finally, the winding layout of all three phases is shown in Figure 5.

**Magnetic Circuit Model**

Next, the motor is modeled as a magnetic circuit. Reluctance elements are included in the model for the back iron, stator teeth, air gap, and rotor yoke. MMF sources in the model include the PMs and the windings enclosing the stator teeth. The PM MMF is determined using the demagnetization curve for the magnetic material, and the windings that enclose each tooth are
summed together to determine the magnitude of the tooth MMF. The schematic of the motor magnetic circuit is shown in Figure 6.

This magnetic circuit is solved by mesh currents. Summing the MMF around the loop containing tooth 1 and tooth 2 yields:

\[-\phi_1 R_{\text{backiron}} - F_{\text{tooth2}} - F_{\text{PM2}} - (\phi_1 - \phi_2) (R_{\text{tooth2}} + R_{\text{gap2}}) - \phi_1 R_{\text{yoke1}} - (\phi_1 - \phi_{24}) (R_{\text{tooth1}} + R_{\text{gap1}}) + F_{\text{PM1}} + F_{\text{tooth1}} = 0\]  \hspace{1cm} (3)

Rearranging Eq. (3) yields:

\[-(R_{\text{backiron}} - R_{\text{tooth2}} - R_{\text{gap2}} - R_{\text{yoke1}} - R_{\text{tooth1}} - R_{\text{gap1}}) \phi_1 + (R_{\text{tooth2}} + R_{\text{gap2}}) \phi_2 + (R_{\text{tooth1}} + R_{\text{gap1}}) \phi_{24} = F_{\text{tooth2}} + F_{\text{PM2}} - F_{\text{PM1}} - F_{\text{tooth1}}\]  \hspace{1cm} (4)

Using Eq. (4), a generalized expression for loops 2 to 23 can be derived:

\[-(R_{\text{backiron}}(n) - R_{\text{tooth}(n+1)} - R_{\text{gap}(n+1)} - R_{\text{yoke}(n)}) - (R_{\text{tooth}(n)} - R_{\text{gap}(n)}) \phi_n + (R_{\text{tooth}(n+1)} + R_{\text{gap}(n+1)}) \phi_{n+1} + (R_{\text{tooth}(n)} + R_{\text{gap}(n)}) \phi_{n-1} = F_{\text{tooth}(n+1)} + F_{\text{PM}(n+1)} - F_{\text{PM}(n)} - F_{\text{tooth}(n)}\]  \hspace{1cm} (5)

Rewriting Eq. (5) for the case of loop 24 yields:

\[-(R_{\text{backiron24}} - R_{\text{tooth1}} - R_{\text{gap1}} - R_{\text{yoke24}} - R_{\text{tooth24}} + R_{\text{gap24}}) \phi_{24} + (R_{\text{tooth1}} + R_{\text{gap1}}) \phi_1 = F_{\text{tooth1}} + F_{\text{PM1}} - F_{\text{PM24}} - F_{\text{tooth24}}\]  \hspace{1cm} (6)

Using Eq. (5), define the following lumped reluctance quantities for the first 23 loops:

\[X_n = -R_{\text{tooth}(n)} - R_{\text{gap}(n)} - R_{\text{backiron}(n)} - R_{\text{tooth}(n+1)} - R_{\text{gap}(n+1)} - R_{\text{yoke}(n)}\]  \hspace{1cm} (7)

\[Y_n = R_{\text{tooth}(n+1)} + R_{\text{gap}(n+1)}\]  \hspace{1cm} (8)

\[Z_n = R_{\text{tooth}(n)} + R_{\text{gap}(n)}\]  \hspace{1cm} (9)

Using Eq. (6), the lumped reluctance quantities for loop 24 are:

\[X_{24} = -R_{\text{tooth24}} - R_{\text{gap24}} - R_{\text{backiron24}} - R_{\text{tooth1}} - R_{\text{gap1}} - R_{\text{yoke24}}\]  \hspace{1cm} (10)

\[Y_{24} = R_{\text{tooth24}} + R_{\text{gap24}}\]  \hspace{1cm} (11)

\[Z_{24} = R_{\text{tooth24}} + R_{\text{gap24}}\]  \hspace{1cm} (12)

Using Eq. (7) through (12), we now define the reluctance matrix as:

\[
\begin{bmatrix}
X_1 & Y_1 & 0 & \cdots & \cdots & 0 & Z_1 \\
Z_2 & X_2 & Y_2 & 0 & \cdots & \cdots & 0 \\
& \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
Y_{24} & 0 & \cdots & \cdots & 0 & Z_{24} & X_{24}
\end{bmatrix}
\]  \hspace{1cm} (13)

Now define the following MMF’s for the first 23 loops:

\[F_n = F_{\text{tooth}(n+1)} + F_{\text{PM}(n+1)} - F_{\text{PM}(n)} - F_{\text{tooth}(n)}\]  \hspace{1cm} (14)

and the loop 24 MMF is

\[F_{24} = F_{\text{tooth1}} + F_{\text{PM1}} - F_{\text{PM24}} - F_{\text{tooth24}}\]  \hspace{1cm} (15)

We now define the following two vectors

\[\phi = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_{24} \end{bmatrix}\]  \hspace{1cm} (16)
\[
F = \begin{bmatrix}
F_1 \\
F_2 \\
\vdots \\
F_{24}
\end{bmatrix}
\]  
(17)

Using Eq. (3) through (17), our model is described as

\[\mathfrak{R}\phi = F\]  
(18)

and by rearranging Eq. (18), solution for flux is thus

\[\phi = \mathfrak{R}^{-1}F\]  
(19)

Because reluctances and MMF source values are dependant on flux density, this model is solved iteratively. First, an initial value of flux is assumed, and flux densities are calculated. Using the B-H curve for the metal parts (Figure 7) and knowledge of the mechanical gap size, reluctances for the metal parts and the air gap are calculated. The demagnetization curve (Figure 8) is used to generate the PM MMF at this flux density level, and the user-selected phase currents are used to calculate the tooth MMF. Equation 19 is then solved using the calculated source and reluctance values, providing new values of loop fluxes, and the process is repeated until the flux converges to a satisfactory limit.

**Simulation Results**

The motor simulation approach described in the previous section was used to calculate flux linkages and back EMF, motor torque and power, and rotor force. Details of these simulations are presented in the following sections.

**Flux Linkage and Back EMF**

The flux linkages are calculated by first running the simulation described in the previous section. Flux linking every individual coil is calculated, and individual coil fluxes in a phase belt are added together to obtain the flux linkages for each phase. A plot showing flux linkages for each phase is shown in Figure 9.

By performing time derivatives of the flux linkages, back EMF for each phase is then calculated using Faraday’s law of induction:

\[v_{\text{emf}} = -\frac{d\phi}{dt}\]  
(20)

The simulation results for the motor operating at the maximum flywheel speed of 60kRPM, displaying the back EMF for each phase, are shown in Figure 10. Note that, as desired, the third harmonic has been virtually eliminated.
Motor Power and Torque

The instantaneous power delivered to the back EMF can be expressed as follows:

\[ p = i_a \text{emf} + i_b \text{emf} + i_c \text{emf} \]  

(21)

This power is equivalent to the mechanical power delivered to the rotor.

\[ p = p_{\text{mech}} = \omega_{\text{mech}} \tau \]  

(22)

where \( \omega_{\text{mech}} \) is the mechanical angular velocity of the rotor and \( \tau \) is the torque applied to the rotor. Combination of Eq. (21) and (22) yield the following expression for torque

\[ \tau = \frac{i_a \text{emf} + i_b \text{emf} + i_c \text{emf}}{\omega_{\text{mech}}} \]  

(23)

In order to calculate torque as a function of electrical angle, the simulation was run under constant phase current, with phase A current set to 1.0A (which automatically sets phase B and C currents at –0.5A). Individual simulations were run over an entire electrical revolution at five degree increments, using an arbitrarily selected zero degree point. The resultant torque plot is shown in Figure 11. Note that the maximum torque/amp generated by the motor is ~0.018 Nm/amp, and that this maximum will be achieved when the motor controller is aligned at about 255 electrical degrees. The ripples in the torque plot are due to back EMF perturbations, which are caused by the stator teeth.

Next, we wish to calculate motor power. Given the selected slot size and the current density of copper, and assuming the slot is filled with copper at a packing factor of 0.5, the maximum phase current the motor can draw is 122 A. Based on this, the previous simulation was run once again with constant phase A current of 122A (phase B and C at –61A). The result of this simulation, motor power as a function of electrical angle, is presented in Figure 12. Note that the maximum motor power under these conditions is ~18KW.
Rotator Force Calculation

The forces the stator produces on the rotor are equal and opposite to the forces on the stator tooth tips. The magnitude of the forces on the tooth tips is given by [2]:

\[ |F_{\text{tooth-tip}}| = \frac{A_{\text{tooth-tip}} \cdot B_{\text{tooth-tip}}^2}{2 \cdot \mu_0} \]  
(24)

Since there are 24 teeth in the rotor, the rotor mechanical angle for each of the tooth locations can be expressed as

\[ \theta_n = \frac{(n - 1) \cdot 2 \cdot \pi}{24} \]  
(25)

where \( n = 1.24 \). The general vector expression for force on the rotor at any of the tooth tips, from Eq. (24), is

\[ F_n = \frac{A_{\text{tooth-tip}} \cdot B_{\text{tooth-tip}}^2}{2 \cdot \mu_0} [i \cdot \sin(\theta_n) + j \cdot \cos(\theta_n)] \]  
(26)

Force on the rotor caused only by the PMs is calculated next, for the case of a rotor which is slightly displaced from center. This simulation is run with zero phase currents, with the rotor displaced in the +x direction, for an entire electrical revolution, using Eq. (26). Note that although the air gap for the motor is 200 mils total, the actual rotor motion is limited to ±10 mils by the magnetic bearing system; thus, the displacement for this simulation was set to 10 mils. The result is presented in Figure 13. The x component of force has an average value of 13.45 lbs, with harmonics at 12 times the electrical speed, which corresponds to harmonics that are at 24 times the mechanical spin speed (as expected with 24 teeth). Note also that the y component of force is zero; this means that all of the force on the rotor is in the radial direction.

Next, the previous simulation was run seven more times, each with a ten mil rotor displacement from center, at 45, 90, 135…315 mechanical degree locations around the rotor radial center point. With each simulation, force vectors on the rotor were calculated for an entire revolution. Each of these points, along with the data from Figure 13, are plotted in Figure 14. Note that the ripple caused by the teeth is present in each of these data sets, which explains why each of the data sets appears as a line. Note also that the force on the rotor in each of these simulations is in the radial direction.
Next, force versus position plots were generated, for small and large displacement values. Once again, the simulation was run with no phase current; separate simulations were run as the rotor was moved in the +x direction. Figure 15 shows the force versus displacement plot for small displacements (less than 2 mils) off center, and each plotted value is the average value of force over a 360 electrical degree rotation. This plot shows that the negative stiffness of the motor with the rotor close to the center is about 700 lbs/in. Note that for small displacements the negative stiffness is linear. Figure 16 shows force versus displacement for larger displacements.

Motor Design Summary

A summary of the motor design described in this paper is provided in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Design value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poles</td>
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</tr>
<tr>
<td>Turns/coil</td>
<td>1</td>
</tr>
<tr>
<td>Number of slots</td>
<td>24</td>
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<tr>
<td>Inner rotor diameter</td>
<td>1.0</td>
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<tr>
<td>Outer rotor diameter</td>
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</tr>
<tr>
<td>Outer stator diameter</td>
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<tr>
<td>Magnet thickness</td>
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<td>Mechanical air gap</td>
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<td>Slot depth</td>
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<td>Axial stack length</td>
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<td>Back iron thickness</td>
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<td>Tooth width at air gap</td>
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<td>Maximum speed</td>
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<tr>
<td>Motor power (kW)</td>
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</table>

Conclusions

This paper describes a detailed motor-generator circuit model and simulation approach. A sample design and simulation are presented, and the simulation predicts motor back EMF, power, and torque versus electrical angle for the specified design. Additionally, force due to the permanent magnets (zero motor currents) is calculated versus rotor displacement, yielding negative stiffness relationships; for small displacements of the rotor (<2 mils), the negative stiffness was found to be linear. Also, simulations at 10 mil displacements all around the rotor center revealed that only radial forces were exerted on the rotor by the permanent magnets.

References

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