NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

TECHNICAL MEMORANDUM 1411

STUDY OF THE MICRO-NONUNIFORMITY OF THE
PLASTIC DEFORMATION OF STEEL

By B. B. Chechulin

Translation of "Issledovanie mikroneodnorodnosti plasticheskoj
deformatsii stali." Fizika metallov i metallovedenie,

Washington
August 1957
The plastic flow during deformation of real polycrystalline metals has specific characteristics which distinguish the plastic deformation of metals from the deformation of ordinary isotropic bodies. One of these characteristics is the marked micro-nonuniformity of the plastic deformation of metals.

P. O. Pashkov (ref. 1) demonstrated the presence of a considerable micro-nonuniformity of the plastic deformation of coarse-grained steel with medium or low carbon content. Analogous results in the case of tension of coarse-grained aluminum were obtained by W. Boas (ref. 2), who paid particular attention to the role of the grain boundaries in plastic flow. The nonuniformity of the plastic deformation in micro-volumes was also recorded by T. N. Guikova and others, on the alloy KhN80T. N. F. Lashko (ref. 3) pointed out the nonuniformity of the plastic deformation for a series of pure polycrystalline metals and one-phase alloys. In his later reports, P. O. Pashkov arrives at the conclusion that the nonuniformity of the distribution of the deformation along the individual grains has a significant effect on the strength and plastic characteristics of polycrystalline metals in the process of plastic flow.

However, until now there has not existed any systematic investigation of the general rules of the microscopic nonuniformity of plastic deformation even though the real polycrystalline metals are extremely simple with regard to structure.

In the present report, an attempt is made to study the micro-nonuniformity of the flow of polycrystalline metals by the method of statistical analysis of the variation of the frequency diagrams of the nonuniformity of the grains in the process of plastic deformation.

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METHODS OF THE STUDY

In investigations conducted up to the present, the fact was made sufficiently clear that the process of plastic deformation along individual microvolumes (particularly, along the grains) of various polycrystalline metals is nonuniform. An estimate of the degree of nonuniformity, by means of application of some criterion of the nonuniformity of plastic deformation was not available. It is perfectly obvious that the assumption of a physically basic parameter, which determines the degree of nonuniformity of the plastic flow of metals, must considerably broaden the possibilities of studying the mechanism and the characteristics of the plastic deformation of ordinary polycrystalline metals.

It is necessary to characterize the micro-nomnonuniformity of the plastic deformation so as to study two basic aspects of that phenomenon: (1) the nonuniformity of the individual grains in deformed state which arises as a result of the nonuniform plastic flow; (2) the nonuniformity of the process of flow of the metal along the individual grains.

The first of the aspects mentioned characterizes the result of nonuniform plastic deformation and must be determined by the mean difference of the true strains of the individual grains for a given magnitude of the mean strain. It is perfectly evident that the nonuniformity of the deformed state necessarily determines to a high degree the strength and other physical and mechanical properties of the metal.

The second aspect - the nonuniformity of the process of plastic flow - reflects the rate of growth of the nonuniformity of the deformed state at the moment of plastic flow and must be determined by the first derivative with respect to the mean strain of the quantity which characterizes the nonuniformity of the deformed state.

The most complete micro-nomuniformity of the plastic deformation is characterized by the distribution curves of the true strains along the individual elements of the general volume of the metal, for instance, by the frequency-distribution curves of the true strains along the grains. Along such a curve, the mean strain is determined by its mean statistical magnitude; the nonuniformity of the deformed state is, in our opinion, best characterized by the dispersion (diffusion or width) of the distribution curve. For the determination of the nonuniformity of the process of plastic flow, the dispersion of the distribution curve must be represented in the form of a function of the mean strain. The first derivative of this function with respect to the magnitude of the mean strain must be assumed as the characteristic of the nonuniformity of the plastic-flow process of the metal.
In order to obtain the distribution curve of the true strains of the individual grains or in order to construct the frequency diagrams of the true strains of the grains, one must have data regarding the true strain of every grain in the flow process of the metal, which presents considerable experimental difficulties. The main difficulty consists in measuring the true local strains of the internal micro-volumes of the metal. However, the problem is considerably simplified if in the polyhedral structure of the metal only comparative measurements are performed, for instance, of the ratio of the grain diameters along and across the main direction of the strain, as was done by P. O. Pashkov (ref. 1). It is possible to show that these ratios depend in a regular way on the true strain of the grains. For reasons of simplicity, we shall represent the undeformed grain in the shape of a sphere of the diameter \( d \). After the deformation the sphere is transformed into an ellipsoid which will have the axes: \( a, b, c \). The true strain of the grain along the axes \( x, y, z \) will be, correspondingly

\[
\varepsilon_x = \ln \frac{a}{d} \tag{1}
\]

\[
\varepsilon_y = \ln \frac{b}{d} \tag{2}
\]

\[
\varepsilon_z = \ln \frac{c}{d} \tag{3}
\]

From the condition of conservation of volume in the case of plastic deformation, we have \( 4/3\pi d^3 = 4/3\pi abc \) or \( d^3 = abc \)

\[
\ln d = \frac{1}{3} \ln abc \tag{4}
\]

Substituting the value \( \ln d \) from formula (4) into equations (1), (2), and (3), we have

\[
\varepsilon_x = \frac{1}{3} \ln \frac{a^2}{bc} \tag{5}
\]

\[
\varepsilon_y = \frac{1}{3} \ln \frac{b^2}{ac} \tag{6}
\]

\[
\varepsilon_z = \frac{1}{3} \ln \frac{c^2}{ab} \tag{7}
\]
In the absence of strain in a certain direction, for instance, along the axis of $z$, $\ln d = \frac{1}{2} \ln ad$, the true strains of the grain will be

$$
\begin{align*}
\varepsilon_x &= \frac{1}{2} \ln \frac{a}{b} \\
\varepsilon_y &= \frac{1}{2} \ln \frac{a}{b} \\
\varepsilon_z &= 0
\end{align*}
$$

(8)

If it is known beforehand that the strains in the $y$- and in the $z$-direction will be equal ($b = c$), the true strains will be

$$
\begin{align*}
\varepsilon_x &= \frac{2}{3} \ln \frac{a}{b} \\
\varepsilon_y &= \varepsilon_z = \frac{1}{3} \ln \frac{a}{b}
\end{align*}
$$

(9)

It can be shown that the derived ratios of the true strains of the grain and the ratio of the diameters of the grain along and across the main direction of the deformation are correct not only for a spherical shape of the grain. The ratios $\frac{a}{b}$, $\frac{b}{c}$, and $\frac{a}{c}$ can be determined according to any plane cross section of the grain which is parallel to the coordinate surfaces $xOy$ and $yOz$ (fig. 1).

However, it is necessary to take into consideration that all derived ratios are correct only if the grains are completely equiaxial before the deformation. Ordinarily the undeformed grains have very diverse forms. If we determine according to the structure of the undeformed metal the ratio of the diameters of the grain along and across the determined direction $\alpha = \frac{a}{b}$ and estimate the percentile ratio of the grains with a different value of that ratio, we may note that the number of grains with the same value of $\alpha$ depends on the magnitude of $\alpha$ according to the determined distribution curve with the maximum for the nonequality of the axes of the grains (for $\alpha = 1$).

In practice it is convenient to represent such a curve in the form of a diagram. If we use the derived formulas, we may assume as the
characteristic of nonequality of the axes of grains not the ratio
\( \alpha = \frac{a}{b} \) but its natural logarithm \( \ln \frac{a}{b} \), which is in many cases the
quantity (eqs. (8) and (9)) which is, as to mathematical form, identical
with the true strain of the grains. Such a frequency diagram will
represent the distribution of the initial nonequality of the axes
("conditional strain") of the grains and must naturally be close to
Gauss's distribution curve.

Frequency diagrams of the nonequality of axes of grains can be
plotted also according to the structure of a deformed polycrystalline
metal. In that case the diagrams of the distribution of the nonequality
of axes of grains, expressed in "true conditional strains" will represent
the superposition of two diagrams of distribution: that of the initial
frequency distribution curve of the nonequality of axes of the grains,
and that of the frequency distribution curve of the true strains of the
grains of a metal. It is known from mathematical statistics that, in the
case of superposition of two principles, each of which gives rise to a
definite dispersion of the given quantities according to a normal rule,
the overall dispersion of the given quantities also must obey that normal
rule according to which the dispersion of the overall distribution curve
is equal to the sum of the dispersion, but the mean statistical quantity
is equal to the sum of the mean statistical quantities of the superimposed
curves. This rule of statistics will be applied in our case in view of
the character of the distribution of the true strains of the grains being
independent of the initial distribution of their axial inequality. Then
the dispersion of the distribution of the true strains of the grains will
be equal to the difference of the dispersions determined according to the
frequency diagrams of the "conditional strains" of the grains before and
after the deformation.

\[ \sigma_\varepsilon = \sigma_y - \sigma_0 \]  \hspace{1cm} (10)

where
- \( \sigma_\varepsilon \) is the dispersion of the distribution of the true strains of the grains
- \( \sigma_y \) is the dispersion of the distribution of the conditional strains in the grains
- \( \sigma_0 \) is the dispersion of the initial distribution of the conditional strains in the grains (before deformation)
Correspondingly, the local mean strain will be

$$\varepsilon_{cp} = \varepsilon_y - \varepsilon_0$$  \hspace{1cm} (11)

where

$\varepsilon_{cp}$ is the true mean strain

$\varepsilon_y$ is the mean conditional strain

$\varepsilon_0$ is the mean initial axial inequality of the grains.

The numerical values of the dispersions and of the mean values of the conditional strains in the grains from the experimental distribution diagrams are calculated according to formulas of mathematical statistics. For the normal distribution rule, the mean value is found according to the formula

$$\varepsilon_y = \frac{1}{n} \sum_{i=1}^{i=m} \varepsilon_i m_i$$  \hspace{1cm} (12)

and the dispersion according to the formula

$$\sigma_y = \frac{1}{n} \sum_{i=1}^{i=m} (\varepsilon_i - \varepsilon_y)^2 m_i$$  \hspace{1cm} (13)

where

$\sigma_y$ and $\varepsilon_y$ have their former meaning

$n$ is the general number of the measured grains

$m$ the number of grains with conditional strain

The applicability of the formulas (10), (11), (12), and (13) for determining the dispersion of the distribution of the true strains of the grains according to the frequency diagrams of the conditional strains may be checked experimentally. It is easy to verify that the frequency diagrams of the distribution of the conditional strains are close to the normal rule after estimation of the asymmetry coefficient and the excess which must be only slightly different from zero. The structure of the
frequency diagrams of the conditional strains (figs. 2, 3, 4, 5) for low-carbon steel of three different melts (0.04 to 0.05 percent C), for carbon steel (0.15 and 0.26 percent C), and for austenitic steel showed the closeness of the frequency diagrams to the normal distribution. Thus the asymmetry coefficient fluctuated within the limits of -0.2 to +0.28, the excess within the limits of -0.156 to 0.18 even on measuring only 150 to 200 grains. An increase in the number of measured grains led to a still lesser magnitude of the coefficient of asymmetry and of the excess. The structure of the diagrams for different parts of a microsection of the undeformed metal showed a satisfactory coincidence of the frequency diagrams with respect to the average statistical quantities as well as to the dispersion upon measurement of 150 to 200 grains.

The independence of the distribution of the true strains of the grains, at the different stages of the mean strain in a metal, on the distribution of the axial inequality of the grains before the deformation is shown in figure 6, which represents the results of determination of the dispersion of the distribution of the true strains of the grains of low-carbon steel of three different melts. The dispersions of the distribution of the axial inequality of the grains before the deformation in all three melts differed from one another ($\sigma_0' = 0.378$, $\sigma_0'' = 0.358$, $\sigma_0''' = 0.337$); nevertheless for all three melts the values of the dispersion of the true strains of the grains as functions of the mean strain coalesce satisfactorily to a common curve.

Table 1 presents a comparison of the results of the calculation of the mean strain determined according to formula (11) and by the usual method for measuring the elongation due to rolling. The measurements were made after cold-rolling of strips of low-carbon steel with a content of 0.16 percent C according to the individual thickness.

From the table it is clear that the strains determined by the two methods are generally close to one another. The discrepancies, noted especially in the case of small deformations, may clarify the micro-uniformity of the plastic deformation which always takes place in the case of rolling and is most noticeable in the case of small (rolling) reductions.

In this manner, the analysis of the frequency diagrams of the axial inequality of the grains of a polycrystalline metal before and also during the process of plastic deformation gives the possibility of determining the average local strain as well as the dispersion of the distribution of the true strains of the grains for different stages of deformation of a metal of any volume from which one can produce a metallographic microsection containing no less than 150 to 200 grains.
For the present report, we had decided to make use of the frequency diagrams of the axial inequality of the grains which were constructed according to the dimensions of the diameters of the grains in the structure of polycrystalline metals for investigation of the axial inequality of the plastic deformation between the grains of the metal. Rolled steel of four different types served as subject of the investigation:

1. Three melts of low-carbon steel (0.04 to 0.05 percent C)
2. Carbon steel (0.15 percent C)
3. Carbon steel (0.26 percent C)
4. Austenitic steel of the type 18-8, heat-treated for pure austenite

The deformation was carried out by means of two methods: by tension of cylindrical specimens, and by cold-rolling of plates of 20 x 50 x 300 mm size, with different (rolling) reduction. The frequency diagrams were constructed for 200 to 300 grains (a part of a microsection with a surface of from 0.1 to 0.5 mm²). The grains of carbon steels were measured at 150-fold magnification, those of austenitic steel at 600-fold magnification. The conditional strains in the case of rolling were determined according to formula (8), and in the case of tension according to formula (9).

In various specimens, the microsection was taken through the axis of the specimen, but the measurements were made in regions distributed along the axis of the specimen. In the case of rolled plates, the microsections were prepared from a middle layer through the thickness of the section.

**BASIC RESULTS AND THEIR DISCUSSION**

Figures 2, 3, 4, and 5 show, in the form of frequency diagrams, the results of measurements of the axial inequality of the grains of various steels at different stages of deformation. On the axis of abscissas lies the characteristic of the axial inequality of the grains which represents the conditional true strain of the grains in the direction of the principal strain; on the axis of ordinates lies the relative number of grains of an axial inequality given in percent. Comparing the series of diagrams of various steels, one can at once note the change in their form upon increase of the mean plastic deformation, which corroborates the observed axial inequality of the process of the plastic deformation between separate grains and completely confirms the earlier established fact of the micro-nomuniformity of the extension of plastic deformation. We can note also several differences in the transformation of frequency diagrams according
to the increase in mean strain of various steels. Even from the mere outer appearance of the diagrams, a relatively large "widening" of the diagrams is noticeable, in accordance with the increase in mean strain of the carbon steels (0.15 to 0.26 percent C).

Figure 6 represents for all steels investigated the change in dispersions of the distribution of true strains of the grains, calculated according to the frequency diagrams using formulas (10), (12), and (13), as functions of the mean strain. The curves of figure 6 characterize even more completely the micro-nonuniformity of plastic deformation at the separate stages of the flow. As can be seen from the figure, austenitic steel has the least micro-nonuniformity in plastic deformation. For this steel there originates, in the case of small deformations, an insignificant nonuniformity of the deformed state which changes only slightly if the deformation is further increased. All three melts of the low-carbon steel, the structure of which consisted of pure ferrite (0.04 to 0.05 percent C), have only an insignificant nonuniformity of the deformed-state grains at the beginning of the flow, but this nonuniformity increases continuously in accordance with the increase in mean plastic deformation. It is characteristic that the experimental values in all three melts coalesce well to a common curve. The increase of carbon content in the steel and the appearance of pearlitic grains in the structure somewhat change the character of the development of the micro-nonuniformity of the plastic deformation. The nonuniformity of the deformed state at one and the same stage of the plastic flow increases considerably in the presence of pearlitic grains. An increase in the nonuniformity of the deformed state is ordinarily observed in the case of relatively small values of the general strain. The increase of carbon content in the steel from 0.14 to 0.26 percent somewhat increases the general nonuniformity of the deformed state, although it does not alter the character of its change in accordance with the increase in the magnitude of the general plastic deformation. It must be noted that the character of the curves of figure 6 does not change upon transition from deformation by means of tension (steel with 0.26 percent C) to deformation by rolling (steel with 0.15 percent C). Thus, the character of the stressed state does not show an essential influence on the development of the micro-nonuniformity of the plastic deformation, although this conclusion still requires additional verification.

It is of considerable interest to estimate, by means of the experimental data obtained, the nonuniformity of the process of plastic flow of the separate grains of a polycrystalline metal, inasmuch as a nonuniformity of the deformed state can originate only as a result of the nonuniformity of the process of plastic flow.

Figure 7 represents curves from which we may estimate the micro-nonuniformity of the process of plastic flow of the steels investigated for varying amounts of plastic deformation. Along the ordinate axis is
plotted the magnitude of the first derivative of the dispersion of the distribution of the true strains of the grains with respect to the mean deformation: \( \frac{\partial \sigma}{\partial \sigma_{cp}} \), that is, the quantity which can sufficiently completely characterize the nonuniformity of the process of plastic flow of a polycrystalline metal. The numerical value of \( \frac{\partial \sigma}{\partial \sigma_{cp}} \) is computed by means of a graphic differentiation of the curves depicted in figure 6. The curves of figure 7 show that, similar to the nonuniformity of the deformed state, the nonuniformity of the process of plastic flow in steels of different types varies differently as a function of the mean strain.

In austenitic steel, the process of plastic deformation proceeds comparatively uniformly, especially associated with large plastic deformations. Steel with ferritic structure shows a nonuniformity of the plastic-flow process which is the greater, the further the plastic deformation has progressed. The greatest amount of nonuniformity in the flow of carbon steels with mixed ferrite-pearlite structure is observed at the moment where the deformation starts. The increase in plastic deformation reduces the nonuniformity of the process; however, its magnitude remains considerable up to the moment of fracture. From a comparison of the nonuniformity of the flow of carbon steels (0.15 and 0.26 percent C), we may draw the conclusion that an increase in the carbon content (raising the amount of pearlitic grains in the structure) increases the nonuniformity of the plastic deformation in the entire range of measurement of the deformation.

By the method of analysis of the frequency diagrams we cannot only estimate the general nonuniformity of the deformation, but can also carry out a parallel comparison of the intergranular nonuniformity of the deformed state of the structural components (in our case ferrite and pearlite), and also determine the magnitude of mean plastic deformation separately for the pearlite and ferrite grains. Figure 8 represents correspondingly, for steels with 0.15 percent (a) and 0.26 percent (b) of carbon, the variation of the dispersion of the distribution of the true strains, determined separately according to the frequency diagrams of the pearlite and ferrite grains, as a function of the mean strain. It can be seen from the figure that the nonuniformity of the deformed state pearlite and ferrite grains in the case of equal mean strains is very close: The test points for both steels lie practically on common curves. Comparing the course of the curves of figure 8 with the course of the curves for low-carbon steel (fig. 6), we can observe that the curve showing the variation in dispersion of the true strains of ferrite grains as a structural component differs from the corresponding curve for steel with purely ferritic structure. This difference consists in
the increase of nonuniformity of the flow process of the ferritic grains at the initial stage of the deformation in the presence of pearlite; it increases as the pearlite content in the structure is augmented (increase of carbon content). The variation of the mean strains calculated separately for the ferritic and pearlitic grains as functions of the mean strain which is common to pearlite and ferrite is represented in figure 9.

It can be seen from the figure that an abrupt variation in the mean deformation of pearlitic and ferritic grains can be observed before the common strain of approximately 0.17 ($\psi = 15$ percent); a further increase in strain reduces the difference in the magnitude of the mean deformation of the two structural components. The magnitude of the common strain, corresponding to the most abrupt deviation in the mean strain of pearlitic and ferritic grains, is close to the strain which corresponds to uniform reduction in area in the case of tension.

**BASIC CONCLUSIONS**

1. A new method is presented for the investigation of the micro-nomuniformity of the plastic deformation of polycrystalline metals.

2. It is shown that, with the aid of the frequency diagrams of the axial-inequality of the grains of an ordinary polycrystalline metal before and after the plastic deformation, we can determine the true local strain of a metal of any volume, if in a plane cross section of this volume a sufficiently large number of grains is present (not less than 150 to 200 grains).

3. Investigation, by means of the suggested method, of the non-uniformity of the deformation of the separate grains of steels of different structure showed that the smallest micro-nomuniformity of the deformed state and of the plastic-flow process in the case of plastic deformation is observed for steel with austenitic structure. Low-carbon steel with ferritic structure is deformed with a micro-nomuniformity (with respect to the deformed state as well as to the flow process) which increases with the mean deformation. Upon plastic deformation of tempered carbon steels, whose structure consists of pearlite and ferrite grains, the micro-nomuniformity of the deformed state increases continuously; at the same time, the micro-nomuniformity of the flow process for these steels continuously decreases, having its highest magnitude at the initial moment of the deformation.

4. The different structure of the frequency diagrams for the axial inequality of the pearlitic and ferritic grains of carbon steels made clear the increase in micro-nomuniformity of the deformation of ferritic grains (ordinarily in the case of small deformations), in comparison
with the nonuniformity of the deformation recorded while investigating low-carbon steel with purely ferritic structure.

Finally, the author expresses his deep gratitude to Prof. Dr. of Technical Sciences, P. O. Pashkov, for valuable direction which was utilized in this report.

Translated by Mary L. Mahler
National Advisory Committee
for Aeronautics

REFERENCES


### TABLE I

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aNACA reviewer's note: Correct equation as referred to in text is equation (11).
Figure 1. - Variation of the parameters of the sphere during deformation.

Figure 2. - Frequency diagrams of the distribution of the conditional strains of the grains of low-carbon steel, containing 0.06 percent C (melt 1) at different mean strains (tension).
Figure 3. - Frequency diagrams of the distribution of the conditional strains of the grains of austenitic steel at different mean strains (tension).
Figure 4. - Frequency-distribution diagrams of the conditional strains of carbon steel containing 0.15 percent C, at different strains (rolling).
Figure 5. - Frequency-distribution diagrams of the conditional strains of the grains of carbon steel containing 0.28 percent C, at different mean strains (tension).
Figure 6.- Variation of the dispersion of the distribution of the true strains of the grains as a function of the mean strain for different steels.

1. Low-carbon steel, containing 0.04 to 0.05 percent C; melt 1: Δ - Δ - Δ, melt 2: o - o - o, melt 3: x - x - x.

2. Carbon steel (0.15 percent C).

3. Carbon steel (0.26 percent C).

4. Austenitic steel.
Figure 7. - Variation of the first derivative with respect to the mean strain of the dispersion of the distribution of the true strains of the grains as a function of the mean strain.

1. Carbon steel (0.26 percent C).
2. Carbon steel (0.15 percent C).
3. Low-carbon steel (0.04 to 0.05 percent C).
4. Austenitic steel.
Figure 8. - Variation of the dispersion of the distribution of the true strains of pearlite and ferrite grains as a function of their mean strains.

(a) For steel, containing 0.1 percent C.

(b) For steel, containing 0.26 percent C.
Figure 9. - Dependence of the mean strain of the structural components of carbon steels, containing 0.15 and 0.26 percent carbon on the overall strain.