INTRODUCTION: A simplified analysis is presented to extend a previous work [1] on flame extinction in a quiescent microgravity environment to a more likely situation of a mild opposing flow. The energy balance equation, that includes surface re-radiation, is solved to yield a closed form spread rate expression in terms of its thermal limit, and a radiation number that can be evaluated from the known parameters of the problem. Based on this spread rate expression, extinction criterions for a flame over solid fuels, both thin and thick, have been developed that are qualitatively verified with experiments conducted at the MGLAB [2] in Japan. Flammability maps with oxygen level, opposing flow velocity and fuel thickness as independent variables are extracted from the theory that explains the well-established trends in the existing experimental data [3].

Thermal Regime: An energy balance for the solid phase control volume of Fig. 1 can be written as.

\[
\lambda_g \left( \frac{T_f - T_v}{L_g} \right) - \frac{\varepsilon \sigma}{\rho_s c_s} = \left( \frac{T_v^4}{T_s^4} \right) L_g \sim \frac{V}{\rho_s c_s} \left( T_v - T_s \right)
\]

where, \( T_f \) and \( T_v \) are characteristic flame and vaporization temperature, \( \tau_h \) is the thickness of the heated layer, and \( L_g = \alpha_g / (V_f + V_g) \) is the gas-phase length scale. For thin fuels in the thermal limit, \( \tau_h = \tau \) and \( \varepsilon = 0 \) produces the de Ris solution \( V_{f,th,thn} = \left( \frac{\varepsilon}{\rho_s c_s} \right) F \), where \( F \equiv \left( \frac{T_f - T_s}{T_v - T_s} \right) \). Using \( V_{f,th,thn} \) to non-dimensionalize \( V_f \), \( \eta_f = \frac{V_f}{V_{f,th,thn}} \) Eq. (1) can be expressed in non-dimensional form as follows.

\[
\left( \eta_f^2 + \eta_s \eta_g \right) \frac{\tau_h}{\tau} - \left( \eta_f + \eta_g \right) + \Omega_0 \sim 0 ; \quad \text{where,} \quad \Omega_0 = \frac{1}{F^2} \frac{\rho_s c_s \varepsilon \sigma \tau}{\lambda_g} \left( \frac{T_v^4}{T_s^4} \right) \eta_g \equiv \frac{V_g}{V_{f,th,thn}}
\]

The thermal thin limit \( \eta_{f,th,thin} \sim 1 \) is recovered when \( \Omega_0 = 0 \) and \( \tau_h = \tau \). To obtain a more general solution \( \tau_h / \tau \) for a thick fuel can be scaled as

\[
\frac{\tau_h}{\tau} \sim \frac{\alpha_f L}{\tau} \sim \frac{\Omega}{\sqrt{\eta_f \left( \eta_f + \eta_g \right)}} \sim \frac{1}{\sqrt{\varepsilon \sigma}} \frac{\lambda_g}{\rho_s c_s} \Omega where, \Omega = \frac{\lambda_s \rho_s c_s}{\rho_g c_g c_g}
\]

Substituting this into Eq. (2) and still ignoring radiation, we obtain the thermal limit for semi-infinite fuel beds.

\[
\eta_{f,th,thick} \sim \frac{F^2}{\Omega^2} \eta_g \left( 1 - \frac{F^2}{\Omega^2} \right)^{-1} \sim \frac{F^2}{\Omega^2 \eta_g} \left( 1 + \frac{F^2}{\Omega^2} \right) where, \Omega \equiv \frac{\lambda_s \rho_s c_s}{\rho_g c_g c_g}
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\eta_{f,th,thick} \sim \frac{F^2}{\Omega^2 \eta_g} \left( 1 - \frac{F^2}{\Omega^2} \right)^{-1} \sim \frac{F^2}{\Omega^2 \eta_g} \left( 1 + \frac{F^2}{\Omega^2} \right) \text{ if } \tau \geq \tau_h ; \quad \text{or, } \tau \geq \frac{\lambda_p}{\rho_g c_g c_g V_g} \text{ \ or, } \eta_g \geq \frac{\Omega^2}{F^2}
\]

The simplification above is achieved because for both PMMA and cellulose it can be shown that \( F < \Omega \). Equation (4) also provides a criterion for transition between the thin and the thick limit for \( \eta_g \gg 1 \).
is plotted in Fig. 2 showing the transition from the thin to the thick limit in the thermal regime. Not much data in the thick-thin transitional region is available to verify this simple transition criterion.

**Radiative Regime:** The energy balance equation, Eq. (2), is solved in both the thick and thin limit producing

\[
\text{Thin Limit: } \eta_{f,\text{thin}} \sim \frac{1 - \eta_g}{2} + \frac{1}{2} \sqrt{\left(1 + \eta_g\right)^2 - 4\Re_0} \quad \text{Thick Limit: } \eta_{f,\text{thick}} \sim -\frac{F^2}{\Omega^2 \eta_g} \left(1 - \frac{\Re_0}{\eta_g}\right)^2;
\]

(5)

These results are plotted in Figs. 3 and 4 for several values of the radiation parameter \(\Re_0\). A number of important features of the radiative effects on flame spread rates are revealed by these plots. When \(\Re_0 > 0\), the slope of the spread rate curves decreases with opposing velocity for thin fuels while this trend is completely reversed for thick fuels. The MGLAB data [2] for flame spread over thin PMMA, shown in Fig 5, support this predicted trend for thin fuels. The DARTFIRE experiments [4] for flame spread over thick PMMA lends supports to the trends predicted by Fig. 4.

Obviously, for \(\Re_0 = 0\) and/or \(\eta_g \rightarrow \infty\), the thermal limits are recovered with \(\eta_{f,\text{thin}} = 1\) and \(\eta_{f,\text{thick}}\) being proportional to \(\eta_g\). To establish a criterion for the transition between the thermal and radiative regimes, we simplify Eq. (5) assuming \(\eta_g \gg 1\). If the spread rate is non-dimensionalized by the corresponding thermal limit, Eq. Error! Reference source not found. Reference source not found., for both the thick and thin limit can be shown to approach the same form.

**Thin and Thick Fuels:** For \(\eta_g \gg 1\), \(\eta_f' \equiv \frac{V_f}{V_{f,\text{thermal}}} \sim \left(1 - \frac{\Re_0}{\eta_g}\right)^2 \sim 1 - 2\Re_f'\); where, \(\Re_f' \equiv \frac{\Re_0}{\eta_g}\)

(6)

A single parameter \(\Re_f'\), therefore, controls the radiative effects on the spread rate for both thermally thin and thick fuels. \(\eta_f'\) from Eq. (6) is plotted in Fig. 6 against versus \(1/\Re_f'\), so that the abscissa is proportional to \(V_g\). Superposed on this figure are experimental spread rates from MGLAB experiments, only part of which were previously reported [2]. Although the spread of the data around the prediction of Eq. (6) is substantial, the onset of radiative effects seems to be well correlated by the analytical prediction.

**Extinction Criteria:** The spread rate expressions of Eq. (5) can be used to establish criterion for flame extinguishment. As can be seen from Figs. 3 and 4, there are two types of extinction behavior. For \(\eta_g \geq 1\), in both the thin and thick limit, steady flame cannot be sustained provided \(\eta_g > \Re_0\), a criterion that is independent of fuel thickness. For \(\eta_g < 1\), the thick fuel criterion remains unaltered. However for thin fuels, the spread rate assumes complex values, an indication of extinguishment, when \(\eta_g < 2\sqrt{\Re_0} - 1\). For flame spread over PMMA, these criteria are combined in the flammability map of Fig. 7. Note that for a critical thickness can be calculated from the relation \(\eta_g = \Re_0 = 1\), beyond which extinction is independent of fuel thickness, thereby, defining a radiatively thick fuel.

**Conclusion**

In this article we present a simplified analysis to develop for the first time a closed-form expression for the spread rate and extinction criterion for flame spread over condensed fuels in a mild opposing-flow microgravity environment. The results presented are supported by experiments on thin PMMA conducted in the MGLAB.

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Fig. 5. Non-dimensional experimental spread rate [9] as a function of $\eta_g$ for different oxygen mole fractions and fuel half-thickness. Note that in this plot the highest experimental spread rate is used to normalize $V_f$ and $V_g$ instead of the theoretical thermal limit.

Fig. 6. Prediction of the non-dimensional spread rate $\eta_f$ plotted as a function of inverse of $\mathcal{R}_f^{-1}$ from Eq. (10). The prediction is compared with the spread rate data from the MGLAB experiments.

Fig. 7. Flammability map for PMMA fuel at various half-thickness at 1 atm. Radiative extinction happens on the left side of each curve.