INTRODUCTION: A simplified analysis is presented to extend a previous work [1] on flame extinction in a quiescent microgravity environment to a more likely situation of a mild opposing flow. The energy balance equation, that includes surface re-radiation, is solved to yield a closed form spread rate expression in terms of its thermal limit, and a radiation number that can be evaluated from the known parameters of the problem. Based on this spread rate expression, extinction criterions for a flame over solid fuels, both thin and thick, have been developed that are qualitatively verified with experiments conducted at the MGLAB [2] in Japan. Flammability maps with oxygen level, opposing flow velocity and fuel thickness as independent variables are extracted from the theory that explains the well-established trends in the existing experimental data [3].

Thermal Regime: An energy balance for the solid phase control volume of Fig. 1 can be written as.

\[
\lambda_g \frac{(T_f - T_v)}{L_g} L_g - \varepsilon \sigma \left( T_v^4 - T_x^4 \right) L_g \sim V_f \rho_s c_s \tau_h \left( T_v - T_x \right)
\]

where, \( T_f \) and \( T_v \) are characteristic flame and vaporization temperature, \( \tau_h \) is the thickness of the heated layer, and \( L_g = \alpha_g \left( V_f + V_g \right) \) is the gas-phase length scale. For thin fuels in the thermal limit, \( \tau_h = \tau \) and \( \varepsilon = 0 \) produces the de Ris solution \( V_{f,\text{th,thin}} \sim \left( \frac{\lambda_g}{\rho_s c_s \tau} \right) F \), where \( F \equiv \frac{(T_f - T_v)}{(T_v - T_x)} \). Using \( V_{f,\text{th,thin}} \) to non-dimensionalize \( V_f \), \( \eta \equiv V_f / V_{f,\text{th,thin}} \) Eq. (1) can be expressed in non-dimensional form as follows.

\[
\left( \eta_f^2 + \eta_g \eta \right) \frac{\tau_h}{\tau} = \left( \eta_f + \eta_g \right) + R_0 \sim 0 ; \text{ where, } R_0 = \frac{1}{F^2} \frac{\rho_s c_s \varepsilon \sigma \tau}{\lambda_g} \left( T_v^4 - T_x^4 \right), \eta_g = \frac{V_g}{V_{f,\text{th,thin}}}
\]

The thermal thin limit \( \eta_{f,\text{th,thin}} \sim 1 \) is recovered when \( R_0 = 0 \) and \( \tau_h = \tau \). To obtain a more general solution \( \eta_f / \tau \) for a thick fuel can be scaled as

\[
\frac{\tau_h}{\tau} \sim \frac{\sqrt{\alpha_f L_g}}{\tau} \sim \sqrt{\frac{\Omega^2 V_f}{F}} \frac{1}{\sqrt{\eta_f \left( \eta_f + \eta_g \right)}} \text{ where, } \Omega = \sqrt{\frac{\lambda_g \rho_s c_s}{\lambda_g \rho_s c_s}}.
\]

Substituting this into Eq. (2) and still ignoring radiation, we obtain the thermal limit for semi-infinite fuel beds.

\[
\eta_{f,\text{thick}} \sim \frac{F^2}{\Omega^2 \eta_g} \left( 1 - \frac{F^2}{\Omega^2} \right)^{-1} \sim \frac{F^2}{\Omega^2 \eta_g} \text{ if } \tau \geq \tau_h ; \text{ or, } \tau \geq \frac{\lambda_g}{\rho_s c_s V_g F^2} ; \text{ or, } \eta_g \geq \frac{\Omega^2}{F^2}
\]

The simplification above is achieved because for both PMMA and cellulose it can be shown that \( F < \Omega \). Equation (4) also provides a criterion for transition between the thin and the thick limit for \( \eta_g \gg 1 \). Prediction from Eq. (4)
is plotted in Fig. 2 showing the transition from the thin to the thick limit in the thermal regime. Not much data in the thick-thin transitional region is available to verify this simple transition criterion.

Radiative Regime: The energy balance equation, Eq. (2), is solved in both the thick and thin limit producing

**Thin Limit:** \( \eta_{f,\text{thin}} = \frac{1 - \eta_g}{2} + \frac{1}{2} \sqrt{\left(1 + \eta_g\right)^2 - 4R_0} \)

**Thick Limit:** \( \eta_{f,\text{thick}} = \frac{F^2}{\Omega \eta_g^2} \left(1 - \frac{R_0}{\eta_g}\right)^2 \)  \( (5) \)

These results are plotted in Figs. 3 and 4 for several values of the radiation parameter \( R_0 \). A number of important features of the radiative effects on flame spread rates are revealed by these plots. When \( R_0 > 0 \), the slope of the spread rate curves decreases with opposing velocity for thin fuels while this trend is completely reversed for thick fuels. The MGLAB data [2] for flame spread over thin PMMA, shown in Fig 5, support this predicted trend for thin fuels. The DARTFIRE experiments [4] for flame spread over thick PMMA lends supports to the trends predicted by Fig. 4.

Obviously, for \( R_0 = 0 \) and/or \( \eta_g \to \infty \), the thermal limits are recovered with \( \eta_{f,\text{thin}} = 1 \) and \( \eta_{f,\text{thick}} \) being proportional to \( \eta_g \). To establish a criterion for the transition between the thermal and radiative regimes, we simplify Eq. (5) assuming \( \eta_g \gg 1 \). If the spread rate is non-dimensionalized by the corresponding thermal limit, Eq. Error! Reference source not found., for both the thick and thin limit can be shown to approach the same form.

**Thin and Thick Fuels:** For \( \eta_g \gg 1 \), \( \eta_f \equiv \frac{V_f}{V_{f,\text{thermal}}} \sim \left(1 - \frac{R_0}{\eta_g}\right)^2 \sim 1 - 2R_f \); where, \( R_f \equiv \frac{R_0}{\eta_g} \)

A single parameter \( R_f \), therefore, controls the radiative effects on the spread rate for both thermally thin and thick fuels. \( \eta_f \) from Eq. (6) is plotted in Fig. 6 against versus \( 1/R_f \), so that the abscissa is proportional to \( V_g \).

Superposed on this figure are experimental spread rates from MGLAB experiments, only part of which were previously reported [2]. Although the spread of the data around the prediction of Eq. (6) is substantial, the onset of radiative effects seems to be well correlated by the analytical prediction.

**Extinction Criteria:** The spread rate expressions of Eq. (5) can be used to establish criterion for flame extinguishment. As can be seen from Figs. 3 and 4, there are two types of extinction behavior. For \( \eta_g \geq 1 \), in both the thin and thick limit, steady flame cannot be sustained provided \( \eta_g > R_0 \), a criterion that is independent of fuel thickness. For \( \eta_g < 1 \), the thick fuel criterion remains unaltered. However for thin fuels, the spread rate assumes complex values, an indication of extinguishment, when \( \eta_g < 2^{1/2} R_0 - 1 \). For flame spread over PMMA, these criteria are combined in the flammability map of Fig. 7. Note that for a critical thickness can be calculated from the relation \( \eta_g = R_0 = 1 \), beyond which extinction is independent of fuel thickness, thereby, defining a radiatively thick fuel.

**Conclusion**

In this article we present a simplified analysis to develop for the first time a closed-form expression for the spread rate and extinction criterion for flame spread over condensed fuels in a mild opposing-flow microgravity environment. The results presented are supported by experiments on thin PMMA conducted in the MGLAB.

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Fig. 1 Control volumes at the flame leading edge in the gas and the solid phases.

Fig. 2. Non-dimensional spread rate in the thermal regime.

Fig. 3. Spread rate as a function of $\eta_g$ and $\mathcal{R}_0$ as predicted by Eq. (7). Opposed-flow flame spread extends down to $\eta_g = -1$.

Fig. 4. Spread rate for thick fuel as a function of $\eta_g$ and $\mathcal{R}_0$ as predicted by Eq. (7). The spread rate is zero (extinction) for $\eta_g < \mathcal{R}_0$. 
Fig. 5. Non-dimensional experimental spread rate [9] as a function of $\eta_g$ for different oxygen mole fractions and fuel half-thickness. Note that in this plot the highest experimental spread rate is used to normalize $V_f$ and $V_g$ instead of the theoretical thermal limit.

Fig. 6. Prediction of the non-dimensional spread rate $\eta_f$ plotted as a function of inverse of $\mathcal{R}_V$ from Eq. (10). The prediction is compared with the spread rate data from the MGLAB experiments.

Fig. 7. Flammability map for PMMA fuel at various half-thickness at 1 atm. Radiative extinction happens on the left side of each curve.