A Well-Balanced Central-Upwind Scheme
for the 2D Shallow Water Equations on Triangular Meshes

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We are interested in approximating solutions of the two-dimensional shallow water equations with a bottom topography on triangular meshes

\[
\begin{pmatrix}
    h \\
    hu \\
    hv
\end{pmatrix}_t + \begin{pmatrix}
    hu \\
    h^2 + \frac{1}{3} h^2
\end{pmatrix}_x + \begin{pmatrix}
    hv \\
    huv
\end{pmatrix}_y = \begin{pmatrix}
    0 \\
    -hB_x \\
    -hB_y
\end{pmatrix}.
\]

Here, \((u,v)\) are the components of the velocity vector and \(h\) is the height of the water above the bottom \(B(x,y)\). A Central-upwind scheme on unstructured grids for hyperbolic systems of conservation laws was recently introduced by Kurganov and Petrova in [2]. At first sight, it seems to be straightforward to extend this scheme to balance laws through a suitable discretization of the source term. However, we prove that for general triangulations it is impossible to discretize the source term in such away that the method of [2] is well-balanced.

We show that there is a certain flexibility in choosing the numerical fluxes in the design of semi-discrete Godunov-type central schemes. We take advantage of this fact to generate a new second-order, central-upwind method for the two-dimensional shallow water equations that is well-balanced. Our construction is inspired by the ideas in [1]. We demonstrate the accuracy of our method as well as its balance properties in a variety of examples.

References
