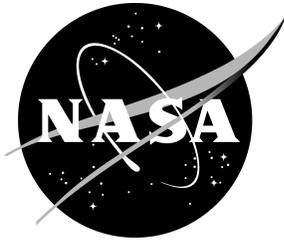
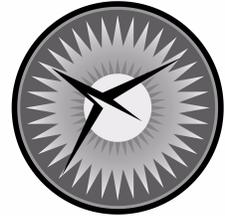


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Parameter Transient Behavior Analysis on Fault Tolerant Control System

Jong-Yeob Shin

National Institute of Aerospace, Hampton, Virginia

December 2003

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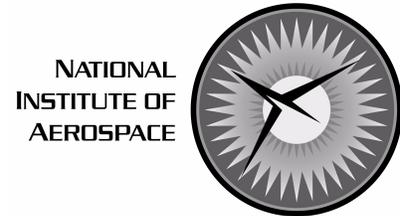
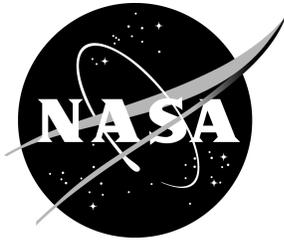
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PARAMETER TRANSIENT BEHAVIOR ANALYSIS ON FAULT TOLERANT CONTROL SYSTEM*

Jong-Yeob Shin[†]

ABSTRACT

In a fault tolerant control (FTC) system, a parameter varying FTC law is reconfigured based on fault parameters estimated by fault detection and isolation (FDI) modules. FDI modules require some time to detect fault occurrences in aero-vehicle dynamics. This paper illustrates analysis of a FTC system based on estimated fault parameter transient behavior which may include false fault detections during a short time interval. Using Lyapunov function analysis, the upper bound of an induced- \mathcal{L}_2 norm of the FTC system performance is calculated as a function of a fault detection time and the exponential decay rate of the Lyapunov function.

1 INTRODUCTION

In the past decades, there has been interest in a FTC system which has ability to detect actuator/sensor faults automatically, and to prevent faults from developing into system failure. Especially, in designing a flight control system, it has been researched for achieving single aircraft accident prevention [1, 2, 3, 4, 5]. An active FTC system requires its control law to react to actuator/sensor faults through reconfiguration and fault detection and isolation (FDI) modules to detect actuator/sensor fault occurrences. FTC law synthesis for aerospace vehicles has been studied since 1980's based on possible faults on actuators and sensors [2]. Recently, using linear matrix inequality (LMI) optimization solutions [3, 4, 5], linear parameter varying (LPV) FTC laws are designed, based on an open-loop system modeled as a function of fault parameters. In the LPV-FTC synthesis procedure, it is assumed to be imminently identified by separate fault detection and isolation (FDI) modules. FDI modules are designed, based on only an open-loop system and applied in a closed-loop system [5, 6, 7]. In general, FDI modules and FTC laws are individually designed, without considering the other dynamics [4, 5].

After individually designing an LPV-FTC law and FDI modules, analysis of a FTC system is required to validate the FTC laws and FDI modules together. A typical way of analyzing a FTC system is nonlinear simulation with the pre-defined command inputs (not all possible command inputs), for possible fault scenario. It is hard to provide a generic criteria of a FTC system via nonlinear simulations. A FTC system analysis frame work is required to provide a certain criteria of a FTC system related with characteristics of FDI modules.

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Usually, FDI modules can not detect faults at the moment a fault occurs. There is always some level of time delay to detect faults regardless of FDI algorithms such as an LPV-FDI filter [7] and an extended Kalman FDI filter [5]. During the delay time interval, the dynamics of the closed-loop system may not be in the pre-defined set of systems used in the LPV-FTC law synthesis procedure [5]. It is also possible that during the time interval, FDI modules may detect false faults which represent faults on healthy actuator/sensors. In this paper, the signal provided by FDI modules is parameterized and dynamics changes due to signal changes during the time interval are modeled as parameter transient behavior of an LPV system. It is possible that the parameter transient behavior includes that the system is locally unstable for a short time interval.

For that case, conventional LPV performance analysis methodologies can not be used since all systems are required to be locally stable for any fixed scheduling parameters. One of the LPV performance analysis methodologies with brief instabilities is developed in Ref. [8], using the main concept of switching system analysis. Modifying the analysis, in this paper, the upper bound of the induced- \mathcal{L}_2 norm of the closed-loop system is calculated in terms of possible delay time interval (a characteristic of FDI modules) and exponential decay rates of the Lyapunov function. A generic analysis framework of a FTC system is presented, including false fault parameter estimation effects on closed-loop system dynamics.

The analysis result of the proposed FTC system in this paper is one of indicators to find out which LPV-FTC law generates less performance degradation due to parameter transient behavior. In this paper, it is demonstrated by applying this FTC analysis frame work to a HiMAT FTC system designed in Ref. [5].

This paper contains the following sections. In section 2, a FTC system analysis problem is stated and an analysis methodology is described in section 3. In section 4, a HiMAT FTC system analysis is demonstrated. In section 5, this paper is concluded with a brief summary.

2 PROBLEM STATEMENT

In this section, analysis problems on a FTC system are described. The structure of a FTC system shown in Figure 1 is briefly described to carry out analysis problem objectives. A FTC system includes a FTC law, FDI modules and supervisory systems (logics). When a fault occurs, FDI modules and a supervisory system (logics) detect it and generate signals for evaluating/reconfiguring a FTC law. Assume that the designed FTC law can be represented as an LPV system or is designed using LPV synthesis methodologies in Ref. [9, 10, 5]. In this paper, FDI modules estimate fault parameters which can be converted into scheduling parameters of the designed FTC law via simple logics (see Fig. 1).

Fault parameters $\rho(t)$ represent actual fault occurrences on aero-vehicle dynamics $P(\rho(t))$. Assume that fault parameters $\rho(t)$ are in a piecewise continuous compact set $\mathcal{F}_\rho \subset \mathcal{R}^{n_s}$ to capture dynamics of abrupt fault occurrence. Since fault parameters are not directly measurable in real time via physical sensors, the parameters are estimated by FDI modules with estimation error bounds (δ_ρ). In practical terms, the size of δ_ρ is decided based on

characteristics of FDI modules and possible fault scenarios. The designed LPV controller $K(\bar{\rho}(t))$ is evaluated based on the estimated fault parameters $\bar{\rho}(t)$ in a piecewise continuous compact set $\mathcal{F}_{\bar{\rho}} \subset \mathcal{R}^{n_s}$. Based on the LPV synthesis methodology [5], the designed LPV controller $K(\bar{\rho}(t))$ can robustly stabilize an aero-vehicle $P(\rho)$ under fault occurrences, when

$$\rho(t) \in \mathcal{S}(\bar{\rho}, \delta_\rho) := \left\{ \begin{array}{l} \rho(t) \mid \|\rho(t) - \bar{\rho}(t)\| \leq \delta_\rho, \\ \rho \in \mathcal{F}_\rho, \bar{\rho} \in \mathcal{F}_{\bar{\rho}}, 0 \leq \delta_\rho \end{array} \right\}. \quad (1)$$

When a fault occurs, FDI modules require some time for estimated fault parameters to satisfy the condition: $\rho(t) \in \mathcal{S}(\bar{\rho}(t), \delta_\rho)$. The required time is called as detection time for FDI modules, hereafter. During the detection time interval, it is possible that fault parameters $\rho(t)$ are not in a set $\mathcal{S}(\bar{\rho}(t), \delta_\rho)$ since δ_ρ is generally chosen as smaller values than the entire range of $\rho(t)$ variation. The performance and stability for the closed-loop system is not guaranteed for the case: $\rho(t) \notin \mathcal{S}(\bar{\rho}(t), \delta_\rho)$ since the LPV control law is designed only for the case: $\rho(t) \in \mathcal{S}(\bar{\rho}(t), \delta_\rho)$. During the interval, parameter transient behavior[5] of the closed-loop system is observed for the case: $\rho(t) \notin \mathcal{S}(\bar{\rho}(t), \delta_\rho)$. When the time interval is long, the performance of the closed-loop system can be noticeably degraded and stability of the system can be also changed.

In order to analyze performance degradation and stability of the system, the closed-loop FTC system $G(\rho, \bar{\rho})$ is modeled as

$$\begin{aligned} \dot{x} &= A(\rho, \bar{\rho})x + B(\rho, \bar{\rho})d, \\ e &= C(\rho, \bar{\rho})x + D(\rho, \bar{\rho})d, \end{aligned} \quad (2)$$

where $x \in \mathcal{R}^{n_x}$, $d \in \mathcal{R}^{n_d}$ and $e \in \mathcal{R}^{n_e}$. All the matrices have compatible dimensions. Note that the fault parameters ρ and estimated fault parameters $\bar{\rho}$ are treated as independent parameters in an analysis frame work.

The entire parameter space is defined as

$$\mathcal{P} := \{(\rho(t), \bar{\rho}(t)) \mid \rho(t) \in \mathcal{F}_\rho, \bar{\rho}(t) \in \mathcal{F}_{\bar{\rho}}\} \quad (3)$$

and is divided into m subspaces \mathcal{P}_i such that

$$\begin{aligned} \mathcal{P} &= \bigcup_{i=1}^m \mathcal{P}_i := \mathcal{P}_1 \cup \mathcal{P}_2 \cdots \cup \mathcal{P}_m, \\ \emptyset &= \mathcal{P}_i \cap \mathcal{P}_j, \quad i \neq j, \quad i \in \mathcal{I}, \quad j \in \mathcal{I}, \\ \mathcal{I} &:= \{1, 2, \dots, m\}. \end{aligned} \quad (4)$$

Local and global stability of the closed-loop system $G(\rho, \bar{\rho})$ in Eq. (2) are defined over parameter subspaces as follows:

Definition 1 *Local and global stability:*

Suppose there exists a positive definite matrix P such that

$$A^T(\rho, \bar{\rho})P + PA(\rho, \bar{\rho}) < 0, \quad (\rho, \bar{\rho}) \in \mathcal{P} \quad (5)$$

then the system is globally stable over the entire parameter set. When the condition in Eq. (5) is satisfied over only subsets \mathcal{P}_i , the system is locally stable.

To analyze system dynamics changes due to parameter transient behavior, it is necessary to introduce duration time of the system over each subspace.

Definition 2 *Duration time T_{p_i} over each subspace:*

$$T_{p_i}(t_o, t) = \int_{t_o}^t \sigma_i(\rho(s), \bar{\rho}(s)) ds, \quad \forall t > t_o > 0, \quad (6)$$

$$\sigma_i(\rho(s), \bar{\rho}(s)) = \begin{cases} 0, & (\rho(s), \bar{\rho}(s)) \notin \mathcal{P}_i, \\ 1, & (\rho(s), \bar{\rho}(s)) \in \mathcal{P}_i, \\ & i \in \mathcal{I}. \end{cases} \quad (7)$$

A duration time is bounded as

$$T_{o_i} + \beta_i(t - t_o) \leq T_{p_i} \leq T_{o_i} + \alpha_i(t - t_o) \quad (8)$$

where $0 \leq \beta_i \leq \alpha_i \leq 1$ and $0 \leq T_{o_i}$. The constant α_i represents a ratio of total time of interest to duration time over the i -th subspace. For example, when the system stays in the i -th subspace for all time, the constant α_i should be one. When the system never stays in the i -th subspace for all time, the constant α_i should be zero.

In this paper, we consider possible local instability over a subspace because FDI modules can generate false fault detection signals during the detection time interval. Without loss of generality, consider the case that the system is locally unstable in the m -th subspace. The constant $\alpha_m < 1$ takes an important role in stability analysis to represent asymptotic stability ratio [8], which is related with stability margin of the system. In this paper, the constant α_m is interpreted as tolerance of the false detection of FDI modules for the closed-loop system [11]. For example, if the system is stable with the larger number of α_m , it implies that the system can stay in the m -th subspace longer.

In this paper, the subspace \mathcal{P}_1 is defined when $\rho(t) \in \mathcal{S}(\bar{\rho}(t), \delta_\rho)$. For practicality, assume that the closed-loop system stays in the subspace \mathcal{P}_1 for the most of the time of interest, since the FTC law is designed for it [5]. During the detection time interval, the closed-loop system stays in the parameter subspaces \mathcal{P}_i , $i \in \mathcal{I} - \{1\}$. The stability margin of the system over the entire parameter space can be described as a function of the constants α_i , $i \in \mathcal{I}$. Also, the performance degradation level can be represented as an induced- \mathcal{L}_2 norm of the closed-loop system over the entire parameter space.

3 ANALYSIS METHOD

In this section, the analysis problems described in the previous section are formulated into LMI forms.

3.1 Stability Analysis

Recall that the closed-loop system $G(\rho, \bar{\rho})$ in Eq. (2) is locally stable in the set of $\bigcup_{i=1}^{m-1} \mathcal{P}_i$ and locally unstable in the parameter subset \mathcal{P}_m .

Proposition 1: *Suppose there exists a positive definite matrix $P(\bar{\rho})$ such that*

$$\begin{aligned} A^T(\rho, \bar{\rho})P + PA(\rho, \bar{\rho}) + \dot{P} &\leq -\lambda_i P, & (\rho, \bar{\rho}) \in \mathcal{P}_i, & \quad i \in \mathcal{I} - \{m\}, \\ A^T(\rho, \bar{\rho})P + PA(\rho, \bar{\rho}) + \dot{P} &\leq \kappa P, & (\rho, \bar{\rho}) \in \mathcal{P}_m, \end{aligned} \quad (9)$$

where

$$0 \leq \kappa, \quad 0 \leq \lambda_i \leq \lambda_1, \quad i \in \mathcal{I} - \{1, m\}. \quad (10)$$

The system is exponentially stable with a decay rate:

$$\lambda = \lambda_1 - \sum_{i=2}^{m-1} (\lambda_1 - \lambda_i) \alpha_i - (\lambda_1 + \kappa) \alpha_m \quad (11)$$

under the condition:

$$\alpha_m < \alpha_t := \frac{\lambda_{\min}}{\lambda_{\min} + \kappa}, \quad (12)$$

where $\lambda_{\min} = \min\{\lambda_i\}$, $i \in \mathcal{I} - \{m\}$.

The proof is follows. Set a Lyapunov function as $V = x^T(t)P(\bar{\rho})x(t)$, $P \in \mathcal{R}^{n_x \times n_x}$. Using Eq. (9), the time derivative of the Lyapunov function is

$$\dot{V} \leq -\lambda_i V, \quad (\rho, \bar{\rho}) \in \mathcal{P}_i, \quad i \in \mathcal{I} - \{m\}, \quad (13)$$

$$\dot{V} \leq \kappa V, \quad (\rho, \bar{\rho}) \in \mathcal{P}_m. \quad (14)$$

From Eqs. (13) and (14),

$$V \leq e^{-\lambda_1(t-t_o) - \sum_{i=2}^{m-1} T_{p_i} - \sum_{i=2}^{m-1} \lambda_i T_{p_i} + \kappa T_{p_m}} V(t_o), \quad \forall t \geq t_o \geq 0. \quad (15)$$

Taking the upper bound of the duration time over each parameter subspace, Eq. (15) is rewritten as:

$$V \leq e^{-\lambda(t-t_o) + \sum_{i=2}^{m-1} (\lambda_1 - \lambda_i) T_{o_i} + (\lambda_1 + \kappa) T_{o_m}} V(t_o), \quad (16)$$

where

$$\lambda := \lambda_1 - \sum_{i=2}^{m-1} (\lambda_1 - \lambda_i) \alpha_i - (\lambda_1 + \kappa) \alpha_m. \quad (17)$$

Without loss of generality, the constant λ_1 can be set as $\max\{\lambda_i\}, i \in \mathcal{I} - \{m\}$. Thus, the term $\lambda_1 - \lambda_i$ is always positive over the parameter subspaces $\mathcal{P}_i, i \in \mathcal{I} - \{1, m\}$. The lower bound of the constant λ is

$$\lambda \geq \lambda_t := \lambda_1 - \sum_{i=2}^{m-1} (\lambda_1 - \lambda_{min})\alpha_i - (\lambda_1 + \kappa)\alpha_m. \quad (18)$$

When the condition Eq. (12) is satisfied, it is easily noticed that the system is exponential stable such that $\lambda_t \geq 0$.

In this paper, the exponential decay rate is of interest to analyze a closed-loop system which may have local instability over a parameter subspace. The constant α_t represents the upper bound of asymptotic instability ratio [8]. In the analysis of a FTC system, the constant α_t can be interpreted as tolerance of instability during the detection time interval. For example, $\alpha_t = 0.1$ implies that the closed-loop system can stay in the parameter subspace \mathcal{P}_m for 10 % of the total interest time without loss of exponential stability.

Given fault tolerant controllers, we can analyze the closed-loop system in terms of a constant α_t value. The stability analysis problem can be formulated into an optimization problem as:

$$\begin{aligned} & \max_{\kappa > 0, \lambda_{min} > 0} \alpha_t, \\ & s.t \quad \text{Eq. (9)} \end{aligned} \quad (19)$$

In this paper, the optimization is solved by checking feasibility of the LMI constraints in Eq. (9) with a line searching method over the constant λ_i and κ values.

3.2 Performance analysis

For the closed-loop system in Eq. (2), the induced- \mathcal{L}_2 norm is defined as:

$$\sup_{(\rho, \bar{\rho}) \in \mathcal{P}, d \in \mathcal{L}_2, \|d\|_2 \neq 0} \frac{\|e\|_2}{\|d\|_2}. \quad (20)$$

It is interesting to determine the induced- \mathcal{L}_2 norm from disturbance d to error e with parameter transient behavior during a detection time interval.

Proposition 2: *Suppose there exists a positive definite matrix $P(\bar{\rho}) \in \mathcal{R}^{n_x \times n_x}$ such that*

$$\begin{aligned} & (\rho, \bar{\rho}) \in \mathcal{P}_i, \quad i \in \mathcal{I} - \{m\}, \\ & \begin{bmatrix} A^T(\rho, \bar{\rho})P(\bar{\rho}) + P(\bar{\rho})A(\rho, \bar{\rho}) + \dot{P}(\bar{\rho}) + \lambda_i P(\bar{\rho}) & P(\bar{\rho})B(\rho, \bar{\rho}) & \gamma^{-1}C^T(\rho, \bar{\rho}) \\ B^T(\rho, \bar{\rho})P(\bar{\rho}) & -I & \gamma^{-1}D^T(\bar{\rho}) \\ \gamma^{-1}C(\bar{\rho}) & \gamma^{-1}D(\bar{\rho}) & -I \end{bmatrix} < 0, \end{aligned} \quad (21)$$

$$\begin{aligned}
& (\rho, \bar{\rho}) \in \mathcal{P}_m, \\
& \begin{bmatrix} A^T(\rho, \bar{\rho})P(\bar{\rho}) + P(\bar{\rho})A(\rho, \bar{\rho}) + \dot{P}(\bar{\rho}) - \kappa P(\bar{\rho}) & P(\bar{\rho})B(\rho, \bar{\rho}) & \gamma^{-1}C^T(\rho, \bar{\rho}) \\ B^T(\rho, \bar{\rho})P(\bar{\rho}) & -I & \gamma^{-1}D^T(\rho, \bar{\rho}) \\ \gamma^{-1}C(\rho, \bar{\rho}) & \gamma^{-1}D(\rho, \bar{\rho}) & -I \end{bmatrix} < 0. \tag{22}
\end{aligned}$$

The induced- \mathcal{L}_2 norm from d to e of the closed-loop system is no larger than M_γ where

$$\begin{aligned}
M_\gamma &= \gamma \sqrt{\frac{e^{\sum_{i=2}^{m-1} (\lambda_1 - \lambda_i) T_{o_i} + (\lambda_1 + \kappa) T_{o_m}} \lambda_b}{\lambda}}, \\
\lambda_b &= \lambda_1 - \sum_{i=2}^{m-1} (\lambda_1 - \lambda_i) \beta_i - (\lambda_1 + \kappa) \beta_m, \\
\lambda &= \lambda_1 - \sum_{i=2}^{m-1} (\lambda_1 - \lambda_i) \alpha_i - (\lambda_1 + \kappa) \alpha_m.
\end{aligned} \tag{23}$$

Under the condition $\alpha_m < \alpha_t := \frac{\lambda_{\min}}{(\lambda_{\min} + \kappa)}$, the constant λ is always positive such that $\lambda \geq \lambda_b > 0$.

The proof is follows. Set $V = x^T(t)P(\bar{\rho})x(t)$, then the time derivative of a Lyapunov function V is

$$\begin{aligned}
\dot{V} &\leq -\lambda_i V + \|d\|^2 - \gamma^{-2} \|e\|^2, \quad (\rho, \bar{\rho}) \in \mathcal{P}_i, \quad i \in \mathcal{I} - \{m\}, \\
\dot{V} &\leq \kappa V + \|d\|^2 - \gamma^{-2} \|e\|^2, \quad (\rho, \bar{\rho}) \in \mathcal{P}_m.
\end{aligned} \tag{24}$$

Eqs. (21) and (22) describe the condition Eq. (24). The Lyapunov function is rewritten as:

$$V(t) \leq e^{f(t_0, t)} V(t_0) + \int_{t_0}^t e^{f(s, t)} (\|d(s)\|^2 - \gamma^{-2} \|e(s)\|^2) ds, \tag{25}$$

where

$$f(s, t) = -\lambda_1(t - s - \sum_{i=2}^m T_{p_i}(s, t)) - \sum_{i=2}^{m-1} \lambda_i T_{p_i}(s, t) + \kappa T_{p_m}(s, t). \tag{26}$$

Since $V(t) > 0$, $\forall t \geq t_0 \geq 0$, the following inequality equation is extracted from Eq. (25).

$$\gamma^{-2} \int_{t_0}^t e^{f(s, t)} \|e(s)\|^2 ds \leq e^{f(t_0, t)} V(t_0) + \int_{t_0}^t e^{f(s, t)} \|d(s)\|^2 ds, \quad \forall t \geq t_0 \geq 0. \tag{27}$$

Using Eq. (8) and the definition of λ and λ_b in Eq. (23), it is easily derived that

$$\begin{aligned}
& \gamma^{-2} \int_{t_0}^t e^{-\lambda_b(t-s)} \|e(s)\|^2 ds \leq e^{\sum_{i=2}^{m-1} (\lambda_1 - \lambda_i) T_{o_i} + (\lambda_1 + \kappa) T_{o_m} - \lambda(t-t_0)} V(t_0) \\
& + \int_{t_0}^t e^{\sum_{i=2}^{m-1} (\lambda_1 - \lambda_i) T_{o_i} + (\lambda_1 + \kappa) T_{o_m} - \lambda(t-s)} \|d(s)\|^2 ds.
\end{aligned} \tag{28}$$

Integrating both sides of Eq. (28) over the interval (t_o, ∞) leads to

$$\frac{\gamma^{-2}}{\lambda_b} \int_{t_o}^{\infty} \|e(s)\|^2 ds \leq \frac{V(t_o)}{\lambda} e^{\sum_{i=2}^{m-1} (\lambda_1 - \lambda_i) T_{o_i} + (\lambda_1 + \kappa) T_o} + \frac{1}{\lambda} e^{\sum_{i=2}^{m-1} (\lambda_1 - \lambda_i) T_{o_i} + (\lambda_o + \kappa) T_o} \int_{t_o}^{\infty} \|d(s)\|^2 ds. \quad (29)$$

Thus, the upper bound of the induced- \mathcal{L}_2 norm M_γ is

$$M_\gamma = \gamma \sqrt{\frac{e^{\sum_{i=2}^{m-1} (\lambda_1 - \lambda_i) T_{o_i} + (\lambda_1 + \kappa) T_{o_m}} \lambda_b}{\lambda}}. \quad (30)$$

The constant M_γ represents the upper bounds of the worst-case performance level under parameter transient behavior which includes possible local instability. Thus, the performance analysis problem is formulated into an optimization problem:

$$\begin{aligned} \min_{\lambda_i > 0, \kappa \geq 0} M_\gamma \left(\min_{P \in \mathcal{R}^{n_x \times n_x}} \gamma \right), \\ \text{s.t. Eq.(21), (22)}. \end{aligned} \quad (31)$$

The minimization γ problem at fixed λ_i and κ is an LMI problem. However, the optimization problem in Eq. (31) is not an LMI problem in terms of λ_i , κ and γ with P . Also, the solution γ is highly related with λ_i and κ values. In this paper, line searching over possible λ_i and κ ranges is used to find a solution of the optimization problem.

Note that this analysis does not require that the positive matrix P is a function of the estimated parameters $\bar{\rho}$. Based on behaviour of estimated parameters by the FDI modules, the matrix P can be set as a constant positive matrix or a function of parameter over the $\bar{\rho}$ variations. The constant positive matrix can allow abrupt change of the estimated parameter but leads to conservative analysis results. It will be demonstrated via the following example.

4 EXAMPLE

In this section, the proposed analysis methodology is demonstrated by application to a FTC system. Using the analysis, it is possible to indicate which FTC law is tolerant of possible false detection signal and generates less performance degradation for parameter transient behaviors during the detection time interval.

4.1 FTC HiMAT System

The FTC HiMAT system [5] consists of an on-line FDI system with simple logics, the dynamics of a HiMAT vehicle shown in Figure 2, and a designed LPV-FTC law. The HiMAT vehicle dynamics is taken from the μ -synthesis Toolbox [12] and is modeled as an LPV system. The LPV model has two inputs: elevons δ_e and canards δ_c ; two outputs: angle of attack α in radians and pitch angle θ in radians; and four states: velocity V in ft/sec, angle

of attack α , pitch rate q in rad/sec, and pitch angle θ . The LFT-LPV model of the HiMAT vehicle is

$$\dot{x} = Ax + B \begin{bmatrix} \bar{\tau}_1 & 0 \\ 0 & \bar{\tau}_2 \end{bmatrix} u + B \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} w, \quad (32)$$

$$z = u, \quad y = Cx, \quad (33)$$

$$w = \Delta z, \quad \Delta = \begin{bmatrix} \delta_1 & 0 \\ 0 & \delta_2 \end{bmatrix} \quad (34)$$

where the detailed elements of the system matrices A , B , and C are in Ref. [5]. The estimated scheduling parameter $\bar{\rho}$ is defined using simple logics as

$$\begin{aligned} 0 \leq \bar{\rho} < 1 & : 0 \leq \bar{\tau}_1 < 1, \bar{\tau}_2 = 1 && \text{Elevon failure} \\ \bar{\rho} = 1 & : \bar{\tau}_1 = 1, \quad \bar{\tau}_2 = 1 \\ 1 < \bar{\rho} \leq 2 & : \bar{\tau}_1 = 1, 0 \leq \bar{\tau}_2 < 1 && \text{Canard failure.} \end{aligned}$$

The on-line FDI model [5] estimates fault parameters $\bar{\tau}_1$ and $\bar{\tau}_2$ using the two-stage optimal Kalman filter [5, 13]. Based on the estimated fault parameters, the estimated scheduling parameter $\bar{\rho}$ is defined via the simple logics.

LPV-FTC laws $K_A(\bar{\rho})$ and $K_B(\bar{\rho})$ are designed without a scaling factor using conventional LPV control synthesis methodology in Ref. [10] and with a scaling factor using a robust LPV synthesis methodology [5], based on the LPV-HiMAT model with estimated error bound 0.1. The closed-loop responses with the controllers K_A and K_B are simulated for the possible fault scenario of the canards failing at 1 sec and are shown in Figure 3, respectively. The right plots in Figure 3 are magnifications of the left plots. Based on Figure 3, there is very little difference between K_A and K_B responses with steady state parameter estimates and pitch angle time responses. The simulation results in Figure 3 show that the detection time interval is about 0.5 sec from $t = 1$ sec to $t = 1.5$ sec. During the interval, a scheduling parameter ρ is 2, which indicates canard failure. The estimated parameter $\bar{\rho}$, however, is not 2 during the interval. It is easily observed that the condition $\rho(t) \in \mathcal{S} := \{\rho \mid |\rho - \bar{\rho}| \leq 0.1\}$ is not satisfied during the interval. However, overall simulation results show that the closed-loop system is stable and achieves the desired performance objective defined in Ref. [5] since the detection time interval is short in the simulation (less than 0.5 sec).

It is noticed, using linear analysis at fixed parameters, that the eigenvalues of the closed-loop system with K_A or K_B are positive at fixed parameters $\bar{\rho} = 0$ and $\rho = 2$. When $\bar{\rho} > 0.1$, the closed-loop system is stable at fixed parameters. This leads to the FTC HiMAT system being locally unstable for the short interval.

constant matrix P				
Controller	λ_1	λ_2	κ	α_t
K_A	0.042	0.041	7.65	5.3×10^{-3}
K_B	0.042	0.038	4.96	7.6×10^{-3}
parameter dependent $P(\bar{\rho})$				
K_A	0.041	0.039	7.6	5.1×10^{-3}
K_B	0.034	0.032	3.7	8.6×10^{-3}

Table 1: *Stability analysis results*

4.2 Stability Analysis

To apply the stability analysis methodology described in section 3, the parameter set \mathcal{P} is divided into three subspaces for this example. Set the subspaces as

$$\begin{aligned}
\mathcal{P}_1 &= \{(\rho, \bar{\rho}) \mid \rho = 2, \quad 0.9 \leq \bar{\rho} \leq 2\}, \\
\mathcal{P}_2 &= \{(\rho, \bar{\rho}) \mid \rho = 2, \quad 0.1 \leq \bar{\rho} \leq 0.9\}, \\
\mathcal{P}_3 &= \{(\rho, \bar{\rho}) \mid \rho = 2, \quad 0.0 \leq \bar{\rho} \leq 0.1\}.
\end{aligned} \tag{35}$$

Note that when $\bar{\rho} \in \mathcal{P}_1$, the condition $\rho(t) \in \mathcal{S}(\bar{\rho}, 0.1)$ is satisfied. Recall that the condition is used in designing an LPV-FTC law. When $\bar{\rho} \in \mathcal{P}_2$, the closed-loop system is locally stable but the condition is not satisfied. When $\bar{\rho} \in \mathcal{P}_3$, the closed-loop system is locally unstable.

The LMI constraints in Eq. (9) are evaluated at grid points $\bar{\rho} \in \{0, 0.1, 0.2, \dots, 2\}$ over the parameter subspaces. In order to solve the optimization problem in Eq. (19) with LMI constraints in Eq. (9), the ranges of λ_1 , λ_2 , and κ are defined as $0.01 \leq \lambda_1 \leq 0.1$, $0.01 \leq \lambda_2 \leq 0.1$, and $5 \leq \kappa \leq 10$, respectively. With fixed λ_1 , λ_2 , and κ values in each range, the feasibility of the LMI constraints is checked with a constant matrix P and a parameter dependent matrix $P(\bar{\rho})$, respectively. Using a parameter dependent matrix $P(\bar{\rho})$, the time derivative $\dot{\bar{\rho}}$ is required to determine $\dot{P} = \dot{\bar{\rho}} \frac{\partial P(\bar{\rho})}{\partial \bar{\rho}}$. In this example, it is assumed that $|\dot{\bar{\rho}}| \leq 1$, based on the FDI module characteristics. The parameter dependent matrix $P(\bar{\rho})$ is described using basis functions.

$$P(\bar{\rho}) = P_o + \bar{\rho}P_1 + \frac{1}{\bar{\rho}}P_2, \quad \text{constant } P_{o,1,2} \in \mathcal{R}^{n \times n}, \tag{36}$$

where n is the closed-loop state order. The optimization results are in Table 1. Recall that the constant α_t represents the ratio of possible duration time over the unstable parameter space to total time. Based on the calculated α_t values, both FTC laws have similar tolerance to possible false fault detection in the stability of the closed-loop system. When two controllers are compared, the controller K_B has slightly better tolerance. It is also shown in Table 1 that

constant matrix P				
Controller	λ_1	λ_2	κ	γ
K_A	3×10^{-3}	1×10^{-3}	9.4	0.42
K_B	6×10^{-4}	1×10^{-4}	5.85	2.14
parameter dependent $P(\bar{\rho})$				
K_A	6×10^{-4}	1×10^{-4}	9.4	0.34
K_B	6×10^{-4}	1×10^{-4}	3.7	0.63

Table 2: *Performance analysis results*

the analysis with parameter dependent P leads to less conservative results on the controller K_B .

4.3 Performance Analysis

It is of interest to know how much performance of the closed-loop system degrades due to parameter transient behaviors. To analyze the closed-loop performance, the desired response function T_i and the performance weighting function W_p in Figure 4 are defined as

$$T_i = \frac{1}{s/0.8 + 1}, \quad W_p = 40 \frac{s/50 + 1}{s/0.05 + 1}. \quad (37)$$

Note that the weight functions are, also, used in an LPV control synthesis in Ref. [5]. Recall that minimizing the upper bound M_γ of the induced- \mathcal{L}_2 norm from d to e is not in convex optimization, since λ_1 , λ_2 , and κ are highly related with γ in Eq. (23). Using line searching over the λ_1 , λ_2 , and κ ranges, the analysis results are calculated in Table 2. When T_{o_3} is fixed as 0.2 sec, the upper bound of the induced- \mathcal{L}_2 norm is calculated as 0.68 for K_A and 2.87 for K_B with constant matrix P and 0.54 for K_A and 0.84 for K_B with parameter dependent matrix $P(\bar{\rho})$.

It is noticed from Eq. (31) that the time constants T_{o_i} are not related with optimized λ_1 , λ_2 , κ , and γ . Thus, with fixed κ , λ_1 , λ_2 , and γ values for each controller, the M_γ variations are calculated due to T_{o_3} changes and are shown in Figure 5. It is observed from Figure 5, that performance analysis results are significantly different using constant matrix P and a parameter dependent matrix $P(\bar{\rho})$. Based on performance analysis result with a parameter dependent matrix $P(\bar{\rho})$, it is easily noticed that when a false fault detection time is short ($T_{o_3} < 0.22$ sec), the M_γ with the K_A controller is less than that with the K_B controller. When a false fault detection time is long ($T_{o_3} > 0.22$ sec), the M_γ values are vice versa. The analysis results imply that K_A controller leads to less performance degradation for the $T_{o_3} < 0.22$ sec case and K_B controller leads to less performance degradation for the $T_{o_3} > 0.22$ sec case.

Recall that M_γ in this FTC system analysis represents the upper bound of the induced norm of $\|\theta - \theta_{ideal}\|_2$ from a given command $\|d\|_2$. In order to validate the FTC system performance analysis results, the closed-loop system is simulated with assumption of false fault detection $T_{o3} = 0.1, 0.2, 0.4$ and 0.8 sec for each controller, respectively.

In Figure 6, the error between pitch angle responses and ideal responses θ_{ideal} are plotted for each case. It is obviously noticed that the error norm of $\|\theta - \theta_{ideal}\|_2$ with the K_A controller is larger than that with the K_B controller at the $T_3 = 0.4$ and 0.8 sec cases. Also, the error norm with the K_B controller is larger than that with the K_A controller at the $T_{o3} = 0.1$ and 0.2 sec cases. The parameter dependent analysis results in Figure 5 correspond to the simulation results in Figure 6. Note that using this analysis tool, we can calculate the upper bound variation of the induced- \mathcal{L}_2 norm due to T_{02} variation which represents parameter transient time. In this example, λ_1 and λ_2 are too small to see effects of T_{02} variations.

5 CONCLUSION

In this paper, the FTC system analysis problem is formulated into the optimization problem subject to LMI constraints which are evaluated at grid points over the stable/unstable parameter subspaces. From the stability analysis, the tolerance of false fault detection which causes local instability can be calculated for a FTC system to achieve exponential stability. From the performance analysis, the upper bound of the induced- \mathcal{L}_2 norm of the FTC system represents worst-case performance during the detection time intervals of FDI modules. The norm bound is calculated as a function of detection time interval and exponential decay rates over each parameter subspace. It indicates performance degradation due to parameter transients during the interval in which the closed-loop system may be locally unstable. In this paper, the usage of the FTC system analysis is demonstrated via the analysis of the FTC HiMAT system.

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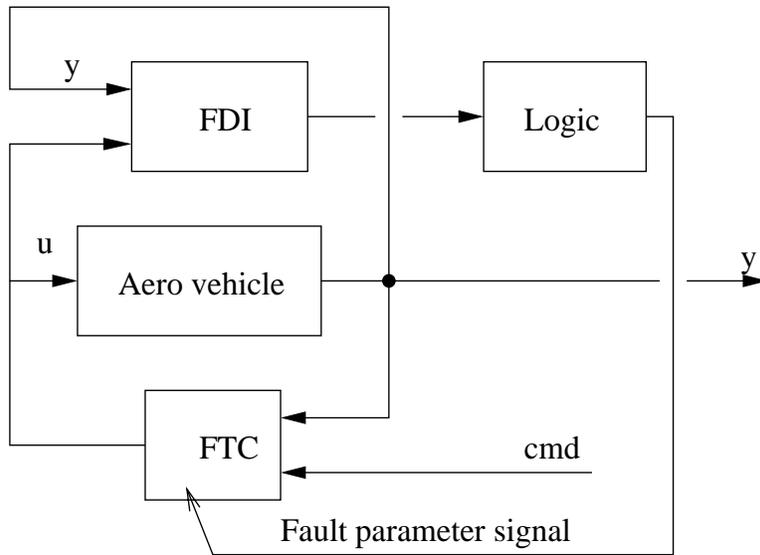


Figure 1: *Structure of a fault tolerant control system.*



Figure 2: *HiMAT vehicle.*

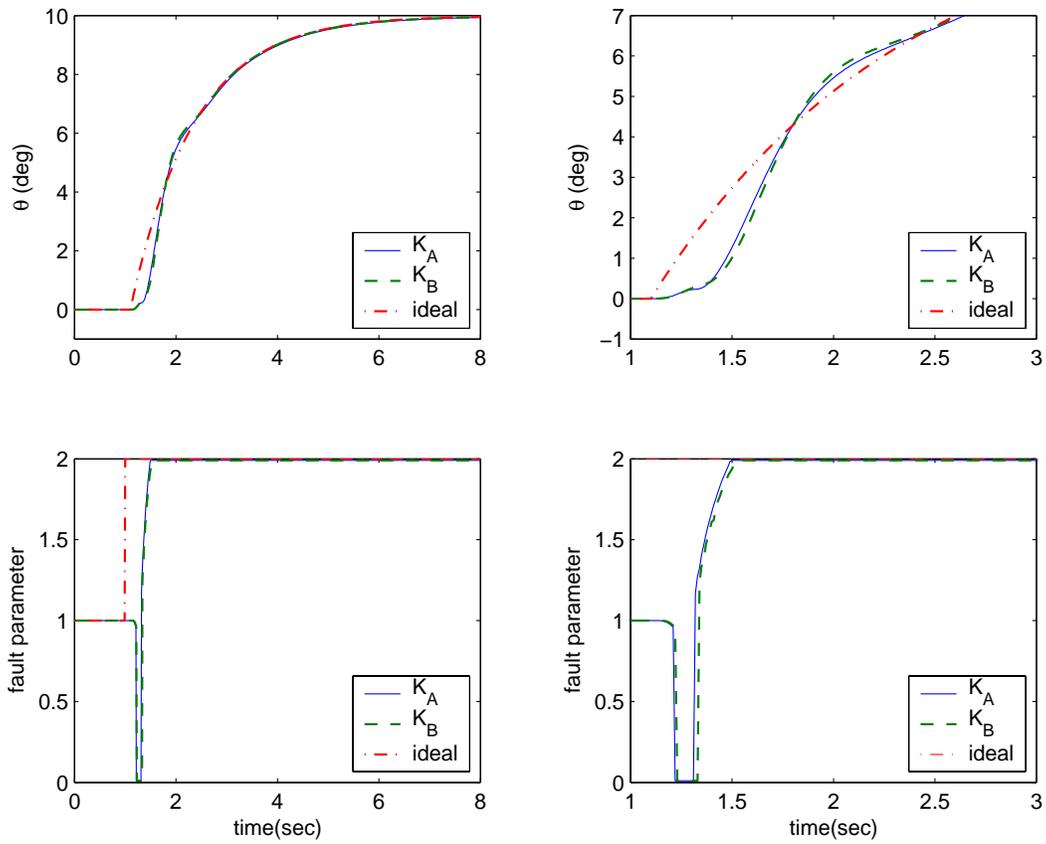


Figure 3: *Nonlinear simulations of a FTC HiMAT Vehicle.*

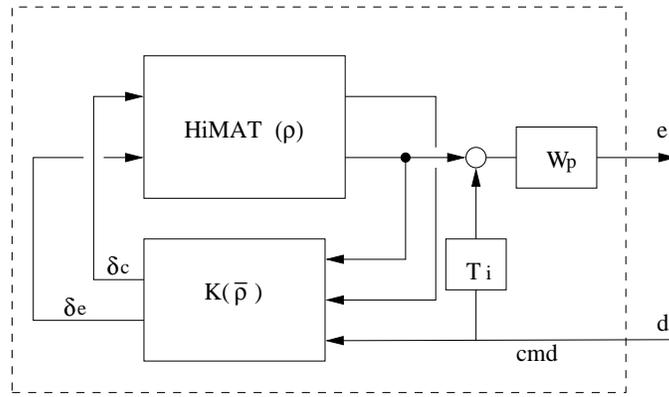


Figure 4: *The weighted closed-loop system of a HiMAT vehicle*

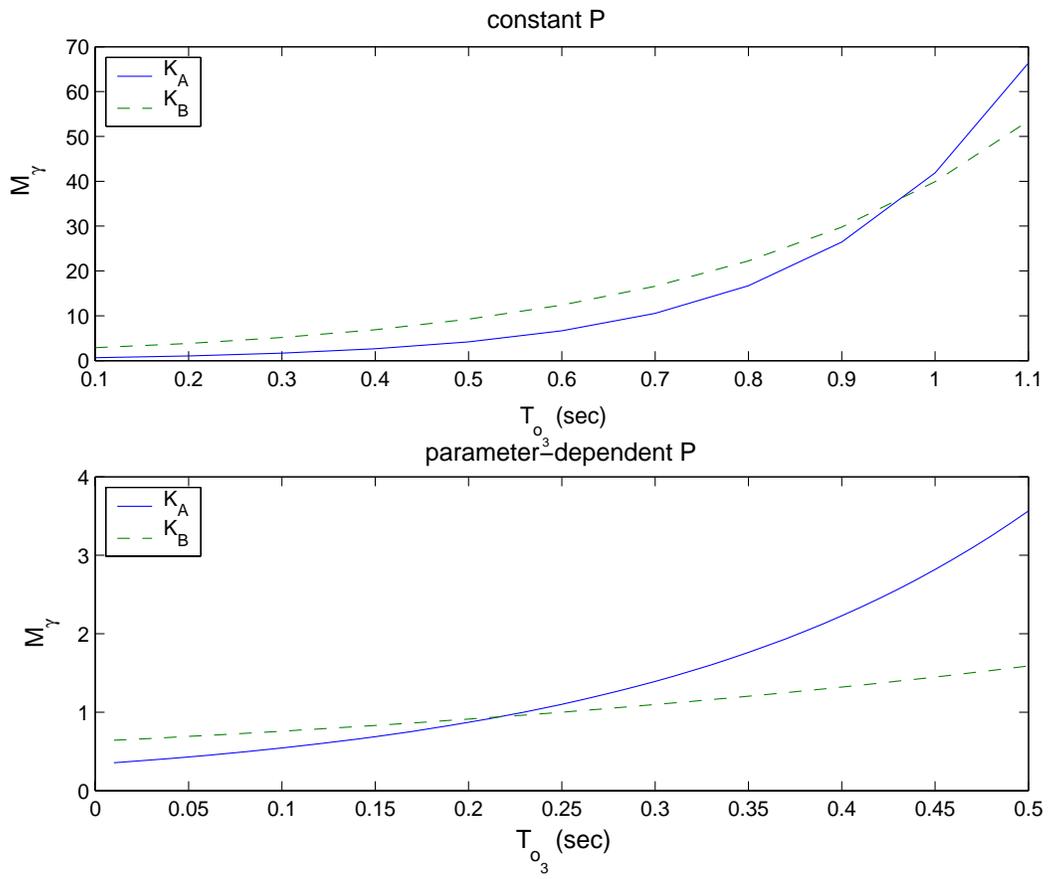


Figure 5: Variations of M_γ due to T_{o_3} changes.

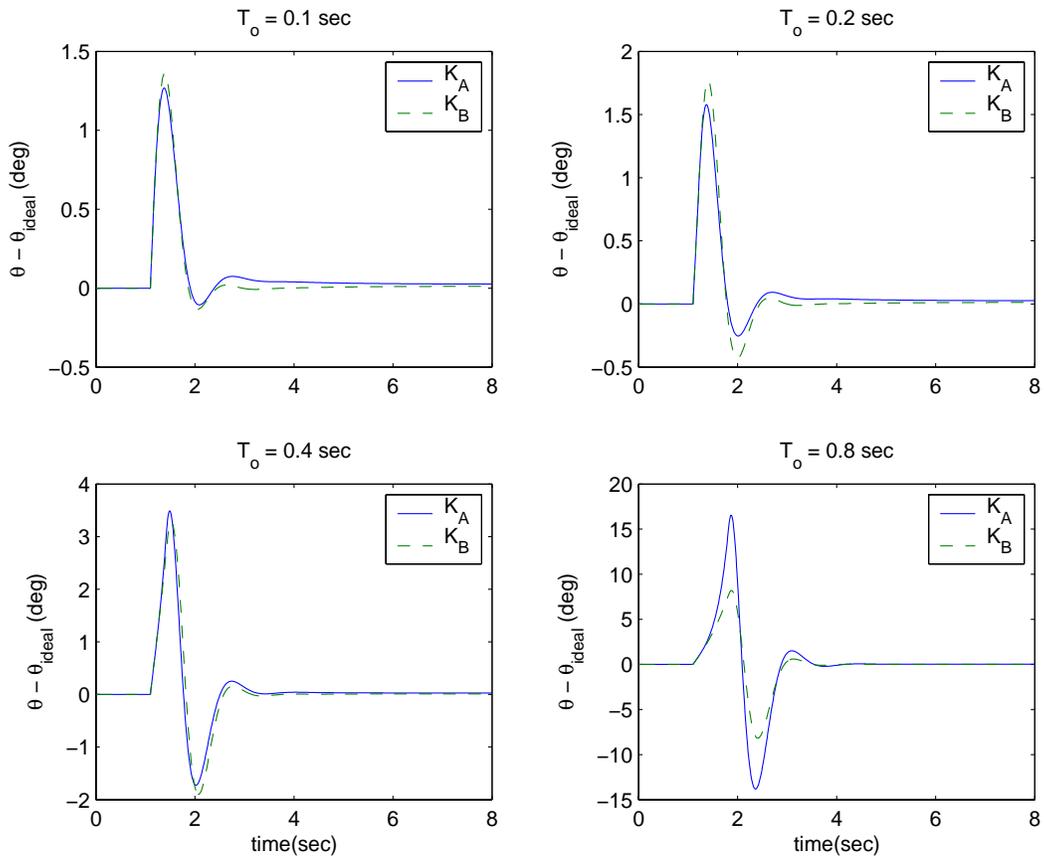


Figure 6: Simulation results with different T_{o3} values: 0.1, 0.2, 0.4 and 0.8 sec.

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