A SUPERIOR KIRCHHOFF METHOD FOR AEROACOUSTIC NOISE PREDICTION: The Ffowcs Williams–Hawkings equation

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Introduction

- Prediction of aeroacoustic noise important
  - all new aircraft must meet noise certification requirements
  - local noise standards can be even more stringent
  - NASA noise reduction goal: reduce perceived noise levels by a factor of two in 10 years

- Several prediction methods available
  - direct computation
    - CFD based methods
    - near field only
    - best coupled with integral method for far-field prediction
  - Acoustic Analogy (Ffowcs Williams–Hawkings Equation)
  - Kirchhoff formula

- Confusion over relationship between methods exists
Comments on Integral Methods

- Technology
  - acoustic formulations and algorithms mature
    - widely used for rotating blade noise prediction
    - potentially useful for airframe noise, engine noise, etc.
  - flow field computation feasible in many cases
    - required for input data
    - provided by CFD
  - high quality experiments aid validation

- This talk will demonstrate the superiority of the FW–H approach over the Kirchhoff method for aeroacoustics
  - analytically
  - numerically
Advantages and Disadvantages

■ FW–H method
  + three source terms (thickness, loading, quadrupole) have physical meaning
  + three source terms are independent
  + mature and robust algorithms
  - quadrupole source is a volume source (more computational resources needed when volume integration included)

■ Kirchhoff method
  + surface sources (only surface integration required)
  + applicable to problems described by the wave equation
  - source terms not easily related to flow physics or design parameters
  - not as much experience with algorithms for Kirchhoff problems

■ Analytical/Numerical comparison needed
Analytical Comparison: FW–H Derivation Procedure

- Embed exterior flow problem in unbounded space
  - define generalized functions valid throughout entire space
  - interpret derivatives as generalized differentiation

\[ \tilde{\rho} = \begin{cases} \rho & f > 0 \\ \rho_o & f < 0 \end{cases} \]
\[ \rho u_i = \begin{cases} \rho u_i & f > 0 \\ 0 & f < 0 \end{cases} \]
\[ \tilde{P}_{ij} = \begin{cases} P_{ij} & f > 0 \\ 0 & f < 0 \end{cases} \]

- Generalized conservation equations:

\[ \frac{\partial \tilde{\rho}}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = (\rho \frac{\partial f}{\partial t} + \rho u_i \frac{\partial f}{\partial x_i})\delta(f) \quad \text{continuity} \]

\[ \frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} + \frac{\partial \tilde{P}_{ij}}{\partial x_j} = (\rho u_i \frac{\partial f}{\partial t} + (\rho u_i u_j + P_{ij}) \frac{\partial f}{\partial x_i})\delta(f) \quad \text{momentum} \]
Analytical Comparison: FW–H Derivation Procedure

- Manipulate conservation laws into form of inhomogeneous wave equation

\[ \square^2 p'(\vec{x}, t) = \frac{\bar{\partial}^2}{\partial x_i \partial x_j} \left[ T_{ij} H(f) \right] \]

\[ - \frac{\partial}{\partial x_i} \left[ (P_{ij} \hat{n}_j + \rho u_i (u_n - v_n)) \delta(f) \right] \]

\[ + \frac{\partial}{\partial t} \left[ (\rho_o v_n + \rho (u_n - v_n)) \delta(f) \right] \]

- Don’t assume integration surface \( f=0 \) is coincident with body
  - given in this form by Ffowcs Williams
  - demonstrated for rotors by di Francescantonio; Brentner & Farassat
Analytical Comparison: Kirchhoff Derivation Procedure

- Use embedding procedure on wave equation
  - define generalized pressure perturbation:
    \[ \tilde{p}' = \begin{cases} 
    p' & f > 0 \\
    0 & f < 0 
  \end{cases} \]

  - use generalized derivatives
  - generalized wave equation is Kirchhoff governing equation:
    \[
    \Box^2 p'(\bar{x}, t) = -\left( \frac{\partial p'}{\partial t} \frac{M_n}{c} + \frac{\partial p'}{\partial n} \right) \delta(f) - \frac{\partial}{\partial t} \left( p' \frac{M_n}{c} \delta(f) \right) - \frac{\partial}{\partial x_i} (p' \hat{n}_i \delta(f))
    \]
    \[ \equiv Q_{kir} \]
Source Term Comparison

- Manipulate FW–H source terms into form of Kirchhoff source terms (inviscid fluid)

\[ \Box^2 p'(\vec{x},t) = Q_{\text{kir}} + \frac{\partial^2}{\partial x_i \partial x_j} [T_{ij} H(f')] \]

\[ - \frac{\partial}{\partial x_j} [\rho u_i u_j] \hat{n}_i \delta(f) - \frac{\partial}{\partial x_j} [\rho u_i u_n \delta(f)] \]

\[ + \frac{\partial}{\partial t} [p' - c^2 \rho'] \frac{M_n}{c} \delta(f) + \frac{\partial}{\partial t} \left( (p' - c^2 \rho') \frac{M_n}{c} \delta(f) \right) \]

- Extra source terms are 2nd order in perturbations quantities
- FW–H and Kirchhoff source terms
  - equivalent in linear region \( (p' \approx c^2 \rho' \ u_i \ll 1) \)
  - NOT equivalent in nonlinear flow region
Inte\nal Formulation of FW–H equation

- New variables put FW–H equation into standard form
  \[ Q = \rho u_n - \rho' v_n; \quad L_i = P_{ij} + \rho u_i (u_n - v_n) \]
  hence
  \[ \Box^2 p'(\vec{x}, t) = \frac{\partial^2}{\partial x_i \partial x_j} [T_{ij} H(f)] - \frac{\partial}{\partial x_i} [L_t \delta(f)] + \frac{\partial}{\partial t} [Q \delta(f)] \]

- Integral representation of solution (formulation 1A)
  \[ 4\pi p'(\vec{x}, t) = \int_{f=0} \left[ \frac{\dot{Q} + \dot{L}_r / c}{r(1 - M_r)^2} \right]_{\text{ret}} dS + \int_{f=0} \left[ \frac{L_r - L_M}{r^2 (1 - M_r)^2} \right]_{\text{ret}} dS \]
  \[ + \int_{f=0} \left[ \frac{(Q + L_r / c)(r\dot{M}_r + c(M_r - M^2))}{r^2 (1 - M_r)^3} \right]_{\text{ret}} dS \]
Kirchhoff Formulation for Moving surfaces

Kirchhoff integral formulation

\[ 4\pi p'(\tilde{x}, t) = \int_{f=0}^1 \left[ \frac{E_1}{r(1 - M_r)} \right]_{ret} dS + \int_{f=0}^1 \left[ \frac{p'E_2}{r^2(1 - M_r)} \right]_{ret} dS \]

where

\[ E_1 = \left( M_n^2 - 1 \right) \frac{\partial p'}{\partial n} + M_n \vec{M} \cdot \nabla \hat{r} p' - \frac{M_n}{c} \hat{p}' \]

\[ + \frac{1}{c(1 - M_r)} \left[ (\hat{n}_r - \hat{M}_r - \hat{n}_M) p' + (\cos \theta - M_n) \hat{p}' \right] + \frac{1}{c(1 - M_r)^2} \left[ \hat{M}_r (\cos \theta - M_n) p' \right] ; \]

\[ E_2 = \frac{(1 - M^2)}{(1 - M_r)^2} (\cos \theta - M_n) \]
Numerical Comparison

- Kirchhoff code (RKIR)
  - numerical implementation of Kirchhoff integration
  - code developed for helicopter rotors (Purdue/Sikorsky/NASA LaRC)

- Prototype code developed (FW–H/RKIR)
  - based on RKIR (Rotating Kirchhoff code - rotor noise prediction)
  - utilizes Farassat’s formulation 1A
  - quadrupole source neglected; could be included

- Cases for comparison
  - hovering rotor
  - rotor in forward flight
  - viscous flow over a circular cylinder
Numerical Comparison: UH-1H hovering rotor

- **UH-1H rotor**
  - 1/7th scale model
  - untwisted blade
- **Test setup (Purcell)**
  - Hover, $M_H = 0.88$
  - inplane microphone, 3.09 R from hub
- **Flow-field computation**
  - full potential flow solver used (FPRBVI)
  - 80 x 36 x 24 grid (somewhat coarse)
Numerical Comparison: Sensitivity to Surface Placement

A principal advantage of the FW–H approach is insensitivity to surface placement.

Kirchhoff

FW–H

(Note difference in pressure scales)
Identification of Noise Components

- Compare components from off FW–H/RKIR with WOPWOP+
  - UH-1H rotor in hover
  - Hover solution from TURNS (Baeder)
- Two predictions necessary with FW–H/RKIR
  - thickness and loading from surface coincident with rotor blade
  - total signal from a surface approximately 1.5 chords away from blade.
- New application of FW–H equation retains advantage of predicting noise components
Numerical Comparison: Forward Flight Case

- **Test setup** (Schmitz et al.)
  - Operational Loads Survey (OLS) 1/7 scale model rotor
  - 3 inplane microphone used for comparison

- **Operating conditions**
  - $M_{AT} = 0.84$
  - $\mu = 0.27$

- **Flow-field computation**
  - flow solver: full potential code for rotors (FPRBVI)
  - 80 x 36x 24 grid
Numerical Comparison: Forward Flight Case

- Advancing-side acoustic pressure underpredicted
- Agreement with data is good
- All three codes agree with each other

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Numerical Comparison: Circular Cylinder Flow

Problem:

- Viscous flow over a circular cylinder
- 2D, unsteady laminar CFD computation, Re = 1000.
- Acoustic calculation 3D, cylinder 40 dia long

Vorticity field from N-S computation

CFD grid 193x97

grid extends out 20 dia.
Noise Generated by Flow Over Cylinder

- Location 128 dia from cylinder, 90 deg from freestream

- FW-H predictions show small sensitivity to surface placement
- Kirchhoff predictions meaningless

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Noise Generated by Flow Over Cylinder

- Location 128 dia from cylinder, downstream

- Differences in FW-H prediction due to:
  - CFD inaccuracy
  - Increased integration error (grid size)

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Conclusions

- FW–H method of choice for aeroacoustic problems
  - conservation of mass and momentum built in
  - unified theory with thickness, loading, and quadrupole source terms
  - insensitive to integration surface placement
- FW–H approach the “better” than linear Kirchhoff because:
  - valid in linear and nonlinear flow regions
  - surface terms include quadrupole contribution enclosed
  - physical noise components can be identified with two surfaces
- The Kirchhoff approach
  - valid only in the linear flow region (not known a priori)
    - input data must satisfy the wave equation
    - wakes and potential flow field can cause major problems
  - solution can be sensitive to placement of Kirchhoff surface