QUALITY FACTOR AND MICROSLIPPING OF FATIGUE CRACKS IN THIN PLATES AT RESONANT VIBRATION

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ABSTRACT

Resonant vibrations have been stimulated in thin metal plates using a non-contacting electromagnetic driver. A sinusoidal force was applied in a swept frequency fashion and the resulting surface displacements were monitored through the use of an acoustic microphone. It has been found that the presence of a fatigue crack in the sample causes a broadening of the second resonance peak. The Q factors of the resonance curves were determined and are directly correlated with the presence of fatigue cracks in the samples. The broadening of the curves is explained in terms of a microslipping at the crack face walls which reduces the amplitude of the resonant vibration by increasing the damping of the system. A comparison is made between the resonance characteristics of fatigue damaged and notched samples, where the stiffness of the two systems is nearly constant while the interaction between crack face walls is eliminated in the latter.

INTRODUCTION

It has previously been shown that resonant modal analysis can provide a fast effective method for inspecting thin metal plates for fatigue cracks [1-4]. A non-contacting electromagnetic driver is used to stimulate resonant vibrations and the amplitude of the plate vibrations are monitored with acoustic emission or audio microphones as a function of the frequency of the applied force. When the frequency of the applied force matches that of a natural frequency of the part under test a peak will occur in the amplitude of the plate vibrations and, correspondingly, in the sensor output. The resonant modes are then detected as peaks in the amplitude of the sensor output versus the frequency of the applied force.

Previous work has concentrated on monitoring the resonant frequencies of thin metal plates. It was found that the presence of fatigue cracks lowers the resonance frequency of a structure. The lowering of the resonance frequency with crack length is illustrated by the linear eigenvalue equation

\[
([K] - \omega^2[M])\{\bar{u}\} = \{0\},
\]

where \([M]\) is the mass matrix, \([K]\) is the stiffness matrix, \{\(\bar{u}\)\} is the amplitude of the degrees of freedom, and \(\omega\) is the circular frequency. The presence of a fatigue crack reduces the stiffness matrix, \([K]\), while the mass matrix, \([M]\), is left unchanged from the unflawed state. A given eigenvalue \(\omega\) must decrease in order for the determinant \((|K| - \omega^2|M|)\) to vanish and provide a non-trivial solution to (1). Finite element models have been used to predict the resonance frequencies as a function of crack length for 1mm thick aluminum alloy plates and the results were found to be in excellent agreement with experimentally determined values.

In addition to changes in the resonance frequency caused by fatigue cracks, it has been found that the interaction of fatigue crack face walls during resonant vibration can produce a characteristic high frequency acoustic emission [1]. The acoustic emission signature was found to be due to an unsticking of the fatigue crack walls, but was difficult to isolate for fatigue cracks less than 1 cm. long. The presence of the high frequency acoustic emission does, however, verify the suspected interaction at the crack face walls.

MICROSLIPPING AT FATIGUE CRACK WALLS

Microslipping at the interface of two materials has long been studied as a damping method to reduce the amplitude of resonant vibrations [5]. Slip damping effects are introduced when loading conditions contain both normal and oscillatory tangential forces at the interface of two materials. Fig. 1 displays a geometry similar to that first studied by Mindlin.
in order to examine the effects of partial slip [5]. The energy dissipation at the interface between the bodies was found to increase as the cube of the displacement and to have an effect even at vanishingly small loads. It was also determined that above a critical vibration amplitude, macro slip, the frictional dissipation at the interface of the bodies could no longer limit the amplitude of the vibrations. This critical vibration amplitude as well as the level of damping were found to depend upon the normal force [5].

The geometry for the present experiment is illustrated in Fig. 2. Aluminum 2024 plates 1 mm thick were clamped in a support frame leaving the front edge of the plates free to vibrate. The vibrational area of the samples was fixed at 25 x 4.5 cm². The non-contact driver explained in [4] was used to apply a sinusoidal force normal to the surface of the plates in a swept frequency manner. The amplitude of the resultant plate vibrations was monitored through the use of a microphone aimed at the sample surface. In order to reduce the effects of background noises, a lock-in amplifier referenced to the frequency of the non-contact driver was used to record the output voltage of the microphone at each step of the drive frequency.

A Material Test System (MTS) load frame was used to grow fatigue cracks in several of the samples used in this study. The cracks were grown from starter notches in the center of the front edge of the sample to lengths of 3 to 20 mm. The samples were then clamped in the test apparatus depicted in Fig. 2. such that the non-contact driver was centered above the crack. The driver was configured so as to produce a force 180° out of phase on either side of the flaw. The resulting forces acting in the fatigue crack region are shown in Fig. 3. The frequency dependent force tangential to the crack faces, normal to the sample surface, is that due to the non-contact driver as explained in [4]. The force normal to the crack face walls, parallel to the sample surface, is a result of the formation of the plastic zone just ahead of the crack tip. The plastic zone forms in response to the high stresses at the crack tip which occur during the fatigue process. The material undergoes plastic deformation as a result of these stresses, and is thus elongated in the applied stress direction [6]. The result of this elongation of material in the plastic zone is a compressive normal force at the crack face walls as depicted in Fig. 3.

QUALITY FACTOR OF RESONANCE CIRCUITS

The quality or Q factor of a resonance system is a measure of the energy stored versus the...
energy dissipated in the system. When damping terms are included the equation of motion for a vibrating body (1) can be written as

\[ \frac{d^2x}{dt^2} + B \frac{dx}{dt} + Kx = F(t) \]  

(2)

where \( B \) is the damping or internal mechanical loss coefficient and \( F(t) \) is the time dependent applied force. For a sinusoidal applied force, \( F(t) = F_0 \sin(\omega t) \), the second order differential equation can be solved for the magnitude of \( x \);

\[ |x| = \frac{F_0}{B} \sqrt{\frac{1}{1 + \left(\frac{\omega M - K}{\omega^2 \omega_0^2} \right)^2}} \]  

(3)

The resonance of the system is given by

\[ \omega_0 = \sqrt{\frac{K}{M}}. \]  

(4)

Equation (3) can be written

\[ |x| = \frac{F_0}{B} \times \frac{1}{\sqrt{1 + \left(\frac{\omega M - K}{\omega_0^2} \right)^2 \left(1 - \frac{K}{\omega^2 M} \right)}} \]  

(5)

The Q factor is then defined as

\[ Q = \omega M / B \]  

(6)

Substituting \( Q_0 = (\omega_0 M) / B \), with \( \omega_0 \) given by (4) into (3) yields

\[ \frac{|x|}{|x_0|} = \left[ 1 + Q_0^2 \left( \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right) \right]^{-1/2}. \]  

(7)

where \( |x_0| \) is the maximum displacement at resonance. Equation (7) is plotted for various values of \( Q_0 \) in Fig. 4, showing the clear relationship between the sharpness of the resonant peak and the Q factor of the curves.

In the present work the effects of fatigue crack face rubbing are investigated by means of a Q factor measurement under resonant vibration. Any slipping at the interface of the crack face walls is expected to increase the damping of the system through the energy loss caused by the microslipping, as previously explained. A broadening of the resonance curves, decrease in the Q factor, is therefore expected as an indication of fatigue damage in the sample.

**EXPERIMENTAL RESULTS**

Fig. 5 displays the output voltage of the microphone at the frequency of the drive signal over a wide frequency range for an unflawed sample. The three peaks in the curve correspond to the second, third, and forth resonant modes of the sample, as verified in earlier work through finite element modeling [3].

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In the present work the Q factor of the second resonance peak was studied. It was previously found that this mode had a strong dependence on fatigue damage in the samples [2]. Data was acquired at 1 hertz intervals over the frequency range containing the second resonance peak for the sample. A 5 point smoothing routine was used to reduce the noise in the data before a chi^2 fitting routine was used to find the best fit of the data to a Lorentzian distribution in order to determine the resonance frequency. After the resonance frequency was determined the data was normalized with respect to the maximum displacement at the resonance frequency. The expression given in (7) was then fit to the smoothed, normalized, experimental data in order to determine the Q factor of the second resonance peak.

![Fig. 4. Sharpness of resonance curves as a function of the Q factor.](image1.png)

![Fig. 5. Experimental data showing resonance peaks for unflawed sample.](image2.png)
Figs. 6 and 7 show the smoothed, normalized experimental data and the Q curve that fits the data most accurately for an unflawed sample and a sample containing a 1.2 cm fatigue crack, respectively. As expected, the resonance curve of the fatigue damaged sample is much broader, with a resulting lower Q, than the undamaged sample. Samples containing fatigue cracks of 0.3 and 2.0 cm were also inspected, and the results showed a decreasing Q with crack length for all samples.

The effect of the crack face rubbing on the Q factor was also studied by inspecting two samples containing clean notches of 0.6 and 2.0 cm lengths. Although the notches were of comparable length to fatigue cracks tested in this study, the clean nature of the notch damaged did not permit rubbing at the interface of the defect walls. The Q factor for these samples did show a decreasing trend with notch length due to the reduced stiffness of the structure, although the Q’s are much greater than for samples containing fatigue cracks of comparable lengths. Fig. 8 is a graph of the Q’s for both the fatigue cracked and notched samples.

### SUMMARY

The presence of fatigue crack face interaction during vibration has been seen to reduce the quality factor of the resonance system. The results have been explained in terms of a microslipping at the crack face walls. It was also shown that reductions in the stiffness of a structure lowered the Q factor, but these effects did not outweigh that due to the increase in the damping of the system due to the presence of microslipping at the interface of a fatigue crack face wall.

### REFERENCES