Volume and Mass Estimation of Three-Phase High Power Transformers for Space Applications

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Spacecraft historically have had sub-1kW electrical requirements for GN&C, science, and communications: Galileo at 600W, and Cassini at 900W, for example. Because most missions have had the same order of magnitude power requirements, the Power Distribution Systems (PDS) use existing, space-qualified technology and are DC. As science payload and mission duration requirements increase, however, the required electrical power increases. Subsequently, this requires a change from a passive energy conversion (solar arrays and batteries) to dynamic (alternator, solar dynamic, etc.), because dynamic conversion has higher thermal and conversion efficiencies, has higher power densities, and scales more readily to higher power levels. Furthermore, increased power requirements and physical distribution lengths are best served with high-voltage, multi-phase AC to maintain distribution efficiency and minimize voltage drops. The generated AC-voltage must be stepped-up (or down) to interface with various subsystems or electrical hardware. Part of the trade-space design for AC distribution systems is volume and mass estimation of high-power transformers. The volume and mass are functions of the power rating, operating frequency, the ambient and allowable temperature rise, the types and amount of heat transfer available, the core material and shape, the required flux density in a core, the maximum current density, etc. McLyman [1] has tabulated the performance of a number of transformers cores and derived a “cookbook” methodology to determine the volume of transformers, whereas Schwarze [2] had derived an empirical method to estimate the mass of single-phase transformers. Based on the work of McLyman and Schwarze, it is the intent herein to derive an empirical solution to the volume and mass estimation of three-phase, laminated EI-core power transformers, having radiated and conducted heat transfer mechanisms available. Estimation of the mounting hardware, connectors, etc. is not included.

Nomenclature

\[ a \quad = \quad \text{core width [m]} \]
\[ k, h, d \quad = \quad \text{ratio of window width, window height, and core depth, respectively, to core leg width} \]
\[ A_c, W_a \quad = \quad \text{core cross-sectional and window areas [m}^2\text{]} \]
\[ A_p \quad = \quad \text{Area product [m}^4\text{]} \]
\[ B \quad = \quad \text{flux density [Tesla]} \]
\[ \phi(t) \quad = \quad \text{flux [Weber]} = \Phi \sin(\omega t) \]
\[ \mu_0 \quad = \quad \text{permeability of free space, } 4\pi \times 10^{-7} \text{ [T-m/A]} \]
\[ A_r \quad = \quad \text{radiating surface area [m}^2\text{]} \]
\[ F \quad = \quad \text{shielding (view) factor} \]
\[ \sigma \quad = \quad \text{Stefan-Boltzman constant, } 5.67 \times 10^{-8} \text{ [W/(m}^2\text{-K}^4\text{)]} \]
\[ \varepsilon \quad = \quad \text{emissivity of surface} \]
\[ T_{\text{max}}, T_a, T_{cp} \quad = \quad \text{transformer maximum, ambient, and cold-plate temperatures, respectively [K]} \]

I. Introduction

POWER transformers are used to change distribution voltage levels or to electrically isolate one system from another. The size and mass of the transformer is a function of many design trade-offs (e.g., core
permeability/size versus efficiency) and design points (e.g., maximum operating temperature, power rating, and frequency). This paper is not intended to be a thorough transformer design treatise, but rather it presents an empirical solution to estimate the volume and mass of a three-phase, EI power transformer, which has only radiation and conduction heat transfer available and will operate at no more than 10kHz. The method can be extended for other core geometries, material, as frequencies.

II. Transformer Equations

To estimate the volume of a transformer, the electrical, mechanical, and thermal performance for a given set of criteria must be derived.

A. Electrical Equations

The general voltage equations for a gapped, laminated EI-core, for which the dimensions are given in Figure 1, are derived for each phase. A voltage on the primary coils of each phase induces a time-varying flux, \( \phi_i(t) \), into the core. This flux links with the other windings and induces voltages in the secondary windings. Each phase winding is on an individual leg (i.e., one primary and one secondary per leg) and equally shares the window area with another phase. Phases 2 and 3 lag phase 1 by 120° and 240°, respectively. With the gaps being equal, the reluctance of each leg is equal and, therefore, the one-half of the flux induced by a primary winding flows in the other two legs. The resultant voltage equations for any winding of n-turns are given in Eq. (1).

\[
\begin{align*}
\Phi_1(t) &= n_1 \frac{d}{dt} \left\{ \frac{1}{2} \phi_1 (t - \frac{2\pi}{3}) + \phi_1 (t - \frac{4\pi}{3}) \right\} \\
\Phi_2(t) &= n_2 \frac{d}{dt} \left\{ \phi_1 (t - \frac{2\pi}{3}) - \frac{1}{2} \phi_1 (t - \frac{4\pi}{3}) \right\} \\
\Phi_3(t) &= n_3 \frac{d}{dt} \left\{ \phi_1 (t - \frac{4\pi}{3}) - \frac{1}{2} \phi_1 (t - \frac{2\pi}{3}) \right\}
\end{align*}
\]

Simplification of Eqs. (1) by use of the following identities

\[
\begin{align*}
\phi(t) &= \Phi \sin(\omega t) \\
\frac{d}{dt} [\sin(\omega t - \alpha)] &= \omega \cos(\omega t - \alpha) \\
\cos(a-b) &= \cos(a)\cos(b) + \sin(a)\sin(b)
\end{align*}
\]

results in expressions for the voltages:

\[
\begin{align*}
V_1(t) &= 1.5 n_1 \Phi \omega \cos(\omega t) \\
V_2(t) &= 1.5 n_2 \Phi \omega \cos(\omega t - \frac{2\pi}{3}) \\
V_3(t) &= 1.5 n_3 \Phi \omega \cos(\omega t - \frac{4\pi}{3})
\end{align*}
\]

From Fig. 1 it is seen that the cross-sectional area, \( A_c \), of the core equals \( da^2 \). For a given core material the maximum, non-saturating flux density, \( \hat{B} \), should not be exceeded. By definition \( \Phi = \hat{B} * A_c \), and the root-mean square (rms) value of the primary and secondary voltages, using Eq. (2), can thus be expressed as

\[
\begin{align*}
V_p &= \frac{1.5 N_c \omega \hat{B} A_c}{\sqrt{2}} \\
V_s &= \frac{1.5 N_c \omega \hat{B} A_c}{\sqrt{2}}
\end{align*}
\]
B. Fundamental parameter derivation

The fundamental dimension, a [m], of the transformer core must be found. Equation 11.10 in McLyman’s handbook relates the current density of the transformer to the area product. No distinction is made between primary or secondary current. Thus the rms-current densities are implied to be equal. \( J_{pri} = J_{sec} = J \) [A/m²]. This implies that each window area, \( W_a \), must be shared equally by the windings within it. Because the primary and secondary of two phases share the window, we have: \( W_{a,p} = W_{a,s} = \frac{W_a}{4} \) [m²].

The ideal cross-sectional area of a winding is a function of the number of turns, \( N \), and the current density allowable within the window area (all four equal). Only a portion of the window can be filled with copper because of insulation and inter-winding gaps. This reduction is denoted as the window utilization factor, \( k_u \). Thus, the ideal cross-sectional area of a single turn of a primary and a secondary winding is:

\[
A_i = \frac{k_u W_a}{4 N_p} \quad \text{(4a)}
\]

\[
A_s = \frac{k_u W_a}{4 N_s} \quad \text{(4b)}
\]

Because Eq. (4) will probably not result in an actual AWG wire areas, the transformer design must be adjusted to accommodate the next-larger, actual AWG diameter, based on the wire diameter relationship for gage sizes stated by Fink & Carroll [3].

The primary and secondary currents are fixed by the phase voltage and by the rated power of the transformer. A further constraint, which must be met, is based upon the relationship between \( J \) and the cross-sectional area of the conductor. Namely, \( I = J A_{\text{conductor}} \), which leads to:

\[
I_p = \frac{J k_u W_a}{4 N_p} [\text{A}_{\text{rms}}] \quad \text{(5a)}
\]

\[
I_s = \frac{J k_u W_a}{4 N_s} [\text{A}_{\text{rms}}] \quad \text{(5b)}
\]

The power of a three-phase transformer, operating in a balanced mode, is simply three times the power in any phase (i.e., \( S_{rms} = 3 I_{rms} V_{rms} \)). By substituting the voltage equations, Eq. (3), the current equations, Eq. (5), and McLyman’s definition for the area product (i.e., \( A_p = 1.5 W_a A_c [m^4] \)) into the power equation results in:

\[
S_p = 3 I_p V_p = 3 \frac{J k_u W_a}{4 N_p} 1.5 N_p \omega \hat{B} A_c \frac{0.75}{\sqrt{2}} = \frac{0.75}{\sqrt{2}} A_p (J \omega \hat{B} k_c) \quad [\text{VA}] \quad \text{(6a)}
\]

\[
S_s = 3 I_s V_s = 3 \frac{J k_u W_a}{4 N_s} 1.5 N_s \omega \hat{B} A_c \frac{0.75}{\sqrt{2}} = \frac{0.75}{\sqrt{2}} A_p (J \omega \hat{B} k_c) \quad [\text{VA}] \quad \text{(6b)}
\]

For an ideal, loss-less transformer, the output power equals the input power and, thus, either expression in Eq. (6) can be used to solve for \( A_p \). The resulting expression for \( A_p \) as a function of the rated power, electrical frequency, actual current density within the conductors, the core utilization factor, and the peak flux density is:

\[
A_p = \frac{4}{3} \frac{\sqrt{2} S_{\text{rated}}}{J \omega \hat{B} k_c} [m^4] \quad \text{(7)}
\]

Having found an expression for \( A_p \), the fundamental core dimension “a” can be found as follows:

\[
A_p = 1.5 W_a A_c [m^4]
\]

\[
= 1.5 (h k a^2) (d a^2)
\]

\[
A_p = 1.5 d h k a^4 [m^4]
\]

\[
\therefore a = \sqrt[3]{\frac{2 A_p}{3 d h k}} [m] \quad \text{(8)}
\]
C. Copper and core losses

The transformer losses are the sum of the copper and core losses. First, the copper losses will be determined. By definition \( R = \frac{\rho L}{A} \), where \( A \) is the cross-sectional area \([\text{m}^2]\) of a conductor, \( \rho \) is the conductor’s electrical resistivity \([\Omega\cdot\text{m}]\), and \( L \) is the total length \([\text{m}]\). Having found the cross-sectional area of the winding per Eq. (4), the total length of a winding must be found. To accomplish this, the Mean Length of a Turn, \( MLT \), of an \( n \)-turn winding (whether primary or secondary) is found according to:

\[
MLT = \frac{\text{Total length}}{\text{turns}} = \frac{l_{\text{max}} + l_{\text{min}}}{2} \quad [\text{m}]
\]  

(9)

From Fig. 2 the MLT can be approximated as follows. Assume that a 0-diameter wire is bent at 90\(^\circ\) angles, then the innermost \( l_{\text{min}} \) and outermost \( l_{\text{max}} \) winding lengths are:

\[
l_{\text{min}} = 2a (1 + d) \quad [\text{m}]
\]

\[
l_{\text{max}} = 2a \{(k + d) + (k + 1)\} = 2a (2k + d + 1) \quad [\text{m}]
\]

Substituting these lengths and Eq. (8) into Eq. (9) and simplifying results in an expression for the MLT:

\[
MLT = 2a (k + d + 1)
\]

\[
= 2(k + d + 1) \left( \frac{2A_p}{3d hk} \right) \quad [\text{m}]
\]

(10)

Figure 2. Transformer dimensions with windings.

Now the total copper loss, ignoring skin effect in this derivation, for the three phases is:

\[
P_{cu} = 3(I_p^2 R_p + I_s^2 R_s) \quad [\text{W}]
\]

\[
= 3 \left( \frac{J k W_a}{4 N_p} \right)^2 \left[ \frac{MLT}{\rho k \frac{k W_a}{4 N_p}} \right] + \left( \frac{J k W_a}{4 N_s} \right)^2 \left[ \frac{MLT N_s}{\rho k_a \frac{k W_a}{4 N_s}} \right]
\]

\[
= 1.5 J^2 W_a k_s \rho MLT
\]

\[
\therefore P_{cu} = 3J^2 k \rho \left( \frac{2h k A_p}{3d} \right) \left( \frac{2A_p}{3d} (k + d + 1) \right) \quad [\text{W}]
\]

(11)

The core loss is the sum of the hysteresis and eddy current losses, and can be approximated over the range of frequencies of interest as [4]:
The lamination thickness, \( t \) [m], is set at 90% of the skin depth, which is a function of frequency and of the relative permeability and resistivity of the material. The minimum value allowed is 0.007" (0.18 [mm]), and is found according to Eq. (14).

\[
t(f) = \sqrt{\frac{\rho}{\pi f \mu_r \mu_0}} \geq 0.18 \text{ [mm]}
\]  

(14)

The core volume is found from Fig. (2) to be:

\[
V_{\text{core}} = (h a + 2a)(3a + 2k a)d a - 2(k a)(h a)(d a) = d (3h + 4k + 6)a^3 \text{ [m}^3]\]

Substitution for the core volume into Eq. (12, 13), summing, substituting Eq. 8, and simplifying leads to the core power loss to be:

\[
P_{\text{core}} = d (3h + 4k + 6) \left( \frac{2A_p}{3dhk} \right)^\frac{3}{2} f k_h B_h^\times.6 + k_e t^2 f B_e^\times \text{ [W]}
\]

(15)

The electrical equations are complete. However, if these were used to solve the transformer volume, without considering the heat dissipation and maximum temperature limitations of a physical device, the solution would be an unrealistically small transformer. Therefore, the heat rejection must be estimated.

**D. Heat Rejection Equations**

A transformer’s size is a function of its efficiency (power loss) and the ability to reject heat to the environment, among other variables. Heat transfer methods are radiation, convection (moving fluids), and conduction (solids). For space applications convective heat transfer to the surroundings is non-existent. For the volume estimation problem, heat rejection is thus limited to radiation and conduction through the core to the cold plate. Furthermore, the surface temperature of the cold plate is assumed to be constant.

The heat transfer for non-black body surfaces is given by:

\[
P_{\text{rad}} = F A \varepsilon \sigma \left( T_{\text{max}}^4 - T_{\text{a}}^4 \right) \text{ [W]}
\]

(16)

It is the assumed that the transformer is enclosed with a shielding factor of 0.9. Transformer surfaces are assumed to not radiate heat to other surfaces of the transformer. The exposed area of the transformer is defined as the total surface area less the surface area in contact with the cold plate. From Fig. (2) the total surface area of the transformer can be found:

\[
A_{SA} = 2(ka + da)(3ka + 3a) + (2ka + 3a)a + 4da^2 + 2hak + 2ha(3ka + 3a) + 2hka(da)
\]

\[
A_{SA} = 2a^2 (3k^2 + 7k + 3dk + 5d + 6 + 4hk + 3h + h) \text{ [m}^2]\]

(17)

And the area of the core in contact with the cold plate is

\[
A_{\text{top,core}} = d a (2k a + 3a) = a^2 (2d k + 3d) \text{ [m}^2]\]

(18)

By subtracting Eq. (18) from Eq. (17), the surface area available for radiation heat transfer is

\[
A_r = a^2 (6k^2 + 14k + 4dk + 7d + 12 + 8hk + 6h + 2hd)
\]

\[
A_r = \sqrt{\frac{2Ap}{3dhk}} (6k^2 + 14k + 4dk + 7d + 12 + 8hk + 6h + 2hd) \text{ [m}^2]\]

(19)

As a result, the expression for radiated heat transfer becomes:
Because the transformer surfaces will not have convective heat transfer available (i.e., no moving air), the cold plate must transfer the heat flux, which is not radiated. Cold plates have two heat transfer modes: conduction and convection. Heat is conducted through the core, across interface, and through the cold plate to the coolant interface. Heat transfer from the cold plate material to the moving coolant is via convection. This is not a cold plate design. Therefore, it will be assumed that the cold plate is designed to maintain the cold plate surface at a constant and uniform temperature, $T_{cp}$. To further simplify the problem, it will be assumed that all of the heat flux conducted within the transformer begins at a surface area of uniform temperature (the maximum allowed). This heat flux is uniformly distributed at one end of an equivalent solid core, having no windows areas and has a height equal to the mean height of the core as shown in Fig. (3). All heat flux must pass through this volume to the cold plate. Lastly, the contact surface area is that given in Eq. (18).

The conducted heat transfer capacity, $P_{cond}$, of this equivalent core mass with a given thermal conductivity, $\tau_c$ [W/m-C], is given by:

$$P_{cond} = \frac{(T_{max} - T_{cp}) \, \tau_c \, A_{cp}}{\text{mean height}} \quad [W]$$

Substituting for the bottom surface area of the cold plate and for the mean height, the expression for the thermal energy transferred by the core to the cold plate is:

$$P_{cond} = \frac{5 \, (T_{max} - T_{cp}) \, \tau_c \, \sqrt{\frac{2A_p}{3d k h}} \, *(2 d k + 3 d)}{3h + 10} \quad [W]$$

The thermal losses must equal the heat transfer system design capacity to limit the fully loaded transformer to its maximum operating temperature. This can be written as

$$P_{core} + P_{Cu} = P_{cond} + P_{rad} \quad [W]$$

E. The system of Equations

The transformer problem consists of a system of equations (i.e., Eqs. 7, 11, 15, 20, and 22) subject to further constraints (i.e., Eq. 23, $h > 0$, $d > 0$, $k > 0$, $J \leq J_{max}$). From the solution of this system of equations all other parameters can be found. For example, “a” is found according to Eq. (8), MLT by Eq. (10), Np from Eq. (3a), etc. Also, having found $N_p$ and $N_s$, the wire gage can be determined according to Eq. (3a) and the transformer ratio, since 2 primary and 2 secondary windings share the window and have a given utilization factor. Now all of the parameters are available to estimate the mass, dimensions, and volume of the transformer copper and core as a function of it power, voltage, and temperature constraints.

III. Sample Estimation Results

A 50kVA, three-phase, 400V:400V isolating transformer is designed to operated with $1.5[T]$, $J_{max} = 3.5 \times 10^6$ [A/m²], and the laminations are a function of frequency, but limited to a minimum of 0.18 [mm]. The core has parameters of $\mu = 1000$, $\rho = 9.579 \times 10^5$ [Ω-m], and a density of 7300 [kg/m³]. The windings are copper for which $\rho = 1.724x \times 10^8$ [Ω-m] and density = 8890 [kg/m³]. The window utilization

| Table 1. 50kVA transformer volume and mass Estimates |
|---------------------------------|-------|-------|-------|
| f [Hz]                         | 50    | 1000  | 1500  |
| volume [cm³]                   | 34525 | 9947  | 8887  |
| Length [mm]                    | 416   | 468   | 458   |
| Height [mm]                    | 316   | 192   | 103   |
| Depth [mm]                     | 262   | 110   | 187   |
| mass [kg]                      | 169   | 28    | 25    |
is assumed to be 40%. The ambient, cold plate, and maximum-operating temperatures are 303K, 293K, and 333K, respectively.

The transformer mass and dimensions—excluding mounting hardware, etc.—is shown as a function of frequency in Fig. (4). Furthermore, a comparison of the estimates for the 50kVA transformer operating at 50Hz, 1000Hz, and 1500Hz are given in Table 1.

### IV. Conclusion

An empirical solution has been derived to estimate the volume and mass of cut-core, E-I, high-power transformers to be used in space applications, and have only radiation and cold-plate cooling available and must be sized to operate within the prescribed operating limits. Mounting hardware and connectors are not estimated, and the estimation is not an optimization of the transformer design. Furthermore, using the derivation steps in this paper, it should be possible to adapt the procedure to other core shapes and materials.

### V. References


Figure 4. 50kVA transformer dimensions and volume
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Transformers; Alternating current; Electric power; Power supplies; Voltage converters; Space power

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