Intelligent, Robust Control of Deteriorated Turbofan Engines via Linear Parameter Varying Quadratic Lyapunov Function Design

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A method for accommodating engine deterioration via a scheduled Linear Parameter Varying Quadratic Lyapunov Function (LPVQLF)-Based controller is presented. The LPVQLF design methodology provides a means for developing unconditionally stable, robust control of Linear Parameter Varying (LPV) systems. The controller is scheduled on the Engine Deterioration Index, a function of estimated parameters that relate to engine health, and is computed using a multilayer feedforward neural network. Acceptable thrust response and tight control of exhaust gas temperature (EGT) is accomplished by adjusting the performance weights on these parameters for different levels of engine degradation. Nonlinear simulations demonstrate that the controller achieves specified performance objectives while being robust to engine deterioration as well as engine-to-engine variations.

Nomenclature

A, B, C, D  State Space Model System Matrices
CLP  Subscript denoting closed-loop system
δ  Deviation from steady state operating condition
Δ  Block diagram representation of model uncertainty
G  Block diagram representation of plant
FN, SM, T5  Engine net thrust, stall margin, and exhaust gas temperature respectively
K  Subscript denoting controller, block diagram representation of controller
L,M  State Space health parameter augmentation matrices
P  Lyapunov Function matrix for unforced system, scheduling parameter space
W  Lyapunov Function matrix for forced system
x, y, p, u  State variable, system output, health parameter, and system input.
v  Controller inputs
w  Exogenous system inputs
z  Unmeasured system outputs related to control system performance

I. Introduction

Turbofan engine performance varies from engine to engine due to manufacturing tolerances, aging, and deterioration caused by use. Generally the control system developed for the engine is robust enough to keep it operating within acceptable boundaries for several thousand flight cycles, even though the degradation will eventually require the engine to be overhauled as limits are reached. These limits include operability constraints such as maximum temperatures, minimum stall margins, and performance constraints such as the FAA’s rise time requirement for thrust in commercial engines. Exhaust gas temperature (EGT) increases as the engine ages and is used as an indicator of the engine’s internal condition, i.e., unmeasurable temperatures that can lead to thermomechanical fatigue (TMF) of engine components. EGT is usually at a maximum during the climb phase of flight, and if it reaches a specified threshold it signifies that the engine needs to be overhauled. Stall margin, or the distance an engine component is operating from the stall line on its performance map, tends to decrease with age, increasing the risk of an engine stall during transient operation.
Turbofan engines typically control Engine Pressure Ratio (EPR) or fan speed to generate the desired thrust, since thrust cannot be measured directly during flight. As the engine is used, manifestations of wear such as turbine blade erosion, larger clearances, etc., degrade the engine’s performance. In order to achieve the same level of thrust as in a new engine, a deteriorated engine must run hotter and/or faster. This shift from nominal operation increases with use, and eventually reaches the point where the engine’s performance cannot be maintained without compromising the life of its components. While regulated variables are maintained at their set points regardless of engine degradation, the non-regulated parameters shift from their nominal values with deterioration. Additionally, in the degraded engine, the actual thrust output, which is indirectly controlled through the regulation of other variables, may be shifted from the expected value.

In this work the objective is to maintain thrust, even in the presence of degradation, which precludes the use of EPR or fan speed as the controlled variable. Here thrust is controlled directly, given the assumption that a reliable estimate is available. Additionally, the controller minimizes exhaust gas temperature, in terms of the steady state value and in excursions during transients, while maintaining acceptable component (estimated) stall margins to overcome the operability problems associated with traditional controllers applied to aging engines. To meet these goals, a robust Linear Parameter Varying (LPV) controller is designed to accommodate a range of degradation using sets of gains scheduled on the engine degradation index (EDI). The Linear Parameter Varying Quadratic Lyapunov Function (LPVQLF) design methodology provides a means for developing unconditionally stable, robust gain scheduled control of nonlinear plants with appropriate Jacobian linearizations across an operating envelope i.e., Linear Parameter Varying (LPV) systems. The LPV controller is applied to a nonlinear simulation of a large commercial turbofan engine. This paper is organized as follows: section II describes the characteristics of engine degradation and its accommodation using LPV control, section III presents an overview of the LPV controller development process via Quadratic Lyapunov Functions, section IV provides the pertinent details concerning the design of LPV thrust/EGT controllers for a large turbofan engine, section V presents the results of nonlinear simulation testing of these controllers and section VI presents conclusions and future work.

II. Engine Degradation and LPV Control

To develop the LPV controller, models of the engine ranging from nominal to fully degraded need to be generated. Off-nominal values of specific internal engine parameters representing component efficiencies and flow capacities are often used to account for these performance variations. These adjustment parameters are called health parameters because they indicate the level of engine deterioration. The equations describing the degraded engine’s behavior are given by

\[ \dot{x}(t) = f(x(t), u(t), p) \]
\[ y(t) = g(x(t), u(t), p) \]  

where \( p \) represents the vector of health parameters. When obtaining a standard linear point model of an engine, the health parameters are treated like inputs.

\[ \delta \dot{x}(t) = A \delta x(t) + B \delta u(t) + L \delta p \]
\[ \delta y(t) = C \delta x(t) + D \delta u(t) + M \delta p \]  

Depending upon how the health parameters manifest themselves, the system dynamics may or may not change with degradation. Equation 2.2 clearly illustrates that steady state is only obtained when the \( x(t) \) and \( u(t) \) vectors shift to compensate for \( p \), and the output equation shows how nonzero values of \( p \) can produce additional steady state shifts in the output variables. These equations also imply that degradation causes shifts in the engine’s trim values, and it is these shifts that can result in unacceptable operation. Figure 2.1 presents the effect of engine deterioration on exhaust gas temperature (EGT) open loop response (deviation from nominal) to a 2% change in fuel flow. A shift in trim is clearly evident, with a less pronounced change in EGT dynamic response.
In general, the health parameters vary slowly enough with time that they are treated as constants in equation 2.2. Trending of the health parameters can be achieved by an estimation algorithm known as a tracking filter. Tracking filters have been successfully used to estimate health parameters from measured variables,\textsuperscript{5} allowing more accurate tracking of unmeasured engine variables such as thrust and stall margin. In this work it is assumed that reliable estimates of the health parameters, thrust and stall margin are available.

As mentioned previously, the LPV controller is scheduled on EDI. Here the health parameters are each assumed to change as functions of EDI, with some variation about a nominal degradation profile. For the present study, a multilayer perceptron neural network is used to map estimated engine health parameter values (assumed to be provided by a tracking filter) back to the scalar EDI to accommodate uncertainties associated with the health parameters. Desired response of engine thrust and exhaust gas temperature (EGT) is accomplished by adjusting the performance weights on these parameters for different levels of engine degradation. Model uncertainty (i.e., engine-to-engine variation) is also incorporated into the design. Figure 2.2 presents an overview of the deterioration-tolerant LPV controller.
III. Linear Parameter Varying Controllers Based on Quadratic Lyapunov Functions

Unconditional stability of parameter dependant controllers is provided via development and solution of a suitable Lyapunov equation, which incorporates the dynamic characteristics of the closed loop system. The Linear Parameter Varying (LPV) system considered here is such that all time dependence in the system is correlated to the parameter dependence on time, with each set of state space matrices used for controller design a function of the EDI, \( \rho(t) \).

Consider first an unforced, time-invariant nonlinear system of the form

\[
\dot{x}(t) = f(x(t))
\]

with origin \( x(0) \). The above is Lyapunov stable if a positive definite function, \( V(x) \), exists such that

\[
\dot{V}(x) = \frac{dV(x)}{dx} f(x)
\]

is negative semi-definite. If equation 3.2 is negative definite, the origin is asymptotically stable. For linear time invariant (LTI) systems

\[
\dot{x}(t) = Ax(t)
\]

and a candidate Lyapunov function takes the form

\[
V(x) = x^T P x
\]

with

\[
\dot{V}(x) = x^T (A^T P + PA) x
\]
If a matrix $P$ can be found such that $(A^T P + PA)$ is negative (semi) definite then the system is asymptotically (Lyapunov) stable. For forced LTI systems

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t)$$  \hspace{1cm} (3.6)$$

the Bounded Real Lemma provides a Lyapunov-like stability criterion i.e., the system is internally exponentially stable and

$$\|C(sI - A)^{-1}B\| < 1$$  \hspace{1cm} (3.7)$$

if and only if the matrix $X = X^T > 0$ provides a solution to

$$A^T X + X A + X B B^T X + C^T C < 0$$  \hspace{1cm} (3.8)$$

Consider a nonlinear system approximated by an LPV system, where $\rho(t) \in P$, $P$ being the space of all possible parameter trajectories,

$$\begin{bmatrix} \dot{x}(t) \\ z(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} A(\rho(t)) & B_w(\rho(t)) & B_x(\rho(t)) \\ C_z(\rho(t)) & D_{zw}(\rho(t)) & D_z(\rho(t)) \\ C(\rho(t)) & D_w(\rho(t)) & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \\ u(t) \end{bmatrix}$$  \hspace{1cm} (3.9)$$

with the feedback controller defined as

$$\begin{bmatrix} \dot{x}_K(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} A_K(\rho(t)) & B_K(\rho(t)) \\ C_K(\rho(t)) & D_K(\rho(t)) \end{bmatrix} \begin{bmatrix} x_K(t) \\ y(t) \end{bmatrix}$$  \hspace{1cm} (3.10)$$

The corresponding closed-loop LPV system is represented by

$$\begin{bmatrix} \dot{x}_{CLP}(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} A_{CLP}(\rho(t)) & B_{CLP}(\rho(t)) \\ C_{CLP}(\rho(t)) & D_{CLP}(\rho(t)) \end{bmatrix} \begin{bmatrix} x_{CLP}(t) \\ w(t) \end{bmatrix}$$  \hspace{1cm} (3.11)$$

where

$$x_{CLP} = \begin{bmatrix} x^T \\ x_K^T \end{bmatrix}$$  \hspace{1cm} (3.12)$$

and

$$A_{CLP}(\rho) = \begin{bmatrix} A(\rho) + B(\rho)D_K(\rho)C & B(\rho)C_K(\rho) \\ B_K(\rho)C(\rho) & A_K(\rho) \end{bmatrix}$$  \hspace{1cm} (3.13)$$

$$B_{CLP}(\rho) = \begin{bmatrix} B_w(\rho) + B(\rho)D_K(\rho)D_w(\rho) \\ B_K(\rho)D_w(\rho) \end{bmatrix}$$  \hspace{1cm} (3.14)$$

$$C_{CLP}(\rho) = \begin{bmatrix} C_z(\rho) + D_z(\rho)D_K(\rho)C(\rho) & D_z(\rho)C_K(\rho) \end{bmatrix}$$  \hspace{1cm} (3.15)$$

$$D_{CLP}(\rho) = \begin{bmatrix} D_{zw}(\rho) + D_z(\rho)D_K(\rho)D_w(\rho) \end{bmatrix}$$  \hspace{1cm} (3.16)$$

with the time dependence of $\rho$ implied. If the matrix inequality
\[
\begin{bmatrix}
A_{CLP}^T(p)W(p) + W(p)A_{CLP}(p) & W(p)B_{CLP}(p) & C_{CLP}^T(p) \\
B_{CLP}^T W(p) & -\gamma I & D_{CLP}^T(p) \\
C_{CLP}(p) & D_{CLP}(p) & -\gamma I
\end{bmatrix} < 0
\] (3.17)

and its Lyapunov solution matrix

\[W(p) > 0\] (3.18)

for all permissible trajectories of the scheduling parameter \(\rho \in \mathcal{P}\), then a family of controllers dependant on \(\rho\) (equation 3.10 ) will provide globally stable, induced L\(_2\) norm performance in terms of \(\gamma^7,8\) i.e.,

\[\|\rho\|_2^2 < \gamma^2\|w\|_2^2\] (3.19)

Equation 3.17 is nonlinear and thus, may not be convex. However with \(W\) independent of \(\rho\), equation 3.17 provides a Linear Matrix Inequality (LMI) allowing for numerical solution via state-of-the-art semidefinite programming (SDP) techniques.\(^7,8\) For an LTI system, this reduces to determining the \(\gamma\)-bounded \(H_\infty\) norm. The controller formulas (equation 3.10) are explicit functions of feasible solutions to additional LMI-based constraints.\(^7,8\) For the present study, the parameter space \(\mathcal{P}\) (i.e., the range of EDI under consideration) is discretized with the solution of equation 3.17 constrained to be a single Lyapunov matrix solution for all of the corresponding plants.

### IV. Controller Development

LPVQLF control seeks to provide a globally stable closed-loop system with optimal performance described by Equation 3.19. This is achieved by accommodating uncertainty in the system via 1) the parameter dependence (deterioration dependence) of the state space system described by equation 3.9, and 2) the ability to incorporate model uncertainty at each deterioration point (i.e., engine to engine variation). The interconnection block diagram used for LPV control system design is presented in Figure 4.1. The nominal plant (i.e., the plant used for controller design) is shown as \(G\) and is the state-space model of a large turbofan engine. \(G_p\) is the perturbed plant, which includes the effects of uncertainty due to model inaccuracies (e.g., engine-to-engine variation), and \(G_a\) is the augmented plant incorporating performance weights for controller design. Plant inputs are specified by \(w\) (exogenous) and \(u\) (control). Plant outputs are specified by \(z\) (unmeasured performance measures) and \(y\) (measurements). For the large commercial turbofan engine simulation used in this work, the available actuators are fuel flow (WF), variable bleed valve position (VBV), and variable stator vane (VSV) position. An exhaust gas temperature measurement, as well as accurate thrust and stall margin estimates are assumed to be available.

Optimal performance is achieved by specifying a set of performance weights included in the augmented plant, \(G_a\), of the system along with the nominal plant \(G\). By satisfying equation 3.19 for a specified value of \(\gamma\) of the augmented plant, the induced \(L_2\) norm of the disturbance-to-performance measure transfer matrix is maintained below \(\gamma\) for all plants specified by \(\rho \in \mathcal{P}\). Robustness to plant uncertainty (i.e., in-band model error or neglected dynamics) and minimization of input control energy are also addressed by including their effects or allowed limits via suitably designed frequency-dependant weights (e.g., \(W_{FN}\) and \(W_U\)). The goals of the controller are to provide acceptable thrust response while minimizing the net change in exhaust gas temperature in the presence of engine deterioration and model uncertainty. This is analogous to the so-called mixed sensitivity minimization problem solved via \(H_\infty\)-optimal control.\(^9\)

Shown in Figure 4.1 as an exogenous input to the plant, \(G\), are norm-bounded perturbations in the plant model i.e., uncertainty. Since a detailed uncertainty representation may add significantly to the complexity of the controller (i.e., controller order), an unstructured diagonal input uncertainty model is used with the upper bound set as the frequency dependent weight \(W_{in}(s)\). A simplified transfer function is assigned to envelop the frequency-dependant error between the “actual” and reduced order plants. \(W_{in}(s)\) essentially represents the normalized difference between the actual (or perturbed) and the nominal (reduced order) plant i.e.,
To reduce the order of the controller, the frequency-dependant weights were set to constants, effectively representing 300% modeling error across the entire frequency range of interest. Plant interconnection and controller design was performed using the MATHWORKS Mu Analysis and Synthesis and LMI Toolboxes as part of the MATLAB software package.\textsuperscript{10,11}

The deterioration index, itself being a function of engine health parameters such as component efficiencies and flow capacities, provides the controller scheduling parameter $\rho$. The uncertainty associated with the deterioration index is primarily due to the estimation of the health parameters i.e., each deterioration index is associated with a specific set of health parameters. However, in service the likelihood of a specific set of health parameters corresponding to one of the parameter-dependant plants used in the controller design may be extremely low. Thus, a mapping from health parameters to deterioration index, capable of accommodating slight variations in health parameters from the values specifically used in the controller design, is required for calculation of the scheduling parameter $\rho$. As an initial approach, the simple feedforward neural network shown in Figure 4.2 was chosen to map the estimated (uncertain) health parameter set to the scheduling parameter, EDI.
The set of sixteen health parameters, known to vary with engine deterioration and engine to engine variation, is fed through input layer weights to hidden layer neurons, which provide an output based on the degree of activation of Sigmoidal transfer functions. The neuron’s weighted outputs are fed to the single output layer neuron, whose linear activation function determines the network’s final output i.e., the EDI. A training set consists of an expected set of health parameters corresponding to a specified level of deterioration, which assumes an overall correspondence among the health parameters with respect to engine deterioration. The network was trained on health parameter sets corresponding to 0%, 10%, 20%, 30%, 40%, 50%, 60%, 70%, and 80% levels of deterioration using the MATHWORKS Neural Network Toolbox as part of the MATLAB software package. Refinement of the training sets (e.g., additional training sets and adding slight amounts of noise) may be accomplished in the future to enhance the functionality of the network.

V. Nonlinear Simulation Testing

The following presents the results of testing the LPV controller on a nonlinear simulation of a large turbofan engine. Nine interconnected plants, corresponding to 0%, 10%, 20%, 30%, 40%, 50%, 60%, 70%, and 80% levels of deterioration, were provided to an LMI-based convex optimization algorithm, which searches for a feasible solution to equation 3.17 through 3.19 – the output being the Lyapunov function matrix, equation 3.17. Additional LMI constraints incorporated into the optimization provide the controller state-space matrices $A_k, B_k, C_k,$ and $D_k$.

Figures 5.1, 5.2, and 5.3 present a comparison of a thrust-only controller, (EGT varying uncontrolled) and a controller incorporating EGT control as well and thrust control. The nonlinear engine model is 40% deteriorated (deterioration index of 0.4) at standard day cruise conditions, with the expected set of health parameters corresponding to this level of deterioration. Thrust is controlled primarily via the fuel flow, $WF$, while tight control of EGT requires manipulation of $VBV$ and $VSV$ as well. For the non-EGT controlled case (fig. 5.1), a 3% increase in thrust demand results in an 11.5 degree increase in EGT (fig. 5.1b). $VSV$ and $VBV$ are unactuated for this situation (fig. 5.1c). In contrast, for the same thrust demand input, the EGT-controlled engine (fig. 5.2) attains a increase in EGT of 7.6 degrees – a net reduction of 3.9 degrees in EGT relative to the uncontrolled case. Figure 5.2c presents the corresponding changes in $VSV$ and $VBV$ for the transient, illustrating the effectiveness of these
actuators in EGT control. The positions of these actuators are typically scheduled on engine operating conditions, thus the deviations presented would be added to the positions dictated by the schedule. Figure 5.3 presents the corresponding component stall margin deviations (SMD) for the non-EGT controlled and EGT controlled cases. As shown in the figure, the SMD show no significant change – all values are well above the stall limits for each component.

Figure 5.1.—Thrust-only control of 40% deteriorated engine (EDI of 0.4).
   a) Normalized thrust response, b) EGT deviation from steady state,
   c) VSV and VBV deviations from steady state.

Figure 5.2.—Thrust and EGT control of 40% deteriorated engine (EDI of 0.4).
   a) Normalized thrust response, b) EGT deviation from steady state,
   c) VSV and VBV deviations from steady state.
As mentioned previously, the parameter-dependant controllers are designed to accommodate plant uncertainty due to engine deterioration (i.e., controller scheduled on EDI), with considerable uncertainty incorporated into the design due to engine-to-engine variation. Other control methods e.g., $H_\infty$ control, while accommodating plant uncertainty at a specific operating point in a similar fashion, are intended for LTI systems and fail to adequately accommodate modeling errors due to significant plant nonlinearities across an operational envelope. Consider the LPV-controlled engine response shown in Figure 5.4. For the nominal engine, the corresponding LPV controller, robust to engine-to-engine variations, achieves the thrust/EGT response objectives.

Figure 5.3.—Stall Margin Deviation (SMD) response of thrust and EGT control of 40% deteriorated engine (EDI of 0.4). a) Thrust-only control, b) Thrust and EGT control.
Figure 5.5 presents a 70% degraded engine controlled by the nominal engine controller (thrust and EGT controlled). Although the thrust response is acceptable, EGT appears to be uncontrolled. The need for gain scheduling of robust controllers is clearly evident due to the nonlinear effects of engine deterioration.

Figure 5.5.—70% degraded engine response with LPV nominal engine controller
a) Thrust response, b) EGT deviation response, c) VSV and VBV response.

Figure 5.4.—Nominal (undeteriorated) engine response with LPV controller scheduling parameter (EDI) equal to 0.0.
a) Thrust response, b) EGT deviation response, c) VSV and VBV response.

Figure 5.4.—Nominal (undeteriorated) engine response with LPV controller scheduling parameter (EDI) equal to 0.0.
a) Thrust response, b) EGT deviation response, c) VSV and VBV response.
An example of controller robustness to engine-to-engine variation is shown in Figure 5.6. The base values of the health parameters correspond to 70% engine deterioration – which are taken to be fleet averages. A Monte Carlo approach, using standard deviations calculated from fleet data, is used to determine a health parameter set for each specific engine. As with the example presented in Figures 5.1 and 5.2 (the 40% deteriorated case), the net effect is a steady-state reduction in EGT. For the engines considered, EGT steady-state reductions range from 2 to 4 degrees, with the corresponding thrust response virtually identical for each engine. Compressor stall margin deviation is also shown to illustrate that component margins remain well above their critical values for the engines considered.

![Figure 5.6.—Deteriorated engine response with LPV controller showing robustness to engine-to-engine variation about an EDI of 0.7 (dashed line).](image)

a) Thrust response, b) compressor SMD response, c) EGT deviation response.

**VI. Conclusions**

An initial study, via testing with a nonlinear engine simulation, has demonstrated the ability of LPVQLF-based control to simultaneously control engine thrust and exhaust gas temperature while being robust to engine deterioration and engine-to-engine variation. By reducing EGT for a given thrust level, it might be possible to reduce TMF damage and thus extend the on-wing life of the engine. An LMI-based optimization, searching for feasible solutions to quadratic Lyapunov functions based on engine state space dynamics, provides a means for designing unconditionally stable gain scheduled controllers across a full range of deteriorated engines. As well, the quadratic Lyapunov function approach guarantees stability for instantaneous scheduling parameter variations. Traditionally gain scheduling is based on an ad hoc approach, requiring multiple pre-designed LTI controllers. These techniques, while employing controllers shown to be stable at the points of plant linearization, are not guaranteed to be stable for plants between those for which the controllers were specifically designed.

With the LPVQLF approach, system (i.e., engine-to-engine) uncertainty is handled by operating condition dependant weights while other, more significant uncertainties due to the highly nonlinear nature of the plant, are addressed by use of the gain scheduled LPV controller. The control system is designed for cruise conditions, using a feedforward neural network to reduce the set of estimated health parameters to a single parameter for gain scheduling, the engine deterioration index EDI. Future work may include controller design over a larger region of the flight envelope, possibly targeting more critical phases of operation such as the climb phase of flight. In addition, as is the case in many robust controller designs, the models used to synthesize the controllers were not pre-scaled. Future work may also investigate pre-scaling as a means to enhance the performance of the LPV controller.
The LPV controller is shown to be stable for all scheduling parameter trajectories, but the trajectories are still based on a nominal degradation profile. Although the controller is robust to wide health parameter variation around the nominal trajectory, it does not take into account large off-nominal deviations of individual health parameters. Likewise, the health-parameter-to-EDI mapping employed (i.e., a feedforward neural network) was developed using the nominal degradation profile. Values of EDI produced by the neural network as a result of off-nominal health parameter input will be invalid, potentially leading to the scheduling of an inappropriate controller. Thus future work should look at the issue robustness of the EDI mapping to off-nominal health parameter variation, and its relationship to the scheduled controller to understand the limitations imposed by the EDI on the effectiveness of the LPV controller implementation.

Due to the use of quadratic Lyapunov functions and the corresponding guaranteed stability for instantaneous scheduling parameter variations, combining the technique described with a method for assessing engine condition e.g., use of finite state machines or discrete event systems\textsuperscript{13} to provide a switching signal, may prove to be extremely powerful for implementing supervisory robust control of engines that incur damage – especially for those damage scenarios that may not be fully (or correctly) hypothesized.

VII. References


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### Abstract
A method for accommodating engine deterioration via a scheduled Linear Parameter Varying Quadratic Lyapunov Function (LPVQLF)-Based controller is presented. The LPVQLF design methodology provides a means for developing unconditionally stable, robust control of Linear Parameter Varying (LPV) systems. The controller is scheduled on the Engine Deterioration Index, a function of estimated parameters that relate to engine health, and is computed using a multilayer feedforward neural network. Acceptable thrust response and tight control of exhaust gas temperature (EGT) is accomplished by adjusting the performance weights on these parameters for different levels of engine degradation. Nonlinear simulations demonstrate that the controller achieves specified performance objectives while being robust to engine deterioration as well as engine-to-engine variations.