CAA For
Jet Noise
Physics

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Dr. Mankbadi summarized recent CAA results. Examples of the effect of various boundary condition schemes on the computed acoustic field, for a point source in a uniform flow, were shown. Solutions showing the impact of inflow excitations on the result were also shown. Results from a large eddy simulation, using a fourth-order MacCormack scheme with a Smagorinsky sub-grid turbulence model, were shown for a Mach 2.1 unheated jet. The results showed that the results were free from spurious modes. Results were shown for a Mach 1.4 jet using LES in the near field and the Kirchhoff method for the far field. Predicted flow field characteristics were shown to be in good agreement with data and predicted far field directivities were shown to be in qualitative agree with experimental measurements.

Dr. Mankbadi also presented results using linearized Euler equations. Comparison of predicted directivities agreed well with measurements for supersonic jet. Agreement was not as good for jet with Mach numbers less than 0.9. Results from very large eddy simulation were also presented. Dr. Mankbadi concluded his presentation with observations that CAA can provide:

- Prediction of sound propagation.
- Physics of the very large flow structures.
- Correlation for other approaches, such as MGB.
- Numerical experiments with various ideas for identification and control of the noise sources.

There are still many challenges:

- LES is CPU intensive
- Shock-acoustic wave interaction is difficult
- Coupling to the engine requires curvilinear meshes, grid generations, and a much faster code.
CAA FOR JET NOISE PHYSICS

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OUTLINE
1. Numerical Issues
2. LES/VLES for Jet Noise
Numerical Issues

• Large Computational Domain (60 D x 40 D)

• Three-Dimensional Problem

• Large Range of Length Scales
  • Acoustics: Acoustic Wavelength
  • Large Scale Structures: Jet Diameter
  • Shear-Layer Scales: Momentum Thickness
  • Mean Flow

• High-accuracy numerical scheme is needed to resolve these scales in large domain

• Dispersion and dissipation errors must be small for amplitude and phase accuracy

• For linear problems, several schemes are available.

• For mixing noise (nonlinear), few schemes are suitable: DRP, High-accuracy MacCormack, Compact, Prefactored Compact

• Boundary Conditions are still the major difficulty
Technical Highlight:

Evaluation of Boundary Conditions on Benchmark Cascade Case

- In this benchmark problem, a periodic vortical gust impinges on a flat plate cascade. The close proximity of the computational boundaries makes this problem extremely difficult. The results of several boundary conditions are shown.
EFFECT OF BOUNDARY TREATMENT ON ACOUSTIC RADIATION

POINT SOURCE IN A UNIFORM FLOW
AXISYMMETRIC COMPUTATION FOR ACOUSTIC RADIATION FROM AXISYMMETRIC LARGE SCALE STRUCTURE
Boundary Conditions

Acoustic Radiation

Thompson Inflow (hydrodynamic regime)

Centerline condition

Tam and Webb outflow

Acoustic Radiation

\[ M_j = 2.1 \]
\[ u_c/u_j = 0 \]
\[ \text{Re} = 70000 \]
\[ T_t = 294^\circ \text{k} \]
Instantaneous Pressure Distribution For M=2.1 Axisymmetric Jet

Single Frequency, $\varepsilon=0.04$

Single Frequency, $\varepsilon=0.001$

Bi-Modal Excitation, $\varepsilon=0.04$

Random Frequency, $\varepsilon=0.04$
INFLOW EXCITATIONS

• Single Frequency:

\[
\begin{bmatrix}
  u' \\
  v' \\
  p' \\
  \rho'
\end{bmatrix} = \epsilon \Re \left\{ \Phi(\alpha x - \omega t) \right\}
\]

\(\epsilon = \) input excitation level, \(\alpha = \) eigenvalue.

\(\Phi\) is the corresponding eigenfunctions

\[
\Phi(\alpha) = \begin{bmatrix}
  \hat{u}(\alpha) \\
  \hat{v}(\alpha) \\
  \hat{p}(\alpha) \\
  \hat{\rho}(\alpha)
\end{bmatrix}
\]

• Bi-Modal Excitation:

\[
\begin{bmatrix}
  u' \\
  v' \\
  p' \\
  \rho'
\end{bmatrix} = \epsilon \Re \left\{ \Phi(\alpha x - \omega t) \right\} + \epsilon \Re \left\{ \Phi_s(\alpha x - \omega_s t + \beta) \right\}
\]

where the subscript \(s\) represents subharmonic frequency.

\(\beta = \) initial phase difference between the fundamental and the subharmonic.

• Random Disturbance:

\[
\begin{bmatrix}
  u' \\
  v' \\
  p' \\
  \rho'
\end{bmatrix} = \epsilon A(t) \exp \left[ -\ln(2) \left( \frac{r - h(0)}{b(0)} \right)^2 \right]
\]

\(A(t)\) is a random function in time, created by random function generator.

\(b(0)\) and \(h(0)\) are the half-width and potential core radius of the shear layer at jet inflow.
AXISYMMETRIC SIMULATIONS

- A fourth-order scheme is appropriate
- Clean solution can be achieved via careful attention to boundary treatment
- The solution is dependent on the inflow disturbances
- The wavelike nature of the large-scale structure
LARGE-EDDY SIMULATIONS

Fourth-Order MacCormack Smagronisky Model
AXISYMMETRIC JET, unheated, fully expanded, 
M=2.1. St=0.2 & 0.4

COMPUTATIONAL DETAILS:

Uniform grid spacing in the axial & Azimuthal 
directions.

Stretched grid in the radial direction
The domain: 5<x/R<70, 0<r/R< 32
theta 0 to pi
CFL=0.5
25 points per wavelength

675 time steps per cycle, 100 time steps per 
cycle

AXIAL (196) radial (385), x=391 r=300
x=196, r=385, n=12

RESULTS:

Though viscous and damping effects are 
damping the solution is free from spurious 
modes.
M=2.1 Supersonic Jet Excited by First Helical Mode

Pressure

Axial Velocity

Vorticity
Technical Highlight

M=1.4 supersonic round jet excited by multiple frequencies at nozzle exit: Large–Eddy Simulation in the near field + Kirchhoff Method in the far field.

Enhanced mixing and the breakdown of organized structures downstream of jet are observed. The far field noise directivity shows qualitative agreement with the experimental measurement.
3D LES

- Direct, 3D predictions is expensive
- Need to split the problem into a source regime in the near field and a propagation regime and use physically based approximation
CAN LEE APPROXIMATE BOTH THE NEAR AND FAR FIELD?
Snapshot of Oscillating Pressure Field

RMS Values of the Pressure Disturbance
Frequency Spectra in the Shear Layer

\[ \text{Amplitude vs. Strouhal number} \]

- \( x/D = 2.5 \)
- \( x/D = 7.3 \)
- \( x/D = 34.8 \)

Frequency Spectra in the Far Field

\[ \text{Amplitude vs. Strouhal number} \]

\( (x/D = 10; r/D = 11.75) \)
Far Field Radial Decay
Noise levels for Axisymmetric Mode
(Symbols: current numerical calculation, Lines: Analytical solution)

Directivity of Jet Noise (R/D = 24)
Overall Comparison (St = 0.2)
Directivity of Jet Noise \( (R = 24) \)
Directivity of Jet Noise \((R = 24)\)
Figure (7) Directivity of Sound Intensity at $St=0.3$, 
((a)$M=0.4$, (b)$M=0.65$, (c)$M=0.9$)
SOUND PROPAGATION - LEE

Assuming the mean flow is assumed to be given, Linear Euler Equations (LEE) can be used for prediction of sound propagation and for approximate prediction of the sound source.

PHYSICS

- Errors may amplify more in the linear simulations!
- Need to make sure that no spurious modes are generated
- The solution should decay as $1/r$ in the far field
- Excellent agreement with Tam & Burton asymptotic analysis for instability waves
- Reasonable agreement with Trout & McLaughlin $M=2.1$ controlled experiment
- LEE can provide reasonable accurate approximate prediction for the large scale structure as low as $M=0.9$
OUTLINE

- Computational domain is split into two regimes:

  - Non-linear source generation regime---Large-Scale Simulation.

  - Linear acoustic wave propagation regime---Surface-Integral Formulation (SIF).
    Linearized Euler.
    Kirchhoff’s Method.
    Lighthill’s Theory.
Surface-Integral Formulation (SIF)

Consider a cylinder of radius \( a \), and length \( L \) enclosing the jet sound sources.

The mean flow outside this cylinder is stagnant and the disturbances are taken to be purely acoustic in nature, described by the simple wave equation

\[
\nabla^2 p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0
\]

The Fourier transform with respect to \( t \) is defined as

\[
p(\omega) = \int_{-\infty}^{\infty} p(t) e^{-i\omega t} dt
\]

Substituting in the wave equation, we obtain

\[
\nabla^2 p + k^2 p = 0 \quad k = \frac{\omega}{c} = 2\pi S M_j \quad S = \frac{fD}{U_j}
\]

The integral solution is given by

\[
p(X_o, \omega) = -\int [G_p \frac{\partial p}{\partial n} - p \frac{\partial G_p}{\partial n}] dS
\]

where \( p(X_o, \omega) \) is the acoustic pressure at the observation point \( X_o = (x_o, r_o, \phi_o) \). \( G \) is the Green function, \( n \) is the normal to the surface \( S \) and \( p(X, \omega) \) is the pressure distribution on the surface at the point \( X = (x, r, \phi) \).
and
\[ \frac{\partial G}{\partial r} = \pi \int_{-\infty}^{\infty} e^{ik_x(x_0 - x)} \frac{H_m(qr_o)}{H_m(qa)} q \left[ J_m(qr) H_m(qa) - J_m(qa) H'_m(qr) \right] dk_x \]

For \( r=a \), we can make use of the following relation
\[ H'(u) J(u) - J'(u) H(u) = \frac{2i}{\pi u} \]
where \( u=qa \). Substituting, we obtain
\[ \frac{\partial G}{\partial r} = \frac{1}{4\pi^2} \sum_{m=0}^{\infty} \epsilon_m \cos(m(\phi - \phi_0)) \int_{-\infty}^{\infty} e^{ik_x(x_0 - x)} \frac{H_m(qr_o)}{H_m(qa)} dk_x \]

With \( G = 0 \) at the surface, the integral solution reduces to
\[ p = \int p \frac{\partial G}{\partial r} \, adx \, d\phi \]

Substituting, we obtain the acoustic field as
\[ p(x, r, \phi, \phi_0) = \frac{1}{4\pi^2} \int p(x, a, \phi) \sum_{m=0}^{\infty} \epsilon_m \cos(m(\phi - \phi_0)) \sqrt{m} \int_{-\infty}^{\infty} e^{ik_x(x_0 - x)} \frac{H_m(qr_o)}{H_m(qa)} dk_x \, dx \, d\phi \]

This is the final formula describing the relation between the acoustic far field and the pressure distribution on a cylindrical surface surrounding the jet noise sources.
Effect of grid spacing $dr$ on pressure amplitude, $L/a=30$, $dx=0.5$. 
Directivity of jet noise at a circle of radius $R=48$. 
OTHER TECHNIQUES FOR SOUND PROPAGATION

- Lighthill Sound source may be noncompact
- Kirchoff solution can be used to calculate the Fairfield sound given the pressure distribution over a cylindrical surface enclosing the source
- An Analytical solution in terms of Hankel function can be used for extending the near field solution to the Fairfield-no pressure derivative term
THE SOURCE REGIME- VLES

IMPROVED NUMERICAL SCHEME
USE A COURSE MESH TO RESOLVE ONLY THE VLS
MODEL THE UNRESOVED SCALES USING A HIGH-ORDER
TURBULENCE MODEL (K-epsilon)
Initial Comparison of Very Large Eddy Simulation and Large Eddy Simulation

Components of Jet Exhaust Flow and Noise

(1) Unsteady exhaust plume generates large and small turbulent eddies.

(2) Noise radiates to Far Field in a preferred direction from Large Eddies. (VLES Code)

(3) Noise radiates to Far Field in all directions from Small Eddies. (KMGB Code)

(4) Two-Equation turbulence model accounts for the effect that the unresolved Small Eddies have on the jet exhaust flow. (VLES Code)

Mach Number (LES on VLES Grid)  Mach Number (VLES)
Initial Comparison of Very Large Eddy Simulation and Large Eddy Simulation (2)

Instantaneous Pressure
(LES on VLES Grid)

Instantaneous Pressure
(VLES)
Very Large Eddy Simulation Method for Fast Prediction of Jet Exhaust Noise

Centerline Axial Velocity

Lip Line Axial Velocity

Axial Velocity Profile at $x/R = 2.29$

Axial Velocity Profile at $x/R = 8.32$

Axial Velocity Profile at $x/R = 16.45$
Snapshot of Very Large Eddy Simulation Pressure Field

Instantaneous Pressure (VLES)
**Technical Highlight**

3–D, Large–Eddy Simulation of a M=1.4 supersonic round jet excited by multiple frequencies at nozzle exit

Enhanced mixing and the breakdown of organized structures downstream of jet are observed. This near–field solution will be used to obtain far–field noise through the use of Kirchhoff–type methods.
Technical Highlight

3-D, Large-Eddy Simulation of a M=1.4 supersonic round jet excited by multiple frequencies at nozzle exit

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NUMERICAL ISSUES

- Extensive Computational Domain 60D x 40D
- Helical modes- 3D
- Various length scales:
  - Acoustic: Acoustic Wavelength
  - large-scale: diameter of the jet
  - Shear layer scales: momentum thickness
  - Mean flow
- High-order scheme is needed to cover these scales in this large domain
- Amplitude and phase accuracy (Dispersive and dissipation errors need to be minimized)
- Several Schemes are available for linear problems-Linear sound propagation can be handled.
- A suitable-scheme for acoustics-shock wave interaction is still being sought
- Boundary Conditions still represent the major difficulty
R. Mankbadi References


Dr. Jonathan Freund of UCLA presented results from DNS calculations. Flow field properties and far field noise were predicted. Comparisons made with the data of Stromberg for a low Reynolds number flow showed good agreement.

Dr. Ray Hixon of NASA Glenn presented results from the application of a high order CAA code which used body-fitted curvilinear grids to compute nonlinear flows about complex geometries. Results for test cases from the 3rd CAA workshop were presented. Good results were obtained.

Dr. Anastasios Lyrintzis from Purdue University presented an overview on Integral Methods for Jet Aeroacoustics. Methods that can be used for this type of analysis include: Lighthill’s acoustic analogy, which requires a volume integral of the source terms; Kirchhoff’s method, which requires a surface integral with a good flow solver to get a good solution on that surface; and the porous Ffowcs Williams - Hawkings equation. Dr. Lyrintzis recommended that work continue to improve refraction corrections and high frequency predictions.

Dr. K. Viswanathan of Boeing presented an assessment of existing jet noise prediction methods. In particular, the MGBK method and the method of Tam were evaluated. A total of seven test cases, covering a range of jet Mach numbers and temperatures were used to evaluate these methods. Predicted spectra were compared with measured spectra for selected angles for each of the test cases. The same CFD solution (based on work of Thies and Tam) for the mean flow and turbulent energy was used for both noise prediction procedures. The method of Tam appeared to predict overall levels better than the MGBK method. Also, the MGBK method tended to under-predict the high frequency portion of the spectra while Tam’s method predicted the spectral shape very well at angles away from the jet axis. Tam asserts that large scales are responsible for the noise near the axis and thus would not expect his method to predict the spectra shape at those angles. The MGBK method attempts to predict the spectra at all angles. In discussions after the presentation it was suggested that the better agreement of Tam’s method with overall levels may be due to the fact that this method has been “calibrated” using aerodynamic input generated using the same method used in this study, whereas the MGBK code was “calibrated” using aerodynamic data from a different code. Though the choice of the CFD solution would play a role, as pointed out by Dr. Morris, there could be more fundamental issues with the acoustic analogy approach that would cause the observed discrepancy.

Dr. Philip Morris of Penn State University presented information comparing the source terms in the Tam and MGBK methods. Dr. Morris’ main conclusion was that the difference in predicted spectral shape between the two methods may be due to the frame of reference used to model the source auto-correlation function and the choice of model for the turbulent statistics.