An Integrated Centroid Finding and Particle Overlap Decomposition Algorithm for Stereo Imaging Velocimetry

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Abstract

An integrated algorithm for decomposing overlapping particle images (multi-particle objects) along with determining each object’s constituent particle centroid(s) has been developed using image analysis techniques. The centroid finding algorithm uses a modified eight-direction search method for finding the perimeter of any enclosed object. The centroid is calculated using the intensity-weighted center of mass of the object. The overlap decomposition algorithm further analyzes the object data and breaks it down into its constituent particle centroid(s). This is accomplished with an artificial neural network, feature based technique and provides an efficient way of decomposing overlapping particles. Combining the centroid finding and overlap decomposition routines into a single algorithm allows us to accurately predict the error associated with finding the centroid(s) of particles in our experiments. This algorithm has been tested using real, simulated, and synthetic data and the results are presented and discussed.

1. Introduction

Stereo Imaging Velocimetry (SIV) is a technique used to determine the 3–D velocity vectors of seed particles entrained in a flow field. SIV is achieved by combining the information from two or more cameras positioned at different viewing angles (fig. 1). SIV is most frequently used to analyze moving objects in a defined volume. There are many components of a SIV system and one of the most difficult problems to solve is the overlapping particle problem. One large source of error in solving the problem of overlapping particles is centroid error or the error associated with accurately identifying each particle’s center of mass. Contributing to this error are data acquisition hardware, spatial quantization, and gray-level quantization. Overlapping particles are a function of the degree to which the flow is seeded (data density). They cause inaccurate centroid locations if multi-particle objects are not properly identified as consisting of more than one particle. This improper identification not only loses particles, but the object’s centroid is not accurate for any of its constituent particles. Hence, any miscalculations of centroid locations will be a direct miscalculation of two dimensional velocities at the least, and may lead to incorrect matches when performing stereo matching, which will lead to large errors when 3D analysis is performed.

There are a number of approaches to finding centroids of particles in stereo imaging applications. Adamzak and Ramai (ref. 1) use a symmetrical approximation routine to locate centroids for tracking particles over time. Wernet (ref. 2) presents an intensity-weighted center of mass technique for particles in particle imaging velocimetry. Miller, Meyer, and Bethea (ref. 3) discuss using an intensity weighted centroid approach for stereo imaging velocimetry experiments. Tian and Huhns (ref. 4) use a technique for finding centroids with subpixel accuracy. The centroid of a particle is analogous to the center of mass of a solid body. We use the centroid of particles as a consistent means to label the location of a multi-pixel particle as a single (x, y) coordinate pair. This paper deals with the boundary detection and centroid calculations for objects consisting of multiple particles (i.e., particles that overlap due to depth of field...
locations). These particles are classified as singles (no-overlap), doubles (two overlapping particles), triples (three or more overlapping particles), etc. Since the data density in our experiments is low (5 to 10 percent of field of view taken up by particles), we only consider up to three overlapping particles.

Using seed particles having a diameter of approximately 168 µm, the particle images in our experiments are three to five pixels in diameter. Objects less than this size are considered to be noise. Additionally, the particles are small enough that quantization errors in imaging their edges can have a significant effect: as a result, the locations of their centroids can be determined accurately only by using the intensity weighted information, where digitization effects are quantified. Because the sizes of our particle images fall into this critical region, we are concerned with both an accurate centroid finding and overlap decomposition routine.

Overlapping of particles resulting in multi-particle objects occurs when particles that are close to the camera partially obscure particles further away. Particle overlaps must be resolved for two reasons: 1) to preserve the total number of particles; 2) unless the object can be properly decomposed into its constituents, the centroid error of the object can be as large as the particle radius (ref. 5). Induced errors of this magnitude would effectively negate the accuracy of any stand-alone centroid finding algorithm. Thus, in SIV image processing, it is necessary to integrate centroid finding and particle overlap decomposition. This generally occurs in a logical, sequential manner. One such published technique is template matching. A single, average-sized particle template is convolved with suspected overlaps and the correlation peaks represent the positions of particles (ref. 6). This method requires a great deal of computational overhead for high seeding densities. Additionally, the benefits of this method are limited due to inherent variations of particle sizes. We have developed an alternative approach which is feature based (ref. 7), integrated (not sequential), and has proven to be more tolerant of imaged particle size variations as well as being less computationally intensive. By expanding upon existing centroid techniques as well as developing a particle overlap decomposition routine from scratch, an integrated algorithm has been developed and is being used in SIV experiments at the NASA Glenn Research Center.

This paper concentrates on an algorithm that accurately traces the edges of a particular multi-pixel object and identifies its centroid(s). This data will be used in later stages of the SIV image processing software in order to predict the trajectory of particles in the flow field as a function of time. Our integrated algorithm is novel in that it combines a modified eight-direction search method, an intensity weighted centroid calculation technique, and an overlap decomposition routine into one efficient algorithm.

2. Intensity-Weighted Centroid Processing

The first problem in determining the centroid of a particle is accurately finding the edges of each particle (refs. 8,9). Our edge finding algorithm is based on using two attributes of the seed particle image, its gray-level and its location in the image field. It is based on a modification of the T algorithm of Gonzalez and Wintz (ref. 10) that uses the left-most-looking rule (LML) to find the edges of an object. Using the LML, one always looks first at the element to the left relative to the direction that one is presently going. As the seeding density increases, however, the chances of having all particles perfectly eight-connected are slim at best. In this case, a modification of the LML rule is implemented. Instead of looking left at a 90° angle in reference to the current direction that one is going, one looks backward at a 45° angle in reference to the present direction. This helps when two or more particles are attached by a single pixel and are thus, not perfectly 8-connected. For example, if an edge pixel is at location \((i, j)\), one begins the clockwise search starting at the pixel \((i - 1, j - 1)\) instead of the LML choice \((i, j - 1)\). At each step in the search, three values are tested, an initial value, IV (the location of the first edge pixel located on the boundary of a particle), the current value, CV (the location of the current edge pixel), and the previous value, PV (most recent edge pixel). Once the initial value is found, the modified eight-direction search is
initiated and the current value and the previous are used to traverse the boundary of the particle. During the traverse of the boundary, if the current value is identical to the initial value, we have successfully completed the search for the boundary of a particle. We then search the image field for the initial value pixel of the next particle image.

The centroid algorithm is based on the standard center of mass equation in discrete form.

\[ R_{cm} = \frac{\sum_{j=j_{\min}}^{j_{\max}} \sum_{i=i_{\min}}^{i_{\max}} [i \cdot f(j,i) \hat{u}_x + j \cdot f(j,i) \hat{u}_y]}{\sum_{j=j_{\min}}^{j_{\max}} \sum_{i=i_{\min}}^{i_{\max}} f(j,i)} \]  

(1)

where:

- \( i_{\min} \) = minimum column index of the particle.
- \( i_{\max} \) = maximum column index of the particle.
- \( j_{\min} \) = minimum row index of the particle.
- \( j_{\max} \) = maximum row index of the particle.

When the edges of the particle are found, the \( i_{\min}, i_{\max}, j_{\min}, \) and \( j_{\max} \) values are used to create a rectangular boundary around the particle. The algorithm uses a thresholding technique to determine a cutoff point between the background and the particles in an image. Typically, when thresholding an image, the values below the threshold are set to a specific intensity value and the values above the threshold are set to a different intensity value. This produces a binary image with two distinct intensity values. One of the intensity values represents the object under consideration and the other represents the background. The problem with this method is that potentially useful information about the particle is lost. This may affect the accuracy of locating the particle centroid. For the small particles used in our experiments (3 to 5 pixels in diameter), many of the pixels are edge pixels with intensities corresponding to a partial fill. This information is lost when using a binary thresholding routine and the location of the centroid is less accurate than if the interior intensity information is preserved. The thresholding technique we use consists of setting a minimum threshold value on the image and setting all pixels whose intensity value is above this threshold to white (intensity value 255) while leaving pixels with intensity values below the threshold unchanged. If the threshold is selected carefully, the background will be distinct from the particles while the particles’ interiors are preserved. The center of mass equation is then used to determine the center of the particle. The area of the bounding rectangle not filled by the particle does not contribute to the particle's intensity-weighted center of mass obtained by eq. (1). Once the center of mass of a particle is calculated and stored in a data file, the particle is erased in order to prevent future searches from detecting it more than once. We store and label each particle as it is found. The centroid calculations yield sub-pixel measurements that can be helpful for an error analysis of the particle velocity vectors. The algorithm for determining these velocity vectors will be presented in a future publication.

### 3. Particle Overlap Decomposition

Our feature based approach to overlap decomposition uses the major axis of the bounding ellipse and the circumference of the object to determine the number of constituent particles and their respective locations. Data necessary for extraction of these features is a by-product of our edge detection and centroid processing algorithms. Thus, no additional processing is required to decompose any object into
its constituent particle centroids. We believe this represents a major advancement over the current state-of-the-art in SIV image processing.

In general, the larger the circumference and major axis length the higher the probability that the object is composed of multiple overlapping particles. This provides the basis for our overlapping particle algorithm. We have used many synthetic and real images to obtain the data required to empirically derive equations that describe these functional relationships. In our experiments conducted in a constant volume with reasonable seeding densities, it is statistically improbable that an object will be composed of more than three overlapping particles. Thus, the probability relationships between the major axis/circumference and the number of particles in an object were determined for up to three overlapping particles. Additionally, both the major axis of the bounding ellipse and the circumference vary linearly with respect to the particle radius. This fact is significant because the probability relationships can be “learned” for one size of particle and be transposed to other experiments through functional normalization.

The circumference is the sum of all of the edge pixels on the object boundary (ref. 11). The major axis length requires an elliptical approximation of the object. We accomplish this using the bounding ellipse algorithm presented in Haralick and Shapiro. Although not presented here, the implementation of this algorithm to digital images is straightforward because only the external points on the edge of the object (left-most, top-most, bottom-most, right-most) are used in the computations. In addition to obtaining the major axis length, by-products of the algorithm are the minor axis length and the orientation of the major axis with respect to the column axis of the CCD array. These data are essential to the decomposition of the object into constituent particle centroid locations. Note once again that all of the required data are outputs of our centroid processing algorithm. The probabilities that an object is a single, double, and triple particle can be obtained following the extraction of the feature vector (major axis length, circumference). This is accomplished by entering the features into the empirically derived probability equations described below. In all equations, \( x \) is the value of the feature.

\[
P(\text{single}) = 1 - \frac{1}{x + q} \\
P(\text{triple}) = 1 - \frac{1}{x + b} \\
P(\text{double}) = 1 - P(\text{single}) - P(\text{triple})
\]

We used the following values for \( q, t, b, \) and \( c \):

<table>
<thead>
<tr>
<th>Circumference</th>
<th>Major Axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q = -8.5 )</td>
<td>( q = -4.0 )</td>
</tr>
<tr>
<td>( t = -0.16 * q )</td>
<td>( t = -0.14 * q )</td>
</tr>
<tr>
<td>( b = -16.5 )</td>
<td>( b = -7.3 )</td>
</tr>
<tr>
<td>( c = -t )</td>
<td>( c = -t )</td>
</tr>
</tbody>
</table>

The parameters \( q, t, b, \) and \( c \) are determined based on the evaluation of test images with the general form of the equation remaining constant. These values were determined for 168 \( \mu \)m diameter particles at a distance of 27.5 cm (10.8 in.) from the lens system to the middle of the imaged volume. Thus, the probability curves can be determined for any experimental setup by normalizing these functions with respect to these values. Once obtained, the maximum of the three probabilities can be used to determine
the number of overlapping particles contained in the object. The object can be decomposed into constituent particle centroids utilizing the centroid of the object, major axis length, minor axis length, the number of overlapping particles, and simple geometric relationships.

4. Results and Discussion

We have tested our algorithm using synthetic, simulated, and real data. The accuracy of the algorithm is determined by the relative error between a known shape and center compared with the shape and center calculated by the algorithm. The following definitions explain terms used throughout this section.

**Synthetic data**—(computer generated) Data is generated by the computer and analyzed using our custom algorithms. When using synthetic data, the exact locations of all particle edges and their centroids are known. This data is used as an initial test to validate the algorithm.

**Simulated data**—(test grid to camera to computer) Data is generated by printing a test grid of the synthetic data, then imaged with a camera and analyzed by computer. We used simulated data in order to generate the error associated with the image analysis set-up. This error will vary from set-up to set-up based on the quality of equipment being used.

**Real data**—(experiment to camera to computer) Data from a real experiment is imaged by the camera, then captured and analyzed by computer. We used real data to test the reliability of the algorithm in an actual experiment.

We tested the synthetic data (figs. 2 and 3—100 and 180 particles) and simulated data (figs. 4 and 5—100 and 180 particles) on a random sample of (50, 100, 150, and 180 particles) each run five times for an average centroid yield. The results are given in table 1 and table 2. The high particle identification yields dramatically improve the results of particle tracking and stereo matching in SIV experiments.

We also tested our algorithm using real image data generated in our flow visualization laboratory. We use a 3-in. by 3-in. cylindrical chamber filled with water (fig. 1) and seeded with 100, 150, 200, 250, and 300 tracer particles (fig. 6).

5. Summary and Conclusion

We have developed an integrated algorithm decomposing overlapping particles into constituent particle centroids and are using it for Stereo Imaging Velocimetry applications at the NASA Glenn Research Center. Our algorithm accurately determines the location of the centroid of the components of any multi-particle object in an image. A 512 x 512 pixel image with 512 x 480 pixels viewable is used to determine the particle boundaries and their centroids. Off-the-shelf cameras, a standard video monitor, and an image analysis subsystem were used to develop this algorithm. The only requirements for the algorithm are a way to determine the intensity values of the images and an ASCII data file of the intensity values. Our integrated algorithm is novel in that it combines a modified eight-direction search method, an intensity weighted centroid calculation technique, and an overlap decomposition routine into a single efficient algorithm.

Testing of the algorithm against known targets shows that the algorithm yields accurate and precise results. The experiments were performed in a controlled environment in order to provide an accurate basis for comparison. The methods described in this paper are general in nature and can be applied to any experiment using similar image analysis techniques.
References

### TABLE 1.—SYNTHETIC DATA (AVERAGE OVER 5 TEST RUNS)

<table>
<thead>
<tr>
<th>Number of Random particles</th>
<th>Yield with Overlap Decomposition</th>
<th>Data Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>49.8 (99.6%)</td>
<td>1.6%</td>
</tr>
<tr>
<td>100</td>
<td>99.6 (99.6%)</td>
<td>3.3%</td>
</tr>
<tr>
<td>150</td>
<td>148.8 (99.2%)</td>
<td>4.9%</td>
</tr>
<tr>
<td>180</td>
<td>178.2 (99.0%)</td>
<td>5.7%</td>
</tr>
</tbody>
</table>

### TABLE 2.—SIMULATED DATA (AVERAGE OVER 5 TEST RUNS)

<table>
<thead>
<tr>
<th>Number of Random particles</th>
<th>Yield with Overlap Decomposition</th>
<th>Data Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>49.6 (99.2%)</td>
<td>1.7%</td>
</tr>
<tr>
<td>100</td>
<td>98.4 (98.4%)</td>
<td>3.3%</td>
</tr>
<tr>
<td>150</td>
<td>148.2 (98.8%)</td>
<td>4.7%</td>
</tr>
<tr>
<td>180</td>
<td>177.6 (98.7%)</td>
<td>5.3%</td>
</tr>
</tbody>
</table>

### TABLE 3.—REAL DATA (AVERAGE OVER 5 TEST RUNS)

<table>
<thead>
<tr>
<th>Number of Random particles</th>
<th>Yield with Overlap Decomposition</th>
<th>Data Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>94.4 (94.4%)</td>
<td>1.2%</td>
</tr>
<tr>
<td>150</td>
<td>140.0 (93.3%)</td>
<td>1.9%</td>
</tr>
<tr>
<td>200</td>
<td>188.6 (94.3%)</td>
<td>3.1%</td>
</tr>
<tr>
<td>250</td>
<td>236.8 (94.6%)</td>
<td>4.0%</td>
</tr>
<tr>
<td>300</td>
<td>270.8 (90.3%)</td>
<td>4.2%</td>
</tr>
</tbody>
</table>

Figure 1.—Experimental setup.
Figure 2.—Synthetic image data (100 pts).

Figure 3.—Synthetic image data (180 pts).

Figure 4.—Simulated image data (100 pts).

Figure 5.—Simulated image data (180 pts).
Figure 6.—Five-frame sequence of approximately 200 particles (real image data).
An integrated algorithm for decomposing overlapping particle images (multi-particle objects) along with determining each object’s constituent particle centroid(s) has been developed using image analysis techniques. The centroid finding algorithm uses a modified eight-direction search method for finding the perimeter of any enclosed object. The centroid is calculated using the intensity-weighted center of mass of the object. The overlap decomposition algorithm further analyzes the object data and breaks it down into its constituent particle centroid(s). This is accomplished with an artificial neural network, feature based technique and provides an efficient way of decomposing overlapping particles. Combining the centroid finding and overlap decomposition routines into a single algorithm allows us to accurately predict the error associated with finding the centroid(s) of particles in our experiments. This algorithm has been tested using real, simulated, and synthetic data and the results are presented and discussed.

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