Frequency Bandwidth Optimization of Left-Handed Metamaterial

Christine T. Chevalier
Analex Corporation, Brook Park, Ohio

Jeffrey D. Wilson
Glenn Research Center, Cleveland, Ohio
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Christine T. Chevalier
Analex Corporation
Brook Park, Ohio 44142

Jeffrey D. Wilson
National Aeronautics and Space Administration
Glenn Research Center
Cleveland, Ohio 44135

Abstract—Recently, left-handed metamaterials (LHM’s) have been demonstrated with an effective negative index of refraction and with antiparallel group and phase velocities for microwave radiation over a narrow frequency bandwidth. In order to take advantage of these characteristics for practical applications, it will be beneficial to develop LHM’s with increased frequency bandwidth response and lower losses. In this paper a commercial three-dimensional electromagnetic simulation code is used to explore the effects of geometry parameter variations on the frequency bandwidth of an LHM at microwave frequencies. Utilizing an optimizing routine in the code, a geometry was generated with a bandwidth more than twice as large as the original geometry.

I. INTRODUCTION

In recent years the existence of left-handed metamaterials (LHM’s), as first postulated by Veselago [1], has been demonstrated [2]. LHM’s, also known as Negative Index Material (NIM) and Double Negative Metamaterial (DNM), are an engineered material with a grid of very thin wires that can behave like a plasma medium to provide an effective negative electric permittivity [3], [4] interspersed with a periodic array of split ring resonators to provide an effective negative magnetic permeability [5]. In the narrow frequency range for which both these parameters are negative, the index of refraction has a negative value and the group and phase velocities of electromagnetic radiation within the material are antiparallel. These characteristics have the potential for enabling LHM to be used in a variety of applications, such as superlenses and high pass filters [6]. However, in order to take advantage of these characteristics for practical applications, it will be beneficial to develop LHM’s with increased frequency bandwidth response and lower losses [7].

The commercial codes CST Microwave Studio (MWS) and CST Design Studio (DS) [8] were used to simulate the dispersion and scattering parameters of an LHM. Variations in the geometry were investigated to determine the effects on frequency bandwidth. Using the optimizer of MWS, the LHM geometry was optimized for maximum percent bandwidth. The accuracy of MWS and DS for simulating LHM as well as the effectiveness of the internal optimizer has been previously demonstrated [9], [10].

II. SIMULATION

A. Standard LHM Structure

Initially a geometry similar to the LHM of Shelby, et al. [11], which consists of a grid of metallic split ring resonators (SRR’s) and wires on interlocking sheets of circuit board, was modeled. A semi-infinite grid was simulated in MWS’s Eigenmode Solver by modeling a unit cell as shown in figure 1 with periodic boundary conditions in the x- and y-directions. Periodic boundaries connect two opposite boundaries so that the calculation domain is simulated to be periodically expanded in the corresponding direction. Electric boundaries were used in the z-direction to simulate a metal encasing. Electric boundaries operate like a perfect electric conductor; all tangential electric fields and normal magnetic fluxes are set to zero. A mesh setting of 15 gridlines per wavelength, which yielded a $35\times35\times23$ grid, was determined to offer the best compromise between accuracy and computation time.
Figure 1.—Unit cell of the initial left-handed metamaterial (LHM) geometry. The dimensions (mm) are as follows: \(a_x = 5\), \(a_y = 5\), \(a_z = 3.3333\), \(c = 0.25\), \(d = 0.3\), \(g_i = 0.46\), \(g_o = 0.46\), and \(w = 2.62\). Not shown are circuit board thickness, \(b = 0.25\), dielectric constant, \(\varepsilon_r = 3.4\), and metal thickness, \(t = 0.03\).

The dispersion was calculated with the phase shift in the y-direction held constant at 0° while the phase shift in the x-direction was varied between 0° and 180°. The resulting dispersion is shown in figure 2 with a left-handed pass band from 10.92 – 11.47 GHz, or a 4.9% bandwidth. The negative slope of the dispersion curve demonstrates that the group velocity is negative, a characteristic of an LHM.

It is also necessary to demonstrate transmission through the LHM. To obtain the scattering parameters (S-parameters) of the LHM, the Transient Solver of MWS and DS were used. The Transient Solver calculates the development of fields through time at discrete locations and at discrete time samples. It calculates the transmission of energy between various ports of the investigated structure. DS enables the breaking down of complex systems into smaller components, each described by an S-matrix. All the matrices are then combined in DS to calculate the complete system’s behavior. Waveguide ports were added at the x-boundaries of the LHM in MWS. The x-boundary conditions were open to accommodate the ports, the y-boundary conditions were magnetic, and the z-boundary conditions were electric. A Gaussian stimulation signal was used to excite the input port. The S-parameter results were imported into DS and an array of 20 unit cells was created. The reflection coefficients, \(S_{11}\), and the transmission coefficients, \(S_{21}\), of the array are shown in figure 3 with the transmission centered at 11.15 GHz. Also shown in figure 3 is the frequency range of the pass band from the Eigenmode computation. The dispersion and S-parameter data match within 1% at -30 dB.

B. Geometry Variations and Optimization

Variations to individual dimensions of the geometry of the LHM unit cell were made to determine their effect on the bandwidth of the structure. The most interesting are noted in table 1.

In order to optimize the LHM frequency bandwidth of the structure, it is necessary to consider all the critical dimensions synergistically. This was done by using the following dimensions as variables in a goal function for MWS’s optimizer: unit cell length in the x direction, \(a_x\); unit cell length in the y direction, \(a_y\); unit cell length in the z direction, \(a_z\); distance between the split rings, \(d\); gap in the inner ring of the SRR, \(g_i\); gap in the outer ring of the SRR, \(g_o\); and width of the SRR, \(w\). The optimizer aims to minimize the goal function, \(gfc\), which was defined as

![Dispersion of LHM Unit Cell](image1)

Figure 2.—The dispersion of the LHM unit cell shown in figure 1.

![S-parameters for LHM Array](image2)

Figure 3.—Simulated S-parameters for the original LHM array. The left-handed pass band predicted by the Eigenmode Solver is shown with dashed lines.
\[ gfc = 1 - \%B \]  

with the percentage bandwidth, \( \%B \), given by

\[ \%B = 2 \frac{(m_0 - m_{180})}{(m_0 + m_{180})} \]  

where \( m_0 \) and \( m_{180} \) are the frequencies at 0° and 180°, respectively, determined by the Eigenmode Solver.

Before the optimization run, two changes were made from the basic structure in order to decrease losses. The metal width, \( c \), was decreased from 0.25 mm to 0.1 mm to minimize the metal losses. Also, RT/duroid 5880 [12] was selected for the circuit board, which has a loss tangent a factor of 16 times smaller than the original material used, to decrease the dielectric losses. These two changes resulted in more than a factor of ten decrease to the overall losses at the center frequency of 11.2 GHz.

### Table 1.—Percent Bandwidth for Parameter Variations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Original Value</th>
<th>New Value</th>
<th>( %B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>standard</td>
<td>-</td>
<td>-</td>
<td>4.9</td>
</tr>
<tr>
<td>( a_x = a_y )</td>
<td>5</td>
<td>4</td>
<td>6.2</td>
</tr>
<tr>
<td>( a_z )</td>
<td>3.3333</td>
<td>3</td>
<td>6.6</td>
</tr>
<tr>
<td>( c )</td>
<td>0.25</td>
<td>0.1</td>
<td>5.2</td>
</tr>
<tr>
<td>( d )</td>
<td>0.3</td>
<td>0.45</td>
<td>7.6</td>
</tr>
<tr>
<td>( \varepsilon_r )</td>
<td>3.4</td>
<td>2</td>
<td>5.0</td>
</tr>
<tr>
<td>( g_o )</td>
<td>0.46</td>
<td>0</td>
<td>9.5</td>
</tr>
<tr>
<td>( g_i=g_o )</td>
<td>0.46</td>
<td>0.69</td>
<td>4.8</td>
</tr>
</tbody>
</table>

The resulting optimized geometry operated at a center frequency, \( f_c = 8.32 \) GHz. For comparison purposes we wanted to scale this geometry to operate at the same center frequency as the original design. While holding constant the circuit board thickness, metal thickness, and dielectric constant, the remaining parameters were scaled down to increase the operating frequency. The resulting geometry is shown in figure 4 and the corresponding dispersion of the optimized unit cell is shown in figure 5. The optimized unit cell has a LH pass band from 10.57 – 11.82 GHz, or a percent bandwidth of 11.2%, which is a factor of 2.3 times larger than that of the original geometry. The S-parameters of the optimized array are shown in figure 6. Also shown in the figure is the frequency range of the pass band from the Eigenmode computation. Although agreement is not as close as in the original LHM array, the -30 dB points of the S-parameter data are within 3.5% of the low frequency end of the eigenvalue computation and within 1% of the high frequency end. The S-parameter data indicates the percent bandwidth of the

![Image](image_url)

**Figure 4.** The optimized LHM geometry. The dimensions (mm) are as follows: \( a_x = 3.82, a_y = 3.82, a_z = 2.93, c = 0.1, d = 0.44, g_o = 0.43, g_i = 0.31, w = 2.6, b = 0.25, \varepsilon_r = 2.2, \) and \( t = 0.03 \).

![Image](image_url)

**Figure 5.** The dispersion of the optimized LHM unit cell shown in figure 4.

![Image](image_url)

**Figure 6.** Simulated S-parameters for the optimized LHM array. The left-handed pass band predicted by the Eigenmode Solver is shown with dashed lines.
optimized geometry is a factor of 2.1 times larger than that of the original geometry and confirms the bandwidth enhancement of the optimized geometry.

III. CONCLUSION

The bandwidth of a LHM has been maximized using a commercial 3-D electromagnetic field simulation software. The optimized geometry has a bandwidth more than twice as large as the original geometry. The technique used can be applied to optimize a variety of geometries for any user-defined goal function.

REFERENCES

[8] www.cst.de
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Christine T. Chevalier and Jeffrey D. Wilson

National Aeronautics and Space Administration
John H. Glenn Research Center at Lewis Field
Cleveland, Ohio 44135–3191

National Aeronautics and Space Administration
Washington, DC 20546–0001

Christine T. Chevalier, Analex Corporation, Brook Park, Ohio 44142; and Jeffrey D. Wilson, NASA Glenn Research Center. Responsible person, Jeffrey D. Wilson, organization code RCE, 216–433–3513.

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