THEORETICAL ANALYSIS AND BENCH TESTS OF A CONTROL-SURFACE BOOSTER EMPLOYING A VARIABLE DISPLACEMENT HYDRAULIC PUMP

By

Charles W. Mathews and Harold F. Kleckner

Langley Memorial Aeronautical Laboratory
Langley Field, Va.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS
WASHINGTON
January 28, 1947
THEORETICAL ANALYSIS AND BENCH TESTS OF A CONTROL-
SURFACE BOOSTER EMPLOYING A VARIABLE
DISPLACEMENT HYDRAULIC PUMP

By Charles W. Mathews and Harold F. Kleckner

SUMMARY

The NACA is conducting a general investigation of servomechanisms for use in powering aircraft control surfaces. This paper presents a theoretical analysis and the results of bench tests of a control-booster system which employs a variable displacement hydraulic pump. The booster is intended for use in a flight investigation to determine the effects of various booster parameters on the handling qualities of airplanes. Such a flight investigation would aid in formulating specific requirements concerning the design of control boosters in general.

Results of the theoretical analysis and the bench tests indicate that the subject booster is representative of types which show promise of satisfactory performance. The bench tests showed that the following desirable features were inherent in this booster system:

(1) No lost motion or play in any part of the system.

(2) No detectable lag between motion of the control stick and control surface.

(3) Good agreement between control displacements and stick-force variations with no hysteresis in the stick-force characteristics.

The final design configuration of this booster system showed no tendency to oscillate, overshoot, or have other undesirable transient characteristics common to boosters. The booster may be adjusted to require the pilot to exert any desired fraction of the stick force.
necessary to deflect the control surface or hold it deflected, but the response of the control surface to motion of the stick is independent of forces in the system or on the airplane.

INTRODUCTION

The NACA is conducting a general investigation of servo-mechanisms for use in powering aircraft control surfaces. This investigation has been divided into the following phases:

(1) Study of flight-test data to determine the effects of airplane-handling requirements on the design requirements of control-surface boosters.

(2) Analysis of boosters in use or in the design stage for evaluating each system.

(3) Wind tunnel or bench tests of the more promising booster systems.

(4) Flight tests of airplanes equipped with servopowered control surfaces.

As a contribution to the first phase of this investigation a study of the power required to move control surfaces was made and reported in reference 1. The present paper gives a theoretical analysis and the results of bench tests of a control-booster system which has been developed both as a contribution to the third phase of the investigation and for use in further extending the studies involved in the first phase.

SYMBOLS

\( \delta_c \)  
control-surface displacement from neutral, degrees

\( \delta_s \)  
control-stick displacement from neutral, degrees

\( \dot{\delta}_c \)  
control-surface velocity, degrees per second

\( \dot{\delta}_{cm} \)  
maximum control-surface velocity, degrees per second

\( K_1 \)  
ratio of control-surface displacement to stick displacement from neutral at any static condition.
$K_2$ ratio of the total stick force required to hold the control surface deflected to the stick force held by the pilot

$a_1$ stick displacement required to fully deflect pump control arm with hydraulic cylinder fixed, degrees

$a_2$ control-surface displacement associated with $a_1$, degrees

$a_3$ control-surface displacement required to fully deflect pump control arm with stick fixed, degrees

$t$ time, seconds

$t_m$ maximum time lag possible during steady motions of the booster system, seconds

$H_s$ variation of control-surface hinge moment with control-surface deflection, foot-pounds per degree

GENERAL REQUIREMENTS OF A CONTROL-SURFACE BOOSTER

Until sufficient flight tests of various booster systems are made, specific requirements for a control-surface booster cannot be formulated. It is believed, however, that the following qualitative requirements are necessary:

(1) Reasonably accurate positioning of the control surface with high sensitivity about any equilibrium position.

(2) Freedom from detectable lag in operation.

(3) Freedom from oscillations or excessive overshooting.

(4) Sufficient power available for rapid control movements.

(5) Provision for stick forces of the correct magnitude and with satisfactory variations with control-surface deflection, airspeed, normal acceleration, etc.

In addition to these requirements, the following features would be desirable for any booster system used in aircraft:

(1) Mechanical simplicity and high mechanical reliability.

(2) Light weight.
(3) Low power input.

(4) Ease in maintenance and installation.

The booster system described herein seems capable of meeting these qualitative requirements to a high degree.

DESCRIPTION OF THE CONTROL-SURFACE BOOSTER

The investigated control-surface booster is shown schematically in figure 1. Its basic components consist of an electric motor, a variable displacement hydraulic pump, a hydraulic cylinder, and the necessary control linkages. Although the mechanical details of a typical variable displacement pump and its servo valve are shown in figure 1 only the general operation of the system will be described at this point. The existence of a hydraulic pump whose displacement can be varied by moving a control arm with negligible force will be assumed and a detailed discussion of such a pump will be given when the setup for the bench tests is described.

In the booster system the electric motor operates continuously and drives the pump at approximately constant speed. The quantity of fluid flow from the pump may be varied from zero to maximum in direct proportion to the displacement of a pump control arm from neutral position (see fig. 1) and the direction of fluid flow from the pump is determined by the direction of displacement of the pump control arm from neutral position. Fluid from the pump actuates a hydraulic cylinder which is connected to the control surface. The overall result then is that the velocity of the control surface is approximately proportional to the displacement of the pump control arm. The pump control arm is operated through direct gearing to the stick and is also connected to the control surface through a follow up linkage. By varying the ratio of A to B as indicated in figure 1, the booster system may be adjusted so that the pilot is required to exert any desired fraction of the stick force necessary to hold the control-surface deflected.

THEORETICAL ANALYSIS OF CONTROL-SURFACE BOOSTER

Inspection of the linkages in figure 1 will show that the displacement of the pump control arm and hence the control-surface velocity is proportional to the amount the control surface is displaced from the position called for by the stick. Thus, provided
the lag in operation of the hydraulic pump is negligible, the differential equation of motion for the proposed booster system may be written

\[
\frac{\delta_c}{\dot{s}_m} = \frac{K_1 \delta_s - \delta_c}{K_1 a_1 - a_2}
\]

where the quantities in the equation are defined in the list of symbols.

Other useful parameters of the system may be noted from inspection of figure 1 and examination of the above equation. The maximum time lag possible during steady motions may be obtained from the relation

\[
t_m = \frac{K_1 a_1 - a_2}{\dot{s}_m}
\]

The boost ratio (ratio of the total stick force required to hold the control surface deflected to the stick force held by the pilot) is given by the equation

\[
K_2 = K_1 \frac{a_1}{a_2}
\]

The term \(K_1 a_1 - a_2\) which appears in several of the foregoing relations is the control-surface displacement necessary to fully deflect the pump control arm with the stick fixed and will be designated \(a_3\).

The values to be assigned to the constants other than \(a_1\) which appear in the differential equation of motion depend chiefly upon the characteristics of the airplane in which the booster is installed. The constant \(K_1\) is determined from the desired total stick displacement and the necessary total control-surface displacement. Numerous flight records of control motions under all flight conditions indicate that maximum control-surface rates do not exceed about 100° per
second for large airplanes or possibly 200° per second in other cases such as fighters on fighter aircraft. The constant $a_1$ chiefly determines the accuracy with which the control surface follows the stick motion. Small values of $a_1$ enable better following but undesirable effects of play or stretch in the system are more predominate because of the high gearing between the stick and the pump control arm. It is believed that if $a_1$ is made equal to about 5 percent of the total stick displacement (for conventional airplanes) adequate following and reasonable stick to control arm gearing will be obtained. The value of $a_2$ depends upon the boost ratio desired for any particular design and does not appreciably affect the kinematics of the system.

The linear differential equation of motion may be solved for $\delta_c$ in terms of time and $\delta_m$ (a function of time) by conventional means (reference 2) and gives

$$
\delta_c(t) = e^{a_3 t} \left( \frac{K_1 \delta_m}{a_3} \int_0^t e^{-a_3 \delta_m \delta_d t + C} \right)
$$

where $C$ is a constant of integration which may be determined from existing initial conditions.

The response of the control surface to given motions of the control stick have been computed for the conditions: $K_1 = 1$ (control surface displacement equal to that of the stick at equilibrium), $a_2 = 0$ (infinite boost ratio), $\delta_m = 100°$ per second, and $a_1 = a_3 = 2\frac{1}{2}$. Figure 2(c) shows the response of the control surface when the stick is accelerated at 10,000° per second per second to a velocity of 100° per second, continued at this velocity to a stick displacement of 90°, and then decelerated at 10,000° per second per second to zero velocity. The maximum time lag during steady motion is 0.025 second. Actually, lag in the control motions could not be sensed even with $t_m$ at least twice this value ($t_m = 0.5$ second). Figure 2(b) shows a similar response curve for the same configuration when the stick is accelerated at 10,000° per second per second to a velocity of 100° per second and immediately decelerated at 10,000° per
second per second to zero velocity. In this case 90 percent of the
desired control-surface deflection was obtained within 0.05 second
after the stick motion ceased which indicates that the booster system
should be highly sensitive to small stick displacements.

The curves of figure 2 show that for a given stick motion (stick
fixed) the control-surface displacement will not oscillate or over-
shoot its equilibrium position. The analysis, however, has assumed
a rigid system with no play in the linkages. Presence of play or
stretch in the system may result in tendencies of the booster to
oscillate or overshoot. The pump and other hydraulic components,
however, provide considerable damping which should minimize such
tendencies.

DESCRIPTION OF BOOSTER MECHANISM USED FOR BENCH TESTS

The arrangement of the booster system for bench testing is
shown in the photograph in figure 3. The electric motor and
variable displacement pump for this assembly were taken from a
Sperry type A-1A gun turret used on the upper fuselage deck of
Boeing B-17 airplanes. The hydraulic pump was manufactured by
Vickers, Inc. The operation of this hydraulic pump is shown
schematically in figure 1. The amount of hydraulic fluid displaced
by the pump during each revolution is determined by the angle at
which the cylinder block of the pump is tilted from neutral since
the stroke of the pistons in the pump is varied in proportion to
this angle of tilt. The direction of the pumping action may be
changed by changing the direction of displacement of the cylinder
block from neutral. In order to tilt and hold the cylinder block
with negligible force a small servovalve is used. This type of
valve is not considered satisfactory for direct application as a
control-surface booster because of limitations on its ability to
position accurately, operate smoothly, and be dynamically stable
under high load conditions. The valve is satisfactory in the
present application because it operates under low load conditions
and its motion directly affects the velocity of the control surface
rather than the control-surface displacement. The hydraulic motor
and gear train used in the turret installation were replaced by a
hydraulic cylinder which actuated the simulated control surface.
The arm that simulated the control surface was restrained by shock
poles which gave an approximately linear variation of control
hinge moment with deflection $H_0$. Unless otherwise stated, the
value of $H_0$ used during the tests was about 8 foot-pounds per
degree. This value would correspond to the hinge-moment variation
existing on the elevator of a medium bomber at landing speed.
provided the elevator had 50 percent aerodynamic balance. The linkages of the bench setup were basically the same as shown schematically in figure 1. Aerodynamic damping was simulated in some of the tests by attaching a viscous damper to the arm used as the control surface. Maximum stick displacements were set at 24° back and 21° forward, and the corresponding control-surface displacements were 24° up and 21° down. The stick force existing at maximum rearward stick displacement for a directly linked system (no boost) having the above values of $E_0$ and stick-control surface gearing was about 90 pounds. The maximum control-surface velocity was set at 20° per second.

**RESULTS OF BENCH TESTS OF BOOSTER SYSTEM**

Tests were made with the system setup to provide infinite boost ratio and a boost ratio of three. For both boost conditions two lengths of the pump control arm were tried (1 inch and 2 inch). The values of the parameters $a_1$, $a_2$, $a_3$, $K_1$, and $K_2$ for each configuration are presented in the following table:

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Control-arm length (in.)</th>
<th>$a_1$ (deg)</th>
<th>$a_2$ (deg)</th>
<th>$a_3$ (deg)</th>
<th>$K_1$</th>
<th>$K_2$</th>
<th>$t_m$ (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2</td>
<td>$2\frac{1}{2}$</td>
<td>$\frac{5}{6}$</td>
<td>$\frac{12}{3}$</td>
<td>1</td>
<td>3</td>
<td>0.021</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>$1\frac{1}{4}$</td>
<td>$\frac{5}{12}$</td>
<td>$\frac{5}{6}$</td>
<td>1</td>
<td>3</td>
<td>0.010</td>
</tr>
<tr>
<td>C</td>
<td>2</td>
<td>$1\frac{1}{2}$</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>1</td>
<td>$\infty$</td>
<td>0.031</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>$1\frac{1}{4}$</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>1</td>
<td>$\infty$</td>
<td>0.016</td>
</tr>
</tbody>
</table>

The letters assigned to specific configurations listed in the above table will be used to designate these configurations in the discussion of the tests. The value of the maximum time lag $t_m$ listed in the table were obtained through use of the theoretical
relation previously presented with $\Delta \omega_m = 30^\circ$ per second. It may be noted that for a given boost ratio $t_m$ is directly proportional to the length of the pump control arm.

During the tests records of control-surface position, stick position, pump control-arm position and stick force were obtained through use of standard NACA recording instruments.

Figure 4 presents records for the configurations with a boost ratio of three made during random movements of the control stick. The time lag between motion of the control stick and control surface is difficult to detect visually. Actually a small amount of time lag did exist and, in accordance with theoretical computations, this lag for configuration A (2 inch arm) was about twice that for configuration B (1 inch arm). The stick-force variations for both configurations were in phase with the control motions, and the accuracy with which the desired boost ratio was obtained was exceptionally good as may be seen by comparing the stick force at maximum rearward stick displacements on the figure with the value of 90 pounds for the system with no boost. The sharp peaks in stick force which occasionally occur at maximum stick displacements resulted from the stick hitting its stops and are not to be confused with the forces transmitted through the system.

In figure 5 two similar sets of records are presented for configuration A only, one with the usual value of $H_6$ of 8 foot-pounds per degree and one with this value reduced by one-third. It may be noted that this reduction was accurately represented by a corresponding reduction in the stick forces.

Records for the configurations with infinite boost made during random motions of the control stick are presented in figure 6. Again as would be expected from theoretical considerations the lag for configuration C (2 inch arm) was about twice that for configuration D (1 inch arm) although in either case the lag is difficult to detect visually. The stick forces show little variation resulting from motion of the stick (about 14 pounds). The slight variations present were chiefly due to inertia of the stick and linkages.

Records of control motions following release of the stick from full rearward displacement are shown in figure 7 for the configuration with boost ratio 3. Oscillations of the system resulted with the stick free for both configurations A and B. For configuration A (2 inch arm) these oscillations showed no tendency to damp with a value of $H_6$ of 8 foot-pounds per degree. A slight damping tendency can be noted for configuration B (1 inch arm) and also configuration A
with $H_{0}$ reduced by one-third. These oscillations resulted from a small lag in the operation of the servo-valve that displaces the cylinder block of the pump. This lag combined the effects of control hinge moment, inertia of the stick and linkages, and the zero-centering tendency of the pump control arm to produce the instability. Because the gearing from the pump control arm for configuration B was higher than for configuration A, the magnitude of this lag was less, which accounts for the superior damping of the configuration. The stick free oscillations did not occur for the configurations with infinite boost ratio because the stick was indifferent to displacement and had no centering tendency.

Since the displacement of the pump control arm is proportional to the velocity of the control surface it was felt that adequate damping could be applied to the stick-free oscillations by equipping the pump control arm with light centering springs. This modification is shown in the photograph in figure 8. Three spring stiffnesses were tried with values of applied stick force per unit control surface velocity of 0.03, 0.04, and 0.10 pounds per degree per second when installed on configuration A. These values were reduced by one-half for installation of the same springs on configuration B.

Records of stick releases for configuration A equipped with these springs are presented in figure 9. A large amount of damping was applied to the oscillations for all values of spring stiffness. The amount of damping increased with increasing spring stiffness until only a very slight overshoot existed when the system was equipped with the heaviest springs. Stick-force variations during random motions of the stick for configuration A with the heaviest centering springs are shown in figure 10. The stick force in phase with the control-surface velocity applied by the springs was so small that the total stick-force variations were little affected. The springs produced similar results for configuration B although they were somewhat less effective than for configuration A.

All the tests discussed previously were made with no damping on the control surface. Similar tests were made with sufficient viscous damping on the control surface to simulate the aerodynamic damping which exists in flight. Figure 11 shows the free motions of the directly linked system (no boost) without applied damping and with sufficient damping to cause the oscillation to damp to half amplitude in less than half a cycle. This time to damp to half amplitude represents a condition which usually exists at high speeds on the elevator of a fighter-type airplane as indicated by flight records. Preferably controls should deadbeat on release from a deflected position. Records of motions following release of the stick are shown in figure 12 for the configurations with a boost.
ratio of three and with damping on the control surface. Tests were made without control-arm springs and with the heaviest control-arm springs. For configuration A, without control-arm springs, the oscillation remained undamped but its amplitude was markedly reduced. Under the same conditions, considerable damping was applied to the stick-free motion for configuration B. When the control arm was equipped with the heaviest springs the stick-free motion for configuration A was deadbeat, the ideal condition for a control surface. The springs were less effective for configuration B as the damped oscillation still existed even with the heaviest springs. Because the length of the pump control arm for configuration B was one-half that for configuration A, only one-half the damping force was applied by the springs at a given control-surface velocity.

Figure 13 shows the stick-force variations during random motions of the control stick for configuration A, with the heaviest control-arm springs and with viscous damping on the control surface. As the control-surface damping was small, it had little effect on the stick-force variations.

Further tests were made on configuration A with viscous damping applied to the control stick, without control-surface damping and without control-arm springs to determine whether the springs or stick damping would more effectively prevent the stick-free oscillations. Two magnitudes of stick damping were tried, one with about the same stick-force component in phase with the stick velocity (approximately equal to the control-surface velocity) as the heaviest control-arm springs and one with about twice this amount. Figure 14 presents stick release for the above conditions. The relative effectiveness of the stick damping and the springs may be seen by comparing these records with the corresponding records in figure 9. The stick damper did apply considerable damping to the oscillations, but for both magnitudes tested, the tendency of the control to overshoot was worse than for this configuration with the heaviest control-arm springs. Changing the magnitude of the stick damping appeared to have little effect of this tendency to overshoot.

DESIGN CONSIDERATIONS

As pointed out in reference 1, short period power demands on a control booster may be very high compared to the average power input over long periods of time. Accordingly, reference 1 suggested that the size of the power unit can be materially reduced provided an energy storing accumulator is used to take care of the short period power demands. With the present booster system the maximum demand for instantaneous power from the pump may be high compared to the
power required when the pump control arm is in neutral position. As the pump and electric motor operate continuously, a low-power electric motor might be used if a small high-speed flywheel is incorporated in the system. High power demands may then be tapped from the kinetic energy stored in the flywheel which can be replaced at a slow rate by the electric motor. For example, the work required to fully deflect the ailerons on a heavy bomber flying at a speed of 400 miles per hour at 20,000 feet was computed to be 425 foot-pounds. With this energy requirement it is easily seen that the power necessary for rapid control motions is exceedingly high. However, a small flywheel represented by a 5 pound annulus 6 inches in outside diameter and 1/2 inch thick, which rotates at 7500 rpm, would be capable of deflecting the ailerons at any rate up to the maximum and suffer less than a 10 percent loss in rpm.

Particularly on large airplanes it is desirable to make the mechanical linkages from the pump control arm to the stick and to the control surface as short and rigid as possible. One method of maintaining short linkages is to mount the entire booster unit in the vicinity of the control stick and actuate control cables near the stick rather than the control surface itself. In this case, the follow-up linkage should be run from the hydraulic cylinder to the pump control arm rather than from the control surface. In this way tendencies for the booster system to oscillate or overshoot can be eliminated and the following of the control surface should be as good as in a directly linked system. This arrangement, however, has the disadvantage of requiring a heavier control system since the long cable system to the control surface must have a high load carrying capacity. Short linkages may also be obtained on large airplanes by mounting the booster unit near the control surface and using a small but very accurate electrical servolink between the stick and the pump control arm. As yet, no tests have been made of such an arrangement.

Use of infinite boost ratio and mechanically applied control feel would appear promising for very large airplanes. In this way, high friction and inertia forces could be eliminated and the stick-force variations could be adjusted to the satisfaction of the pilot, regardless of the aerodynamic loads on control surface. This arrangement would also appear promising for use on airplanes designed to operate in the transonic and supersonic speed ranges where large and unpredictable variations in control-surface hinge moment are likely to exist.
CONCLUDING REMARKS

The results of the theoretical analysis and of bench tests of the investigated booster system indicate that the booster should be satisfactory for use in powering aircraft control surfaces. A constant amplitude oscillation which existed in the basic booster system with the stick free was completely eliminated by installing light centering springs on the pump control arm. With these centering springs installed sufficient damping was applied to the stick free motions of the system to cause the control surface to deadbeat when released from a deflected position. At no time during the bench tests was lag in the system detectable to the operator, and the sensitivity of the system to small stick motions was exceptionally good. For all configurations of the basic booster system the variation of stick force with control-surface displacement was considered satisfactory.

Since actual flight conditions cannot be simulated perfectly by ground tests, it will be desirable to conduct a flight investigation of this booster system. Such a flight investigation would determine whether the basic principle of this system is satisfactory for the proposed application, and would aid in formulating specific requirements for control boosters in general.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va.

REFERENCES


Figure 1.- Schematic arrangement of control-booster system.
Figure 2.- Theoretically derived response of the control surface to given motions of the control stick for a typical design configuration of the control-booster system.
Figure 3.- Photograph of setup for bench tests of control-booster system.
Figure 4.- Records taken during bench tests of control-booster system showing time histories of surface position, stick position, and stick force during random motions of the control stick, boost ratio = 3, $H_8 = 8$ foot-pounds per degree.
Figure 5.- Records taken during bench tests of control-boosted system showing time histories of surface position, stick position, and stick force during random motions of the control stick, configuration A. boost ratio = 3.
Figure 6. Records taken during bench tests of control-booster system showing time histories of surface position, stick position, and stick force during random motions of the control stick, boost ratio = \infty, \ H_b = 8\ foot-pounds per degree.
Figure 7.- Records taken during bench tests of control-booster system showing time histories of surface position and stick position following release of the control stick from full back deflection, boost ratio = 3.
Figure 8.- Photograph of bench setup of control-booster system showing installation of centering springs on pump control arm.
Figure 9.- Records taken during bench tests of control-booster system showing the histories of surface position and stick position following release of the control stick from full back deflection, configuration A, centering-springs on pump control-arm, boost ratio = 3, $H_b = 8$ foot-pounds per degree.
Figure 10.—Records taken during bench tests of control-booster system showing time histories of surface position, stick position, and stick force during random motions of the control stick, configuration A, heaviest centering-springs on pump control-arm, boost ratio 3, $H_s = 8$ foot-pounds per degree.
Figure 11.- Records taken during bench tests of control-booster system showing free motions of control surface and control stick with and without viscous damping applied to the control surface, control stick and control surface directly linked (booster disconnected), $H_9 = 8$ foot-pounds per degree.
Figure 12.- Records taken during bench tests of control-booster system showing time histories of surface position and stick position following release of the stick from full back deflection, viscous damping applied to control surface, boost ratio 3, $H_5 = 8$ foot-pounds per degree.
Figure 13.—Records taken during bench tests of control-booster system showing time histories of surface position, stick position, and stick force during random motions of the control stick, configuration A, heaviest centering-springs on pump control-arm, viscous damping applied to control surface, boost ratio 3, $H_5 = 8$ foot-pounds per degree.
Figure 14.—Records taken during bench tests of control-booster system showing time histories of surface position and stick position following release of the control stick from full back deflection, configuration A, viscous damping applied to control stick, no centering springs on pump control arm, boost ratio 3, \( H_b = 8 \) foot-pounds per degree.