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HEAT TRANSFER OF AIRFOILS AND PLATES

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The few available test data on the heat dissipation of wholly or partly heated airfoil models are compared with the corresponding data for the flat plate as obtained by an extension of Prandtl's momentum theory, with differentiation between laminar and turbulent boundary layer and transitional region between both, the extent and appearance of which depend upon certain critical factors. The satisfactory agreement obtained justifies far-reaching conclusions in respect to other profile forms and arrangements of heated surface areas. The temperature relationship of the material quantities in its effect on the heat dissipation is discussed as far as is possible at the present state of research, and it is shown that the profile drag of heated wing surfaces can increase or decrease with the temperature increase depending upon the momentarily existent structure of the boundary layer.

INTRODUCTION

Economical high-speed flight requires aircraft of high aerodynamic quality— that is, aircraft with air frame designed for minimum flight drag and all parasite areas, especially the radiators— reduced to a minimum. In this endeavor attempts have been made in this country as well as abroad to utilize exposed outside surfaces, preferably those of the wings, for heat transfer. Such radiators are hereinafter called "skin radiators."

The heat output of the exposed outside surface of a skin radiator and its distribution across the surface are hardly amenable to prediction from the differential equations for flow and heat exchange, as these equations are not integrable in such generalization. However, the moment theory and its refinement according to Prandtl affords at least very satisfactory practical average values.

**"Wärmeübertragung von Profilen und Platten." Jahrbuch 1938 der deutschen Luftfahrtforschung, pp. II 245—II 256.**
of the heat transfer coefficients. The only stipulation for its validity is that skin friction and heat transfer begin and stop simultaneously. In the practical case that some portion of the outer surface of an airplane, whether on the fuselage, engine nacelles, or wings are designed as skin radiator, the heat effect usually begins after the friction; hence a temperature profile forms progressively in the already existent and still growing frictional boundary layer which unfortunately does not lend itself to representation in closed form, although it is mathematically defined by the equations of motion and heat exchange. The correlation between the Nusselt number $\text{Nu}$ and the Reynolds number $\text{Re}$ for all possible flight conditions must therefore be secured by measurements on models. The subsequent study deals primarily with the processes on wing radiators — that is, radiators mounted in the wings. In this connection the data on the heat dissipation of flat plates are invaluable; indeed, they may even save special measurements on airfoils (see sec. V) if no unusual claims on accuracy are involved.

II. AVAILABLE TEST DATA

The most important publications on investigations of this nature, all of which deal with wing radiators, are:

a) R.&H. Report No. 1311 (1927)
Wind-Tunnel Tests on Gloster and Supermarine Wing Radiators (Supermarine airfoil section RAF 30, chord 15 ft = 1524 mm, model scale 1:1, span = tunnel diameter, of which 34 in. = 876 mm were heated, total upper and lower surface heatable with water separately or collectively).

b) R.&H. Report No. 1163 (1928)
On the Convection of Heat from the Surface of an Airfoil in a Wind Current (model of RAF section 26, 5 in. = 152 mm chord between end plates at 3 in. = 76.2 mm spacing, with 29 span-wise platinum strips over the total surface).

c) R.&H. Report No. 1326 (1930)
Wind-Tunnel Experiments on Steam Condensing Radiators (nose steam-heated, RAF 30 section, 2 ft = 610 mm chord, 7 ft = 2134 mm span,
the radiator occupies 5 ft = 1524 mm; 25.1 percent of the arc length of the upper surface and 15.2 percent of the lower surface or 24 percent and 14 percent, respectively, of the chord (of the symmetrical) profile heatable with saturated vapor at low positive pressure).

d) R.& M. Report No. 1481 (1932)
Estimation of Wing Surface Area for Evaporative Cooling (comparison of R.& M. Reports Nos. 1311 and 1326).

In figure 1 the Nusselt number \( \text{Nu} = f(Re) \) is shown according to R.& M. Reports Nos. 1163, 1311, 1326, and 1481 for different angles of attack \( \beta \).

where

\[
\text{Nu} = \frac{c_D D}{\lambda_0} = \text{Nusselt number}
\]

\[
Re = \frac{vD}{\nu} = \text{Reynolds number}
\]

\( D \) airfoil circumference

\( v \) flying speed

\( \nu \) kinematic viscosity referred to state of air in undisturbed zone

\( \lambda_0 \) heat conductivity of air at surface portion.

\( t_0 \sim 40^\circ \) on RAF 26,

\( t_0 \sim 93^\circ \) on RAF 30;

Air temperature assumed at 20\(^\circ\).

III. CONCLUSIONS

a) The heat emissivity of the surface portions in proximity of the leading edge is greater than
at any other part of the airfoil. It naturally increases with \( \text{Re} \), but varies so much less with the angle of attack \( \beta \) as the area extends less on the upper and lower surface.

b) On the upper and lower surface the heat dissipation decreases along the wing trailing edge for given \( \text{Re} \); but in the transitional zone from laminar to turbulent boundary layer it can increase once more and then decrease, as exemplified in the test data of R.& M. No. 1163.

c) By ascending \( \text{Re} \) and \( \beta \) the heat dissipation on the upper surface increases, the function \( \text{Nu} = f(\text{Re}) \) resembles on the whole a curve at least as steep as that for the nose.

d) The heat dissipation on the lower surface of the wing scarcely varies over the practical range of positive angles of attack; it increases at angles of attack corresponding to zero or negative lift. Depending upon the absolute magnitude of \( \text{Re} \) (and other factors, see pt. IV) the curve can be as flat as that of the nose or about as steep as or steeper than that of the upper surface.

While this dependence of the heat dissipation is in general quite comprehensible, the different slopes of the curves for the heat dissipation of the surface portions cannot be summarily explained. These and other relations are discussed in part IV in the light of the enlarged momentum theory as applied to the flat plate. The comparison of these theoretical relations with the results obtained on the RAF 26 and RAF 30 sections (fig. 1) becomes particularly simple. The numerical agreement is also very satisfactory if allowance is made for the difference in the flow velocity on the upper and lower wing surface areas. The arguments can likewise be applied quite satisfactorily to heated leading edges by a minor conversion. Reliable heat dissipation data on flat plates are available (reference 1) but they do not lend themselves to strict comparison with the ideal case, because the test arrangement causes the heat dissipation to start after the skin friction. In any event they enable to support the theory and, in addition, afford some insight into the expected displacements when only parts of the wing surface area are heated. Very insufficient, however,
are the data available on the effect of the temperature increase of the heated portion or the total surface, respectively, or better, of the dimensionless surface temperature \( \frac{T_0}{T} \) (see pt. IV) on the heat dissipation and, above all, on the drag of the total area. This might be regarded as pure skin friction on the flat plate, while on the airfoil the heating itself can influence the form drag (pt. VII).

IV. HEAT DISSIPATION OF FLAT PLATES IN THEORY AND TEST

On real flat plates—that is, those of finite thickness—the boundary layer is theoretically formed somewhat differently than on ideal plates—that is, on infinitely thin plates in symmetrical flow parallel to its plane. Postulating complete freedom from friction the ideal plate does not affect the flow at all, while the real plate outwardly displaces the individual streamlines, depending upon the shape of the leading edge, and so causes increases of speed relative to the undisturbed flow.

Taking into account the skin friction, the boundary layer on the ideal plate is produced by the decelerating action of the surface particles on the flowing fluid which then continues through it. Hence in two-dimensional flow the velocity profile of a section at right angles to the flow direction at any distance from the plate leading edge must (as on a pipe) be unequivocally defined by the Reynolds number (reference 2) computed with this depth \( x \) and the undisturbed velocity \( v \) and the kinematic viscosity \( \nu \):

\[
\text{Re}_x = \frac{vx}{\nu}
\]

(1)

regardless of whether and how far the plate extends downstream. On the real plate this skin friction is superposed by the displacement process associated with the shape of the body (especially its thickness, over-all length, roundings) which very likely can also become effective upstream. The combined action of this profile effect and the friction favors the creation of the so-called form resistances which cannot occur on the ideal plate.
The velocity field on the real plate must accordingly have a somewhat different aspect, which is, that the discrepancies from the theory are especially great at the beginning of the plate. The effect of the leading edge on the heat transfer is obviously similar. The foregoing logically applies also to airfoils with and without angle of setting, where an even somewhat poorer agreement is to be expected.

On ideal plates and on real thin plates* (with sharpened edge and interference-free inflow) a laminar flow forms first in the boundary layer. Turbulence does not come into being until after traversing a certain entrance length \( x_{cr} \) which can be computed from the so-called critical Reynolds number.

\[
Rex_{cr} = \frac{vx_{cr}}{v}
\]  

(2)

For \( x > x_{cr} \) the laminar boundary layer becomes fully turbulent in a transitional region; \( Rex_{cr} \) according to Ten Bosch (reference 2, pp. 139–140), has been measured up to \( 5 \times 10^5 \) for very small initial disturbances, but may drop to \( 10^5 \) in the presence of major disturbances, the transitional region can extend from \( Rex = 10^5 \) to \( 2 \times 10^7 \) for sharpened edge and steadied flow. With the one-sided surface \( C \) of the plate, the drag coefficient \( C_f \) and the air density \( \rho \) the drag of the ideal plate \( W \) for each side is

\[
W = C_f 0 \frac{V^2 \rho}{2}
\]  

(3)

The amount of heat dissipation from one side of the plate at increase of temperature \( \Delta T \)

\[
Q = \alpha m K \Delta T
\]  

(4)

is, according to simple momentum theory, and as employed by Von Kármán and Latzko (references 3 and 4), to the

*Thus Ten Bosch (reference 2) indicates on page 148 that heat exchange surface of streamline form should have less than \( 33 \times 100/600 = 5.5 \) percent profile thickness, since only from this slenderness ratio on the friction forms the principal portion of the total drag. (See also pp.139–140.)
plate drag as the heat content of some gas mass in the nucleus to its quantity of motion:

\[
\frac{Q}{W} = \frac{(\rho v)(\varepsilon c p m \theta)}{(\rho v) v} = \frac{\varepsilon c p m \theta}{v} \quad (5)
\]

Equations (3), (4), and (5) afford

\[
\alpha_{m K} = \frac{Q}{W} \frac{W}{c_p} = \frac{1}{2} C_f c_p m v \quad (6)
\]

or the mean Nusselt number with the plate chord \( x \) introduced in equation (1):

\[
N_{ux K} = \frac{1}{2} C_f \left( \frac{\varepsilon c p m \eta}{\lambda} \right) \left( \frac{v x \rho}{\eta} \right) = C_f \frac{1}{2} Pr \ Re x \quad (7)
\]

A similar line of reasoning was originally followed by O. Reynolds (reference 5) to the flow in circular pipes. But here, as on the flat plate, this simple theorem enables a satisfactory reproduction of the actual amount of heat transfer only in the specific case where the Prandtl number

\[
Pr = \frac{\varepsilon n c_p}{\lambda}
\]

is equal to unity. In all practical cases for which \( Pr \neq 1 \); hence for air also* divergencies are found which Prandtl attempts to comply with by assuming a special gas film between surface and actual boundary layer within which the gas velocity increases in proportion to the wall distance. Its thickness and velocity distribution existing in it are defined by the proportionality factors \( \Psi \) and \( \varphi \), to be computed from experiments (reference 2). Hence, according to the enlarged Prandtl equation:

\[
\alpha_{m} = \frac{\frac{1}{2} C_f c_p m v \varphi}{\Psi - \varphi(1 - Pr_g)} \quad (8)
\]

*Pr~0.725 for air of surrounding temperature; Pr varies only slightly with temperature.
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\[ N_{\text{ux}} = \frac{\frac{1}{2} \psi}{\psi - \varphi (1 - \text{Pr}_g)} \left( \frac{c_p \text{Nu}}{\lambda} \right) = \frac{1}{2} \text{Pr} \frac{1}{\psi - \varphi (1 - \text{Pr}_g)} \text{Re} \quad (9) \]

with \( \text{Pr}_g \) indicating the Prandtl number for the "boundary layer temperature." Prandtl gives the values \( \psi \) and \( \varphi \) only for the round pipe. In the case of heating of the fluid \( \psi = 1 \), \( \varphi = 1.4 \), \( \text{Pr}^{-0.185} \), \( \text{Re}^{-0.1} \)
and cooling of the fluid \( \psi = 1 \), \( \varphi = 1.12 \), \( \text{Pr}^{-0.185} \), \( \text{Re}^{-0.1} \)
and in both cases \( \text{Re} = \frac{\nu d}{\nu} \), \( d \) = pipe diameter.

Ten Bosch computed \( \psi = 0.89 \) and \( \varphi = 1.4 \), \( \text{Pr}^{-0.185} \), \( \text{Re}^{-0.075} \)
from experiments for the flat plate. The heat transfer factor is further dependent upon the difference between surface temperature and gas temperature. Ten Bosch assumes a turbulence variation \( \lambda \) as factor in the drag coefficients. Lacking more precise data he gives quantity \( \lambda \) as function of the increase of temperature \( \Theta \). Figure 2 holds true for flow in round pipes. Ten Bosch further suspects a probably minor dependence on \( \text{Pr} \). For a more exact exploration of these important interconnections the author recommends in place of \( \Theta \) two variables: the absolute temperature \( T \) of the gas and the ratio \( T_g/T \) — that is, the "dimensionless" boundary layer temperature — where \( T_g \) is the mean absolute temperature of the boundary layer. If, as in the present case, only one gas is involved, the absolute wall temperature \( T_w \) may be substituted for \( T_g \) as determining quantity, and \( \text{Pr} \) discounted; \( \Theta \) is positive for heating of the air.

Since the dependence \( \lambda = f(\Theta) \) for the flat plate is not known, figure 2 will serve as a basis within the framework of the present report.

In relation to equation (7), equation (9) contains the improvement factor

\[ V = \frac{1}{\psi - \varphi (1 - \text{Pr}_g)} \quad (10) \]

which for constant \( \text{Pr} \) and \( \text{Pr}_g \) can be represented as (slightly variable) power function of \( \text{Re} \) (cf. equation (18)).
The equations for $C_f$, $\alpha_m$, and $Nu_x$ in the three zones:

a) In Laminar Boundary Layer (Subscript L):

For isothermic flow

$$C_{fL} = 1.328 \text{Re}^{-0.5}$$

for any increase of plate temperature

$$C_{fL} = 1.328 \xi_L \text{Re}^{-0.5} \quad (11)$$

with equation (11) and the simplification through $Pr_g = 1$ in equation (8), we get

$$\alpha_m = \frac{2 \xi_L \text{Re}^{-0.5} v c p m V}{2} = \frac{0.664 \xi_L \lambda \frac{Pr}{x} \sqrt{\text{Re}}}{\psi}$$

against the approximation according to theoretical solution conformably to E. Pohlhausen (reference 2, p.144)

$$\alpha_o = \frac{0.664 \lambda}{x} \frac{\Theta_1}{\Theta_m} \sqrt{\frac{3}{Pr} \sqrt{\text{Re}}}$$

For the completely free plate, $\frac{\Theta_1}{\Theta_m} = 1$; $\psi$ is unknown in the first equation, and, although the term $\sqrt{Pr}$ in the second equation represents an approximation the reliability of which is limited to the range of $Pr = 1000$, it is nevertheless used for air. Besides, both equations become identical for $\psi = 1$, $\xi_L = 1$ and $Pr = 1$. Experiment alone can indicate which relation comes closest to reality. A variation of the "turbulence" in the laminar boundary layer owing to finite increase of temperature is to be disregarded. It might be assumed that $\xi$ becomes less than 1 for heating (positive $\Theta$), as in turbulent flow in pipes, and greater than 1 for cooling.

Henceforth:

$$Nu_{xL} = \frac{1}{2} C_f \sqrt{Pr} \text{Re}^{0.5}, \quad Pr \sim 0.725, \quad \frac{3}{2} \sqrt{Pr} = 0.449 \quad (12)$$
b) Transitional Region in Boundary Layer
(Subscript Ü)

According to an approximation by Prandtl the drag coefficient of the to~ plate for Rex between $5 \times 10^5$ and $2 \times 10^7$ allows for the increase of temperature $T$:

$$C_f \Pr = \left( \frac{0.455}{(\log \text{Rex})^{8.66}} - 1700 \text{Rex}^{-1} \right) T$$

The figure 1700 is merely the mean value for a range within which the true value can fluctuate, depending upon the initial disturbances. Although all important conclusions could be drawn from the variable slope of this curve from its lower limit of validity (Rex $\approx 5 \times 10^5$), an approximation by Ten Bosch is introduced which, while not quite as close, has the advantage of bringing out the effect of Rexc on the drag coefficient and naturally also on the heat dissipation.

The total frictional drag is visualized as being composed of the forward plate portion $L$ on which the boundary layer is laminar, and of the rest of the plate portion $Ü$, along which the boundary layer transforms, whereby the latter portion might be put at

$$C_f Ü = 0.003$$

independent of Rex. With equation (10) we get in this instance:

$$Nu Ü = \frac{\frac{1}{2} C_f Ü \text{Pr}}{\psi - \varphi (1 - \text{Pr} \epsilon)} \text{Rex} = C_f Ü \nu \frac{1}{2} \text{Pr} \text{Rex}$$

**c) Turbulent Boundary Layer (Subscript T):**

For $\text{Rex} > 2 \times 10^7$

*Ten Bosch used an older form giving almost the same values as equation (13), the first term of which reads $0.072 \text{Rex}^{-0.2}$.*

For further information on the drag coefficients of flat plates at large Reynolds numbers see reference 6.
\[ C_{fT} = 0.072 \frac{1}{\text{Re}} \text{ex}^{-0.2} \]  \hspace{1cm} (16)

\[ \text{Nu}_T = \frac{\frac{1}{2} C_{fT} \text{Pr}}{\psi - \phi (1 - \text{Pr_g})} \]  \hspace{1cm} \text{Re} = C_{fT} V \frac{1}{2} \text{Pr} \text{Re}  \hspace{1cm} (17)

By observance of the above values for \( \psi \) and \( \phi \) and approximation \( \text{Pr} \sim \text{Pr_g} \), the improvement factor according to equation (10) becomes

\[ V \sim 1.36 \quad \text{for} \quad \text{Re} = 5 \times 10^5 \]

\[ V \sim 1.30 \quad \text{for} \quad \text{Re} = 10^7 \]

or, represented as power function:

\[ V = \frac{1.638}{\sqrt{\text{Re}}} = 1.638 \text{Re}^{-0.0143} \]  \hspace{1cm} (18)

This relation needs to be checked by accurate tests and improved accordingly.

Figure 3 shows the drag coefficients \( C_f \) according to equations (11), (13), (14), and (16) and the mean Nusselt number \( \text{Nu}_x \) over the range \( \text{Re} = 2.5 \times 10^4 \) to \( 1.5 \times 10^7 \) for the unheated plate. Postin the respective term for \( C_f \) and of \( V \) in \( \text{Nu}_x \) according to equation (18) (again for \( \Theta = 0 \)) affords
a) for the laminar boundary layer according to equation (12):
   \[ Nux_L = 1.328 \times 0.449 \cdot Rex^{0.5} = 0.597 \cdot Rex^{0.5} \]

b) for the transitional region according to equation (15):
   \[ Nux_T = 0.003 \times 1.638 \times 0.3625 \cdot Rex^{0.9857} = 0.00178 \cdot Rex^{0.9857} \]
   or, according to Prandtl,
   \[ Nux_{Pr} = \frac{0.455 \times 1.638 \times 0.3625 \cdot Rex^{0.9857}}{(log \cdot Rex)^{0.868}} - 1700 \times 1.638 \times 0.3625 \cdot Rex^{-0.0143} \]
   \[ Nux_{Pr} = 0.27 \frac{Rex^{0.9857}}{(log \cdot Rex)^{0.868}} - 1010 \cdot Rex^{-0.0143} \]

c) for the turbulent boundary layer according to equation (17)
   \[ Nux_T = 0.072 \times 1.638 \times 0.3625 \cdot Rex^{0.7857} = 0.0428 \cdot Rex^{0.7857} \]
   according to equation (7):
   \[ Nux_T = 0.072 \times 0.3625 \cdot Rex^{0.8} = 0.0261 \cdot Rex^{0.8} \]

For easier plotting of the individual theoretical lines figure 3 contains several numerical values following from these equations.

The reason for the choice of the specific case of infinitely small increase of temperature on the plate surface was the avoidance of uncertainty in the choice of \( \delta \) which in this instance become equal to 1. The drag coefficient of plates with finite increase of temperature is theoretically analyzed in part VII. For \( Rex < Rex_{cr} \)

\[ C_f = C_{fL}; Nux = Nux_L \]

For \( Rex > Rex_{cr} \) up to the end of the transitional region (at \( Rex \sim 2 \times 10^7 \)), the mean values are:

\[ C_f = \frac{C_{fL}[Rex_{cr}] \cdot Rex_{cr} + C_T (Rex - Rex_{cr})}{Rex} \]

\[ = (C_{fL}[Rex_{cr}] - C_T) \frac{Rex_{cr}}{Rex} + C_T \]

\[ Nux = \frac{Nux_L[Rex_{cr}] \cdot Rex_{cr} + Nux_T (Rex - Rex_{cr})}{Rex} \]

\[ = (Nux_L[Rex_{cr}] - Nux_T) \frac{Rex_{cr}}{Rex} + Nux_T \]
whereby the length ratio $\frac{\lambda_{cr}}{x}$ is at the same time re-
placed by $\frac{\text{Re}_{cr}}{\text{Re}}$. The two cases

$$\text{Re}_{cr} = 10^5 \text{ and } \text{Re}_{cr} = 5 \times 10^5$$

are differentiated.

On plates of such total depth $x$ that $\text{Re}_{x}$ ranges
between $10^5$ and $2 \times 10^6$, $C_f$ and $W_{ux}$ values situated
in the shaded region should be possible to be measured
(up to $\pm 40$ percent referred to some mean value). (See ref-
ereence 7.) For still greater plate depths the uncer-
tainty decreases then rapidly (figure 3). Prandtl's
formula gives a curve similar to that of the linear com-
position under the assumption of especially great freedom
from disturbance.

The heat dissipation (and the temperature and velocity
fields) of a flat plate have been treated in detail by
Elia's (reference 1). The plate was 500 millimeters deep,
250 millimeters wide and 29 millimeters thick with a
wooden frame around it. The strip facing the flow was
$x_1 = 100$ millimeters deep and carefully tapered for the
purpose of minimizing the initial disturhance. Referring
the heat volume $Q$, dissipated from both sides up to a
certain depth $x$, to the unit span, he plotted the (named)
term $\frac{Q}{x \nu}$ against $\text{Re}_{x}$ (or $P_{ex}$). This is then compared
with the theoretical value $\frac{1}{2} C_f \rho \nu \gamma$ following from
equation (6). The displacement of the start of heat dissi-
pation relative to that of friction he attempted to ac-
count for by measuring the depth of the respective section,
first, from the beginning of the thermal reaction as $x_{th}$
and then from the beginning of the hydrodynamic reaction
as $x_0$. But neither of the representations is really able
to give a satisfactory reproduction of the process.

On the other hand, the reasoning on the basis of the
following is very informative despite the marked scatter:
if the plate had been heatable up to the leading edge it
would naturally have dissipated more heat by reason of its
larger area and the added existence of a region of strong
transformation of the velocity profile and hence of best
heat dissipation. In the dimensionless representation
the upper limit value $Nux_{\text{max}}$ theoretically corresponds to the curves $Nux$ according to equation (19). The actually heated portion would, of course, have then been able to dissipate less heat than in the experiment; namely,

$$Nux_{\text{min}} = \frac{Nux(x=x) - Nux(x=x_1)}{x} \left\{ \begin{array}{l}
= Nux(x=x) - Nux(x=x_1) \frac{x_1}{x} \\
= Nux(x=x) - Nux(x=x_1) \frac{Rex(x=x_1)}{Rex(x=x)} \end{array} \right. \right\}$$ (20)

The measured heat or its corresponding $Nux_m$ must lie between these two limits.

For comparison with the test data of $Nux_m$, the value $Nux_T$ is used in figure 4.

For a constant $x_1/x$ equation (20) becomes

$$Nux_{\text{min}} = 0.0428 Rex(x=x) - 0.0428 \left( \frac{x_1}{x} \right) \left[ 1 - \left( \frac{x_1}{x} \right)^{0.7857} \right]$$ (21)

The factor $\omega$ - that is, the ratio of the actual mean Nusselt number referred to that of the total depth

$$\omega = \frac{Nux_m}{Nux_T}$$ (22)

therefore ranges between $1 - \left( \frac{x_1}{x} \right)^{1.7857}$ and 1. The parameters are $x_1/x = 1/2, 1/3, 1/4, \text{and} 1/5$, each of which is applicable to one plate each with a "cold" forward and a "hot" residual portion amounting to 2, 3, 4, and 5 times that of the front portion each in a different air-speed flow $v$ (between 10 and 35 m/s). It is readily seen that,

(a) With exception of the range of small Reynolds numbers the $Nux_m$ are located above the $Nux_T$, as anticipated.
(b) The experimental curves (and the \( w \) factors) are located between the theoretical limiting values.

(c) They are, admittedly, not quite so systematically staggered according to parameter as equation (21) postulates.

(d) With increasing \( \text{Re}_x \) they approach, contrary to equation (21), the curve of \( \text{Nu}_x \) - that is, the factor \( w \) depends on \( \text{Re}_x \) owing to the dissimilarity of correlated velocity and temperature fields.

(e) The marked steepness of the actual \( \text{Nu}_x \) curves from the beginning indicates that the critical Reynolds number according to equation (2) is close to the lower limit \( 10^5 \).

In this comparison, especially for point (d), it should be noted that the function \( \text{Nu}_x \) is not as yet completely defined as to magnitude and steepness. Systematic discrepancies in figure 4, which would have to be interpreted as temperature effect, are, of course, not discernible by reason of the smallness of the chosen \( \Theta \).

V. APPLICATION TO AIRFOILS

These studies on the heat dissipation of flat plates can be applied to airfoils. It is advisable to treat the upper and lower surfaces separately by introducing as characteristic length \( x \) the measured distance around the airfoil from the stagnation point, the "start" of the boundary layer, up to the end of the heated area.

By reason of the variation of the airspeed along the airfoil circumference as a result of the angle of attack and of the finite airfoil thickness, individual mean velocities \( v_x \) for upper and lower surfaces must be introduced, which follow from the undisturbed velocity, say, with the aid of the mean differences \( \Delta p \) of the static pressures on both sides of the airfoil in ratio to the dynamic pressure \( q = \frac{v^2 \Delta p}{2} \). The mean \( \Delta p \) values are
obtainable from pressure distribution curves. Assuming that the acceleration or deceleration follows Bernoulli's equation, we get (for constant air density $\rho$):

$$\left\{ \frac{v_x^2 \rho}{2} - \frac{v^2 \rho}{2} = \Delta p \right\}$$

(23)

$$\left\{ \frac{v_x^2}{v} = 1 + \frac{\Delta p}{\rho} \right\}$$

$\Delta p$ is, of course, posted with its algebraic value (negative for the lower surface at positive lift).

For simplification the slight travel of the stagnation point with the angle of attack and the identical absolute variations of the lengths $x$ are discounted. The pressure distribution need not be known, if satisfied with the admittedly very rough approximation that the velocity on the wing upper surface is greater by the amount $\Delta v$ meters per second, and on the lower surface smaller by the same amount than the undisturbed flow velocity $v$. Actually, local velocities amounting to multiples of the undisturbed flow prevail on the forward portion of the upper surface especially on thick and high incidence airfoils, while the decelerations on the lower surface are substantially less. On the other hand,

$$\left[ (v + \Delta v)^2 \frac{\rho}{2} - v^2 \frac{\rho}{2} \right] + \left[ v^2 \frac{\rho}{2} - (v - \Delta v)^2 \frac{\rho}{2} \right] = \Sigma \Delta p$$

$$4v\Delta v \frac{\rho}{2} = \Sigma \Delta p$$

where $\Sigma \Delta p$ denotes the mean pressure difference between upper and lower surfaces. With wing area $F$ and angle of attack $\beta$ we get

$$\frac{c_a F v^2 \rho}{2} = F \Sigma \Delta p \cos \beta$$

$$\Delta p = \frac{c_a}{\cos \beta} \frac{v^2 \rho}{2}$$

$$\frac{\Delta v}{v} = \frac{c_a}{4 \cos \beta}$$

$$\frac{v_x}{v} \sim 1 \pm \frac{c_a}{4 \cos \beta}$$

(24)
For normal angles of attack \( \cos \beta = 1 \). The functions \( v_x/v \) are shown plotted against \( c_a \) in figure 5.

The Reynolds number used in the following is obtained from the \( Re \) employed in figure 1, by correction of the velocity from \( v \) to \( v_x \) according to equation (23) or approximation (24) and replacement of profile circumference \( D \) by chord \( x \) measured on the circumference:

\[
Re_x = Re \frac{v_x}{v} \frac{x}{D}
\]  

(25)

The conversion of the Nusselt number is simpler

\[
Nux = Nu \frac{x}{D}
\]  

(26)

In figure 6 the association of figures 1 and 3 with the factors of equations (25) and (26) for the flat plate and the two airfoils is compared. It is to be noted that the transition from \( v \) to \( v_x \) shifts the points horizontally, the transition from \( D \) to \( x \) in the direction 1:1.

On RAF section 26 and RAF section 30 \( x/D \) is assumed equal to \( 1/2 \) for upper and lower surfaces. The values for the nose on both airfoils are plotted on the assumption that the undisturbed velocity \( v \) as located between the two \( v_x \) is more suitable; \( x/D \) is averaged at 0.20 for both sides of the wing. The curves in figure 6 cover the test range of figure 1 from beginning to end.

In the comparison it is to be noted that a different temperature \( t_o \) or increase of temperature \( \Theta \) is applied to each of the three cases:

<table>
<thead>
<tr>
<th></th>
<th>( t_o )</th>
<th>( \Theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>RAF 26 . .</td>
<td>(~40^\circ)</td>
<td>(~20^\circ)</td>
</tr>
<tr>
<td>RAF 30 . .</td>
<td>(~93^\circ)</td>
<td>(~73^\circ)</td>
</tr>
<tr>
<td>Flat plate</td>
<td>Air temperature</td>
<td>( 0^\circ )</td>
</tr>
</tbody>
</table>

The fully heated models manifest the following:
a) All straight lines lie practically within the zone bounded by the theoretical curves $N_u x_L$, $N_u x_y$, $N_u x_T$ applicable to the flat plate.

b) On both airfoils the straights representing the heat dissipation of the upper surface at different angles of attack are packed closer together, those for the lower surface a little farther apart.

c) In conformity with the "narrowness" of the zone for large $R_e$ on the (symmetrical) RAF 30 section the systems of curves for both sides of the wing fall especially close together at equal angle of attack.

d) The slope of the separate straights for the RAF 26 section (with exception of those for $U\beta - 0.9^\circ$) rises fairly uniformly with increasing distance from $N_u x_L$ and ultimately approaches that of $N_u x_T$.

e) The laminar boundary layer persists for some time on the lower surface, but the curve for the $R_e$ factors above the upper limit of the test range for RAF 26 section appears to become steeper and, as theoretically stipulated, hugs the straight $N_u x_y$ until it deflects toward $N_u x_T$ in the range of the factors of the RAF 30.

Heated wing leading edge:

f) Here the accord with theory is not so good as already indicated. The deciding velocity is nevertheless very much dependent on the angle of attack as reflected by the fairly great distance between the straights on both airfoils corresponding to the lift coefficients $\sim 0.24$ and $0.64$. Added to this is the effect of especially high increases of velocity in this region. The heat dissipation of the nose portion on the upper and lower surfaces of the RAF 26 section could also be computed separately from the quoted data (pt. II) and evaluated in the discussed manner, whereby the pressure distribution might afford even "better" velocities $v_x$ and perhaps even better arc lengths $x$ by reference to the respective stagnation point.
From d) it can be concluded that the boundary layer on the wing upper surface changes sooner and becomes fully turbulent quicker than expected according to the value of Rex computed with $v + \Delta v$. Undoubtedly this is accomplished by the cited increases of velocity to which larger Reynolds numbers also correspond. The total surface area of the RAF 26 model is "rough" by reason of the glued-on platinum strips and it is surprising that the boundary layer remains laminar over the greater part of the lower surface, as indicated quite plainly by the location of the respective straight lines. However, the more scattered test values for $\theta - 0.0^\circ$ allow it to be inferred that the laminar boundary layer is no longer stable even at low angles of attack and lift coefficients, respectively — that is, even for very small differences relative to the flow velocity. According to the foregoing it may be expected that in the range of $\text{Re}_{\infty} < 10^5$ the laminar boundary layer on the upper surface also extends over continuously greater depths with decreasing Rex, so that the curves $Nux$ become progressively flatter (reference 2, p. 153). The S-shape course is naturally less pronounced on the curves for the wing upper surface than on those for the lower, because they are of themselves quite close to the asymptote $Nux_f$ for maximum Rex.

VI. CHOICE OF MATERIAL QUANTITIES

The studies described under part IIa) to d) were made on rectangular model airfoils (up to scale 1:1) of great aspect ratio, only a small portion of the span being used as heating surface, or on such between end plates (two-dimensional flow closely approximated in both cases). The airfoil circumference $D$ serving as characteristic length, was little more than twice the wing chord.

The material values $\lambda$ and $\eta$ involved in the dimensionless factors are related to the temperature (about proportional to $\sqrt{T}$). In none of the practically important cases of heat-transfer (flow in pipe, flow around pipe, flat plate) has there, up to now, been successfully determined a "decisive mean temperature" dependent on surface temperature, gas temperature, Reynolds number (and Prandtl number) to which the material values were to be referred. In R.&M. Reports Nos. 1326 and 1481 the Reynolds number Re is computed with the kinematic viscosity $v$ for outside air.
temperature $t$. In $\text{Nu}$, $\lambda$ is introduced with the value corresponding to that of the surface temperature $t_0$, since the processes in the boundary layer are justly held as the essential. When, furthermore, Ten Bosch recommends the kinematic viscosity $\nu = \frac{\eta}{\rho}$ corresponding to the surface temperature for the plate in forced flow even in $\text{Re}_x$, since as proved by experiment, it was best "suitable" for the plate in free flow, this procedure should answer the purpose for airfoils as well. It would involve the conversion of the "aerodynamic" $\text{Re}_x$ to the "thermic" $\text{Re}_{x0}$.

$$\text{Re}_{x0} = \text{Re}_x \left( \frac{\frac{\rho_0}{\eta_0}}{\frac{\rho_0}{\eta_0}} \right) = \text{Re}_x \frac{T \eta}{T_0 \eta_0} \quad (27)$$

The conversion factor $\text{U}_{\text{Re}_x} \rightarrow \text{Re}_{x0} = \frac{T \eta}{T_0 \eta_0}$ is simply a function of the two temperatures $t$ and $t_0$ (if this is variable a mean value substitutes). To avoid confusion between $\text{Re}_x$ and $\text{Re}_{x0}$ it would be preferable to use the respective Péclet number $\text{Pe}_{x0} = \frac{\text{Pr}_0 \text{Re}_{x0}}{\text{Pr}}$ in place of $\text{Re}_{x0}$, the Prandtl number $\text{Pr}$ may be averaged at $\sim 0.725$. The conversion factor $\text{U}_{\text{Re}_x} \rightarrow \text{Pe}_{x0} = 0.725 \frac{T \eta}{T_0 \eta_0}$ also depends on $t$ and $t_0$ only. Accordingly, if the temperature effect is taken into account, all straight lines of figure 6 relating to the airfoils shift toward the left, while $\text{Re}_x$ remains $= \text{Re}_{x0}$ for the flat plate. But the Nusselt number for the flat plate must be multiplied by the still unknown factor $\xi$. Assuming that it is smaller than 1 for the case of a heat dissipating surface similar to the flow through a pipe, the curves must shift downward with rising plate temperature and the close agreement of figure 6 is completely lost.

It would afford for the

$$\begin{align*}
\text{RAF 26} & \quad t_0 = 40^\circ \\
& \quad t = 20^\circ \\
\text{U}_{\text{Re}_x} \rightarrow \text{Pe}_{x0} = 0.725 \frac{273 + 20}{273 + 40} \times \frac{1.86}{1.95} = 0.647
\end{align*}$$

and for the

$$\begin{align*}
\text{RAF 30} & \quad t_0 = 93^\circ \\
& \quad t = 20^\circ \\
\text{U}_{\text{Re}_x} \rightarrow \text{Pe}_{x0} = 0.725 \frac{273 + 20}{273 + 93} \times \frac{1.86}{2.18} = 0.495
\end{align*}$$
Accepting, in the absence of other data, the correlation of $\mathcal{F}$ with the increase of temperature of the pipe for the flat plate, the surface temperature becomes

$\theta_0 = 40^\circ, \Theta = 20^\circ, \mathcal{F} = 0.92; \theta_0 = 93^\circ, \Theta = 40^\circ, \mathcal{F} = 0.88$

$\frac{U_{Re} - P_{ex}}{P_{ex}} = Pr = 0.725$

The result is illustrated in figure 7. The percentage of discrepancies between the experimental and theoretical data is readily apparent. They amount to more than 60 percent (on the RAF 30 section). Accordingly, it is very questionable whether Ten Bosch's suggested method actually reproduces the true conditions; in any event it would not explain what effects the theoretical values for the heat dissipation on the recorded, are able to increase. It furthermore conflicts with the view that the formation of the major portion of the boundary layer proceeds the same as in isothermic flow, the absolute temperature of which is, however, $T_0$ rather than $T$. The effect is therefore much too great.

In any case the arguments seem to indicate that the temperature effect for every one of the three boundary layer structures involved is of a different nature. Experiments in this zone will require care to assure laminar boundary layer over the entire depth of the airfoil or plate, and then to secure a turbulent boundary layer on the major part of the surface by using surfaces of maximum depth (with purposely great initial disturbance (reference 7)). In both cases the material values of the undisturbed flow will be introduced in the factors for the evaluation (for $\lambda$ also!). The plotting of the curves $Nux = f(Re)$ with the "temperature effect" as parameter affords a formula represented in the form $(T_0/T)^n$ if limited to a gas (air), whereby the exponent $n_L$ for laminar boundary layer need not be equal to the exponent $n_T$ for fully turbulent boundary layer. Both will be very much smaller than 1. It might even be possible to split these powers wholly or in part in factors and to correlate them with the material values in such a way that these could be coordinated with the afore-mentioned controlling mean temperature, as has been accomplished for horizontal wires and pipes to a certain extent (reference 8).
Form resistance does not contribute directly to heat diffusion, although very high heat transfer has been ascertained on the back of pipes in oblique flow especially at large Reynolds numbers, which are traceable to such vortices. Up to the present no theoretical formula has been published for this type of heat transfer (reference 2, p. 150). The induced drag on the wing is, in this sense, equally to be treated as form drag, that is, it may affect the boundary layer flow very slightly.

The heat transfer on the nose of the airfoil is most easily comparable with that on the pipe in oblique flow, but it is just as difficult to treat theoretically as that on the front of a pipe. However, the following holds true:

a) The heat dissipation of a surface area is so much greater as it is closer to the stagnation point;

b) It varies in relation to the angle of attack and the Reynolds number similar to the dissipation of the side of the wing of which it occupies the major portion;

c) The smaller the heat dissipating area near the stagnation point is, the greater its heat dissipation will be and so much less its dependence on the angle of attack. Fairly small areas fall within the laminar region \((\text{approximately } \text{Re}<10^4)\); hence \(\text{Nu}_x\) varies very slowly with the Reynolds number (with \(\text{Re}^{1/3}\)).

With exception of the tests on the heat dissipation of wing radiators, described in R.&M. Report No. 1163, only mean values for the Nusselt number are known. But within the framework of the necessary experimental data the distribution of heat dissipation along the outer skin of the surface by constant limitation of the skin radiator is of particular interest, because the local cooling conditions and the heat stresses are governed by it. If the analogy between flat plates and airfoils can be extended, the local values of heat-transfer coefficients can be closely approximated in power form \(\text{Nu}_x = M \text{Re}^n\) from the measured mean values by differentiation of the empirical data, analogous to the function represented in equation (19), at least for the case where the heating extends without interruption from the leading edge up to depth \(x\). After that, it is important to
know the appropriate areas on the wing surface for the heat dissipation in order to be able to find the best arrangement for the particular case.

For the heat dissipation of partial surfaces the statements made concerning the flat plate hold true. For every chosen parameter $x_1/x$ a factor $w$ is computed similar to equation (22), with reference to the proper $Nux_T$ or $Nux_L; w$, which again can be a function of $Re_x$, might at the same time include the temperature effect. Accordingly, it will be advisable to use the relations governing the flat plate — extrapolated to increase of temperature $\Theta = 0$ — as "standard" for a basis in form of the curves $Nux_L, Nux_T, and Nux_\Gamma$. The actual variation must, of course, be established by very careful experiments, while the concept "transitional region" requires a more exact interpretation. Comparison might also be made with a "standard function" for which $Nux_\pi$ is probably most suitable, since it is free from empirically defined values. For reduction to other temperature ranges the previously introduced mean value for $Pr$ should be retained and the variation of $Pr$ in the expression $V$ allowed for.

Irrespective of what new information is obtained for practical calculations the representation according to figure 1 should even then be more convenient than that according to figure 6, if the temperature effect is included in the suggested form.

VII. SUPPLEMENTARY DRAG OF SKIN RADIATORS

The transition from the speed of undisturbed flow to zero value of the gas particles adhering to the surface takes place, as is known, within the boundary layer. In accord with measurements we chiefly differentiate between the parabolic velocity distribution in the laminar boundary layer and the distribution in the turbulent boundary layer according to the $1/7$ power law (pt. IV). If the temperature of the surface differs from that of the gas, the statements concerning the temperature variation in the boundary layer hold true. If the relation of density $\rho$, kinematic viscosity $\nu$, and temperature conductivity $a$ to air temperature is discounted, the fields of velocity-
and temperature are exactly similar for the specific case that friction and heat dissipation begin simultaneously and the Prandtl number is equal to 1, the increase of temperature being computed from the surface temperature $T_0$. Many problems can be satisfactorily solved with the aid of this simple, although not exactly correct relationship. For more accurate adaptation to practice the previously cited values $\Psi$ and $\Phi$ deduced from tests we used.

The nature of the temperature effect on the drag consists, accordingly, in the simultaneous influence on the boundary layer thickness and further on the velocity profile within the boundary layer. In the test the two effects can, of course, be measured only concurrently. Application to geometrically similar processes is very reliably possible if the critical values are borne in mind. But it first needs to be proved that the data on the drag of thin, heated plates is applicable to airfoils; the effect on the boundary layer thickness being treated as if it did not change the velocity distribution within the boundary layer.

It may be suspected that with regard to drag a heated airfoil is decidedly thicker by $\Sigma \Delta \xi$ than the cold one, and the conversion of the thereby increased flow velocities to pressure could favor the formation of further form drag. Owing to the variation of the increases of velocity past the curved surface the boundary layer will form somewhat different from that of a flat plate but the discrepancies are small. (Cf. pt. V.) Undoubtedly the form drag (local vortices) is also of some influence.

In order to make the estimation of the hypothetical thickening clear, an airfoil with vanishing form drag but finite skin friction is investigated, in its effect on a potential flow, first cold, then heated to temperature increase $\Theta$. By assumption of the decisive velocities $v_x$ for both surfaces of the wing the boundary layer thickness on all points of the airfoil circumference is (reference 2, pp. 138–139).

$$\frac{\delta}{x} = 5.477 \text{Re}^{-0.5} \text{ For laminar boundary layer} \quad (28)$$

$$\frac{\delta}{x} = 0.37 \text{Re}^{-0.2} \text{ For turbulent boundary layer} \quad (29)$$
An airfoil without skin friction — that is, on which the potential flow remained up to the surface would, at equal pressure — and hence, velocity distribution, have to be just as much thicker as the boundary layer thickness decreased, if the potential velocity $v_x$ prevailed in it. For the thickness $\delta'$ of the substitute layer — if $y$ in figure 8 is the distance of a filament from the surface within the boundary layer — must be

$$\rho u \delta' = \int_0^\delta \rho v_{xy} dy$$

According to general law of velocity distribution

$$\frac{v_{xy}}{v_x} = \left(\frac{y}{\delta}\right)^m$$

where $m = 1/2$ for laminar, and $m = 1/7$ for turbulent boundary layer, we get

$$\delta' = \frac{1}{1 + m} \delta$$

$$\delta - \delta' = \frac{m}{1 + m} \delta$$

The new profile obtained on this basis, which similar to the so-called half-body extends to infinity, can of course, never exactly fulfill the theoretical condition for zero skin friction (sum of all pressure forces equal zero) because of the already mentioned simplifications, and particularly, because the potential velocity $v_x$ is not quite reached on the "outer limit" of the boundary layer.

On the heated profile the boundary layer thickens up as a result of the heat expansion. Since no appreciable acceleration forces can occur in flow direction by reason of the small inclination of the outside limitation of the boundary layer toward the surface, it may be assumed that each infinitely small width $dy$ expands almost exactly in correspondence with its temperature. Hence, according to figure 8:
The substitute thickness \( \delta_\Theta' \) for the boundary layer converted for the potential velocity \( v_x \) follows from

\[
\delta_\Theta' = \frac{\delta}{\int_0^\delta \rho v dx} \left( \frac{v_x y_1}{T} \right) T_y = T_0 - \Theta \left( \frac{y}{\delta} \right)^m
\]

\[
\delta_\Theta = \frac{T_0}{T} \delta - \frac{1}{1 + m} \frac{\Theta}{T} \delta = \left( 1 + \frac{m}{1 + m} \frac{\Theta}{T} \right) \delta
\]

The substitute thickness \( \delta_\Theta' \) for the boundary layer converted for the potential velocity \( v_x \) follows from

\[
\rho v \delta_\Theta' = \int_0^\delta \rho v y_1 \left( \frac{v_x y_1}{T} \right) T_y = \int_0^\delta \rho \frac{T}{T_y} v_y \frac{T_y}{T} dy = \int_0^\delta \rho v y_1 dy
\]

\[
\delta_\Theta' = \frac{1}{1 + m} \delta (\equiv \delta')
\]
\[ \Sigma \Delta \varepsilon = \sum \frac{1}{3} \frac{\Theta}{T} 5.477 \text{Re}^{-0.5} x \]  
For laminar boundary layer \hspace{1cm} (33)

with equation (29)

\[ \Sigma \Delta \varepsilon = \sum \frac{1}{8} \frac{\Theta}{T} 0.37 \text{Re}^{-0.8} x \]  
For turbulent boundary layer \hspace{1cm} (34)

The inferior accuracy of equation (32) justifies, for simplification, the introduction of the undisturbed velocity \( v \) in place of the decisive velocities \( v \) even at greater angles of setting and a mean value instead of the arc lengths \( x \) computed for both wing surfaces from the stagnation point to the point of maximum profile thickness. Lastly, it is assumed that the distance of the point of maximum profile thickness is about 30 percent of the chord length \( t \) from the wing leading edge. Since the profile circumference is slightly greater than \( 2t \) it is approximately

\[ \text{Re} = 0.15 \text{Re} \]
\[ x = 0.3t \]

The thickening \( \Sigma \Delta \varepsilon \) is referred to the actual maximum profile thickness \( d \) itself:

\[ V_d = \frac{\Sigma \Delta \varepsilon}{d} \]

Then the equation for the laminar boundary layer, (32) reads:

\[
\begin{align*}
V_{dL} &\sim 2 \frac{1}{3} \frac{\Theta}{t} 5.477 (0.15 \text{Re})^{-0.5} 0.3 \frac{t}{d} \\
\end{align*}
\]

and for the turbulent boundary layer, equation (34):

\[
\begin{align*}
V_{dT} &\sim 2 \frac{1}{8} \frac{\Theta}{t} 0.37 (0.15 \text{Re})^{-0.8} 0.3 \frac{t}{d} \\
\end{align*}
\]
Equation (35) will serve for small models, equation (36) for full-scale versions. In all practical cases the amounts of $V_d$ lie at 1 percent,* hence the form drag is scarcely affected by the heating in normal flight attitudes. Therefore the test data on flat plates can be applied fairly reliably to airfoils. In this connection it is further of interest to know whether portions of the heated outer skin lower the lift of the profile, especially its maximum, and whether the flow is adversely affected, so as to induce earlier breakaway than on the unheated wing with increasing angle of setting. Such phenomena are said to have been observed in past experiments and they appear altogether plausible according to the foregoing arguments, even though these do no longer permit of safe conclusions in these extreme cases. If the flow still adhes on the cold wing the assumed slight thickening up of the profile due to heating may very well be sufficient to make it separate.

Despite the fact that Ten Bosch's method appears to make the temperature effect excessive, the lack of other data prompts its use for determining the skin friction on a thin, flat plate heated over its total length to uniform increase of temperature $\Theta$ and surface temperature $T_0$ in two-dimensional flow. This merely involves the conversion of $Re_x$ to $Re_{x_0}$ by means of equation (27) and insertion in $C_f$ along with the still unknown factor $\xi$, the numerical values of which are taken from pipe flow. The ratio of resistance of the heated to the cold plate

$$V_c = \frac{C_f\Theta}{C_f}$$

a) $$V_{CL} = \xi \left( \frac{T_n}{T_0^{1.5}} \right)^{0.5}$$ (37)

for "short" plates (predominately laminar boundary layer, $Re_{x_0} < Re_{x_{cr}}$) according to equation (11).

b) $$V_{CL} = \xi$$ (38)

for "medium" plates, (largely transitional boundary layer, $Re_{cr} << Re_{x_0} < 2 \times 10^7$) according to equation (14).

*For the small model cited in the subsequent footnote, we get in round numbers $Re \sim 0.7 \times 10^6$ ($v \sim 20 \text{ m/s}$, as minimum value), $\Theta \sim 75^\circ$, $T \sim 298^\circ$, $a/t \sim 0.13$. According to equation (35) $V_d\alpha = 0.01$ (1 percent). If $Re \sim 10^7$, $\Theta \sim 100^\circ$, $T \sim 223^\circ$, $a/t \sim 0.13$. Equation (38) gives $V_d\alpha = 0.01$ (1 percent).
for "long" plates (predominantly fully turbulent boundary layer, \( \text{Re} \gg 2 \times 10^7 \)) according to equation (16).

With the approximated relation \( \frac{\eta}{\eta_0} = \left( \frac{T}{T_0} \right)^{0.5} \), these facts become even more evident.

The evaluation of equations (37), (38), and (39) is graphically represented in figure 9. On the basis of the little reliable assumptions made, it is found that: on short plates the skin friction drag rises with ascending \( \Theta \) but slower for high temperatures, with an initial drag decrease at low \( \Theta \); the same holds for long plates, but the initial decrease is more distinct, and the curve is subsequently flatter. On medium plates, on the other hand, the frictional drag drops to a minimum value, with ascending \( \Theta \), independent of the actual height of temperature.*

If the fore part of a plate is not heated, its skin friction drag naturally decreases proportionally less than corresponds to the ratio of the heat volumes (factor \( w \)). The exact magnitude of the frictional force exerted by the flow through the heated boundary layer onto a cold portion disposed behind the heated surface must be decided by experiment. Numerically this proportion of the drag lies naturally between that occurring in isothermal flow, if the temperature is, once \( T \), then \( T_0 \).

*The heat dissipation of surface portions distributed over the nose, upper and lower surface to the amount of \( \sim 55 \) percent of the profile circumference was measured on a model wing, heatable to 250 millimeters chord (O. Seibert, Jahr–buch 1938 der deutschen Luftfahrtforschung, pp. II 224 – II 244). Subsequent drag measurements, by O. Schoppe, disclosed as definite increase in the total drag on heating throughout the entire possible range (Re to \( \sim 2 \times 10^6 \)). The average for \( V_c \) was 1.12 at to \( \sim 100^\circ \); to \( \sim 25^\circ \) (\( \Theta \sim 75^\circ \)). The proportion of the laminar flow to the skin friction was certainly still perceptible on the model. Greater geometrical similarity would have perhaps afforded a lower \( V_c \) at these temperatures. (Continued on p. 30)
On a wing with heating surfaces in the forward portion only, the pressure distribution measurement in the rear unheated portion which can be easily made during the heat transfer experiments, itself is likely to afford practical data for the evaluation of the flow and separation phenomena.

Impending measurements may yield smaller drag variations than those deduced. But in any event the tendencies will hold good to some extent. Perhaps this dissimilarly in behavior of plates or airfoils depending upon the momentary Reynolds number enables an explanation of several contradictory observations on the relationship between drag and temperature.

Translation by J. Vanier,
National Advisory Committee for Aeronautics.

*(Continued from p. 29) Figure 9 shows $V_C$ at about 1.06 for the cited conditions. If the finite $\Theta$ were exactly allowed for, it would have had to result in a value higher than that recorded, since completely heated surface had been assumed for the study and the temperature effect was known to be too great. The term $\left( \frac{T_\eta}{T_\text{ref} \cdot T_\eta} \right)^{-0.5}$ is with 1.22 actually greater than the test value $V_C$. This seems to suggest that the unknown factor $\xi$ is considerably closer to 1 for flat plates and airfoils than for the flow in pipes.
REFERENCES


Figure 1.— Representation of $\text{Nu} = f(\text{Re})$ according to R and M Reports 1163, 1311, 1326, and 1481.
Figure 2.- \( \xi \) values for air.

Figure 5.-
\( v_x/v \) (for cos 3-1) total plate plotted against \( c_a \)
(according to equation 24).
Figure 3.— Relation of drag coefficients $C_f$ and the mean Nusselt number $N_u$ for the flat plate with the Reynolds number (increase of temperature $= 0$).

Subscripts:

$L$ = laminar boundary layer
$U$ = transitional region
$Pr$ = (according to Prandtl)
$T$ = turbulent boundary layer
$K$ = (according to Karman-Latzke)
Figure 4.— Comparison of the theoretical relation $N_{ux}$ and $R_{ex}$ of the flat plate with Elias' test data

$$\frac{x_1}{x} = \text{depth of cold fore portion}$$

$$\frac{x_1}{x} = \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$$

$\Delta \theta = 10^\circ C$

$\Delta \theta = 30^\circ C$
Surface temperatures
RAF 26 ~ 40°C
RAF 30 ~ 93°C
Flat plate = air
temperature ~ 20°C

Figure 6.— Reduced actual Nux plotted against the theoretical Nux for the flat plate.
Figure 7. Figure 6 replotted against $Pe_o$ to allow for the temperature effect according to ten Bosch (surface temperature of flat plate the same as that of models).
Figure 8.- Effect of heating on the flow,
\[ \delta; \delta_\theta = \text{actual boundary layer thickness} \]
\[ \delta'; \delta_\theta' = \text{substitute thickness} \]

\[ \epsilon = \delta - \delta' \]
\[ \epsilon_\theta = \delta_\theta - \delta_\theta' \]
\[ \Delta \epsilon = \epsilon_\theta - \epsilon \]

Surface

Potential Flow

Boundary layer

Figure 9.- Drag variation of plate due to heating = \( V_c \) (assumptions)

\[ t_0 = \text{surface temperature of plate} \]
\[ t = \text{air temperature of plate} \]
\[ \theta = \text{increase of temperature of plate} \]