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DETERMINATION OF THE MASS MOMENTS AND RADII OF INERTIA OF THE SECTIONS OF A TAPERED WING AND THE CENTER-OF-GRAVITY LINE ALONG THE WING SPAN

By V. V. Savelyev

Central Aero-Hydrodynamical Institute

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INTRODUCTION

For computing the critical flutter velocity of a wing among the data required are the position of the line of centers of gravity of the wing sections along the span and the mass moments and radii of inertia of any section of the wing about the axis passing through the center of gravity of the section. A sufficiently detailed computation of these magnitudes, even if the weights of all the wing elements are known, requires a great deal of time expenditure. Thus a rapid competent worker would require from 70 to 100 hours for the preceding computations for one wing only, while hundreds of hours would be required if all the weights were included.

With the aid of the formulas derived in the present paper, the preceding work can be performed with a degree of accuracy sufficient for practical purposes in from one to two hours, the only required data being the geometric dimensions of the outer wing (tapered part), the position of its longerons, the total weight of the outer wing, and the approximate weight of the longerons.

The entire material presented in this paper is applicable mainly to wings of longeron construction of the CAHI type and investigations are therefore being conducted by CAHI for the derivation of formulas for the determination of the preceding data for wings of other types.

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*Report No. 452, of the Central Aero-Hydrodynamical Institute, Moscow, 1939.

DETERMINATION OF THE CENTER-OF-GRAVITY LINE OF THE SECTIONS OF THE WING ALONG THE SPAN

Outer-Wing Structure (Tapered Part)

Determination of the centers of gravity of the sections of the wing along the chord.- To obtain the center of gravity of the sections of the tapered part of the wing along the chord* a large number of 2- and 3-longeron wings of the CAHI type were taken and each of them was divided into a number of segments of width Δl_n (fig. 1). By means of a sufficiently detailed computation there was determined for each segment the position of the center of gravity along the chord. By joining all the centers of gravity a somewhat broken line was obtained which may be replaced by a straight line passing through the center of gravity of the entire outer wing.

The chordwise position of the center of gravity may be determined either by computation or statistically. It is generally assumed that the over-all center of gravity of the tapered part of a wing of longeron construction lies at 40 percent of the chord of the section passing through the center of gravity. The computations for a large number of wings did not confirm this. It was found that the most forward position of the center of gravity was at 41.8 percent of the chord and the rearmost position at 43.5 percent. The mean position of the center of gravity of all the wings considered was at 43.0 percent of the chord.

The percentage given the chordwise position of the center of gravity does not remain constant for each segment of the wing but varies, the minimum value always being obtained at the sections toward the fuselage and the maximum value near the tip of the wing. The following values for the chordwise position of the center of gravity were obtained: At the fuselage end the minimum was 41.5 percent of the chord, the maximum was 42.5 percent of the chord, and the mean value was 42.0 percent of the chord. At the tip of the wing (at the section $0.97l_1$) the minimum was 44.5 percent of the chord, the maximum was 50.9 percent of the chord, and the mean value was 47.7 percent of the chord.

*In the parts of the wing considered. whatever does not contribute to the weight of the structure is excluded.

In connection with wing vibration computations the position of the center of gravity along the chord at the wing tip is of most interest. From the preceding data it is observed that the range of fluctuations of the position of the center of gravity at the tip of the wing is very large, amounting to 6.4 percent, and for this reason it is not advisable to take the average. On the other hand, the work of determining the position of the centers of gravity along the chord of the tapered wing requires a worked-out wing construction and consumes very much time. On the basis of available data it was found possible to give an approximate method for the rapid determination of the chordwise positions of the centers of gravity of the wing sections.

Approximate method of rapid determination of the chordwise positions of the centers of gravity of the sections of an outer wing.- The method consists of the following: Knowing the geometric dimensions of the wing, the section of the plane in which lies the center of gravity of the entire wing is determined by the formula

$$l_{c.g.o} = \frac{l_1}{3} \frac{n_\Phi + 2}{n_\Phi + 1} \tag{1}$$

where

$l_1 = l - \Delta l_1$	reduced length of the outer wing (fig. 1)
Δl_1	length of the rejected tip of the wing when the area of the wing is reduced to an equivalent trapezoid
l	length of the outer wing
n_Φ	is determined from figure 2 as a function of

$$n_0 = \frac{c_1 h_1}{c_0 h_0}$$

where

c_1 and h_1	chord and thickness at the wide end
c_0 and h_0	corresponding values at the tip at distance l_1

By laying off a length of 43 percent of the chord along the chord from the leading edge, the center of gravity of the entire outer wing (mean statistical position) is obtained. The line of the centers of gravity of the sections should be drawn with a certain inclination through the center of gravity just obtained. To draw this line the position of the center of gravity at the large end of the outer wing is determined. The latter position is given by

$$\frac{\Delta c_1}{c_1} 100 = 43.0 - 0.25\, l_{c.\,g.\,o} \tag{2}$$

where $l_{c.\,g.\,o}$, determined by formula (1), is expressed in meters.

Assuming that $l_{c.\,g.\,o} = 3.0$ meters, the center of gravity of the wing at the large end will lie at 42.25 percent of the chord and the second point is obtained. Joining this point with the center of gravity of the entire outer wing and prolonging it to the end of the wing, the line on which the centers of gravity of all the sections should lie is obtained. It should be noted that this determination of the center of gravity is correct only for the case where the axis or each longeron is located at the same percent of the chord along the entire length of the taper, the first longeron lying at 14 to 17 percent from the leading edge and the second at 45 to 50 percent.

For a more accurate determination of the position of the center-of-gravity line of the sections along the span, it is necessary to know the position of the center-of-gravity line of the wing without the longerons, the weight of the longerons, and their location.

<u>Position of the line of centers of gravity of the sections of the wing without longerons</u>.- In determining the center-of-gravity line of the wing without the longerons, a number of points are obtained through which a straight line may be drawn, the scatter of the points being small. To draw this line for any wing it is necessary to know the position of the center of gravity at the fuselage end and at the tip. It is not necessary to make any computations but to make use of statistical data according to which the center of gravity at the fuselage end lies approximately at 46.5 percent of the chord and the position of the center of gravity of the

tip (at the distance l_1, fig. 1) is determined from the curve shown in figure 3. Plotting these two points on the wing at the corresponding sections and joining by a straight line, the center of gravity line for the wing without longerons is obtained.

The greatest deviation of the true position of the centers of gravity from the previously mentioned straight line will occur at the sections near the fuselage end where the centers of gravity generally lie nearer the leading edge; but since the rest of the line passes more nearly through the remaining centers of gravity than any other straight line, a detailed computation of the centers of gravity of the sections is not required.

<u>More accurate determination of the chordwise position of the centers of gravity of the wing sections</u>.- Since the curve of the spanwise weight distribution of the wing, the weight of the longerons, the position of the centers-of-gravity line without the longerons, and the position of the longerons is known, the position of the center of gravity of any section of the wing may be determined.

Another manner, however, also may be used. Having determined according to the previously described method the line of centers of gravity of the outer wing without longerons and knowing the location of the longerons and their weight, the center of gravity of the entire outer wing without longerons is determined; assuming that the center of gravity of the outer wing and longerons lies at the distance $l_{c.g.\ o}$ from the fuselage end of the wing (equation (1)) and then making use of equation (2), the center of gravity at the fuselage end is determined. Then joining these two points by a straight line and prolonging to the tip of the wing, the chordwise position of the center of gravity of the wing sections is obtained. From statistical data on tapered wings of longeron and metal skin construction it is found that if the total weight of the wing without the weights of the joints to the center wing is taken as 100 percent, the first longeron will constitute about 16 percent, the second longeron about 15 percent, and the remainder about 69 percent.

Center-Wing Structure

The center wing, in addition to the air load, must support the attached loads (engines) and various loads disposed within it (fuel tanks, etc.). The mounting of these loads makes local re-enforcing of the center wing necessary and therefore the line joining the centers of gravity of the sections of the center-wing structure is not always straight but is usually a broken line so that a sufficiently accurate method for determining this line can not be given. Approximately the line may be determined by the formula

$$l_{c.g.c} = \frac{l_2}{3} \frac{n_c + 2}{n_c + 1} \qquad (3)$$

where

l_2 length of the part of the center wing projecting from the fuselage

$$n_c = \frac{c_2 h_2}{c_1 h_1}$$

where

c_2 and h_2 chord and thickness at the intersection of the center wing with the fuselage

c_1 and h_1 corresponding values at the juncture with the outer wing

The mean chordwise position of the center of gravity for a large number of planes was 39 percent of the chord. Laying off this value at distance $l_{c.g.c}$ from the root of the wing, the center of gravity of the part of the center wing projecting from the fuselage is obtained.

It was found for all of the center-wing sections investigated that the center of gravity of the center-wing section adjacent to the outer wing agreed almost exactly with the position of the center of gravity of the section of the outer wing adjacent to the center wing. Hence by joining with a straight line the center of gravity at the juncture end of the outer wing with the center of gravity of the part of the center wing projecting from the fuselage and prolonging the line to the root of the center wing, approximately the center-of-gravity line of the center-wing structure is obtained.

DERIVATION OF FORMULAS FOR THE DETERMINATION OF THE MASS MOMENTS AND RADII OF INERTIA OF THE OUTER-WING SECTIONS ABOUT THEIR CENTERS OF GRAVITY

The usual method of determining the moments of inertia of the wing sections is that of considering in succession all the elements entering the given segment, determining the moment of inertia of each element, and adding the moments. This method requires a knowledge of the wing structure so that the weight of all the elements entering the given segment and their distances from their centers of gravity to the chosen axis could be determined. Such method of determining the mass moments of inertia of the wing sections requires much time expenditure and can not be applied when the rough design of the airplane is known with only the over-all dimensions of the wings and the statistical weight of the center-wing and outer-wing structures. Since a knowledge of the moments of inertia along the wing is essential for determining the critical flutter velocity, various simplifying methods are used. The investigation of this problem has shown that a knowledge of the weight of the wing (of the center and outer parts separetely) and its geometric dimensions and the position of the longerons is sufficient for determining with a satisfactory degree of accuracy the moments and the radii of inertia of the wing sections.

Derivation of Formulas for The Determination of The Moments of Inertia about Their Centers of Gravity of The Sections of The Outer Wing

Since the total weight of the structure of the outer wing and its geometric dimensions is known, a curve of the weight distribution along the span* can be drawn (fig. 1). From this curve the weight of a segment of the wing of width l_n at any point can be determined.

It should be remembered that the curve of spanwise weight distribution of the outer wing assumes not a

*See author's paper "Span Distribution of The Weight of The Wing Structure," Technika Vozdushnogo Flota No. 1, 1938, or CAHI Report No. 381, under the same title.

concentrated weight of the ribs but a uniform distribution along the span so that the weight of each rib is uniformly distributed to the right and left over a width equal to half the distance between ribs. If the weight of the outer-wing structure was uniformly distributed along the chord, no difficulty would be encountered in determining the moment of inertia of any segment of the outer wing. Actually, however, there is no element in the outer wing the weight of which is uniformly distributed along the entire chord for even a dural skin may have various thicknesses over the profile. The thickness of the stringers may, in general, vary along the wing, and the rib structure is such that its weight is nonuniformly distributed along its length. The longerons, for the purpose of determining the moments of inertia of the wing sections along the y-axis, are considered as concentrated loads.

From an examination of a large number of outer-wing structures it was found that the first longeron as a rule lay along the entire span at a distance from the line of centers of gravity approximately equal to the magnitude of the radius of inertia (fig. 4) and therefore in computing the moments of inertia of the various segments of the outer wing, its weight may be taken as distributed proportionally to the remaining weights. The second longeron was found to lie at a distance not exceeding 12 percent of the chord from the line of centers of gravity (fig. 4) and therefore was of slight effect on the magnitude of the moment of inertia.

Taking account of what was said previously, the following formula for the determination of the moment of inertia at any section of the outer wing (fig. 5) may be written.

$$I_{Bc.g} = \frac{k_1}{g}\left[k_2 \frac{(q_n - \Delta q_{I_n}) c^2{}_n}{12} + (q_n - \Delta q_{I_n})(\Delta_n c_n)^2 + \frac{\Delta q_{I_n} c^2{}_n}{12}\right] + \frac{\Sigma m z^2}{\Delta l_n} \tag{4}$$

where

k_1 coefficient that takes into account the nonuniform chordwise distribution of the weight of such elements as the stringers, ribs, various joints, nonuniform thickness of the skin over the profile contour, and so forth, and also the effect of the moment of inertia of the second longeron

g acceleration of gravity, equal to 9.81

k_2 coefficient that takes into account the change in the magnitude of the moment of inertia of the wing without longerons owing to the displacement of the centers of gravity of the sections at both sides from the center of the profile

q_n weight of a given segment without the second longeron

Δq_{I_n} weight of the first longeron at a given segment

c_n outer wing chord of the segment considered

Δ_n distance between the line of centers of gravity of the segments of the wing with longerons and without them at the given section in percent of the chord c_n

$\frac{\Sigma mz^2}{\Delta l_n}$ moment of inertia of a given segment taking into account its height.

From the polynomial in the brackets (formula (4)) the common factor $\frac{q_n c^2_n}{12}$ is taken out and the coefficients defined

$$\frac{(q_n-\Delta q_{I_n})c^2_n}{12} = k_3\frac{q_n c^2_n}{12}$$

whence

$$k_3 = \frac{q_n-\Delta q_{I_n}}{q_n} \qquad (5)$$

$$(q_n-\Delta q_{I_n})(\Delta_n c_n)^2 = k_4\,\frac{q_n c^2_n}{12}$$

$$k_4 = 12\left(\frac{q_n-\Delta q_{I_n}}{q_n}\right)\Delta^2$$

whence

$$k_4 = 12\Delta^2{}_n k_3 . \tag{6}$$

$$\frac{\Delta q_{I_n} c^2{}_n}{12} = k_5 \frac{q_n c^2{}_n}{12},$$

whence

$$k_5 = \frac{\Delta q_{I_n}}{q_n}$$

or

$$k_5 = 1 - k_3 . \tag{7}$$

The obtained coefficients are substituted in equation (4)

$$Ip_{c.g.} = k_1 \left[(k_2 + 12\Delta^2{}_n - 1) k_3 + 1\right] \frac{q_n c^2{}_n}{12g} + \frac{\Sigma mz^2}{\Delta l_n} .$$

There is set

$$k_1 \left[(k_2 + 12\Delta^2{}_n - 1) k_3 + 1\right] = k . \tag{8}$$

$$\frac{\Sigma mz^2}{\Delta l_n} = \Delta k \cdot k \frac{q_n c^2{}_n}{12 g} . \tag{9}$$

Substituting the new notation in the preceding equation, there is obtained

$$Ip_{c.g.} = (1 + \Delta k) k \frac{q_n c^2{}_n}{12g} . \tag{10}$$

For the outer wings considered

$$(1 + \Delta k) k = 1, \tag{11}$$

whence

$$Ip_{c.g} = \frac{q_n c_n{}^2}{12g} . \tag{12}$$

Determination of q_n.- Directly from figure 5 there is obtained

$$q_n = q_0 + q - \Delta q_{II_n} , \tag{13}$$

where

$$q = q_1 - q_0 - (q_1 - q_0) \frac{l_n}{l_1} .$$

or

$$\Delta q_{II_n} = \Delta q_{II_1} - (\Delta q_{II_1} - \Delta q_{II_0}) \frac{l_n}{l_1} . \tag{14}$$

The values q and Δq_{II_n} are substituted in equation (13)

$$q_n = q_0 + q_1 - q_0 - (q_1 - q_0) \frac{l_n}{l_1} - \Delta q_{II_1} + (\Delta q_{II_1} - \Delta q_{II_0}) \frac{l_n}{l_1} ,$$

whence

$$q_n = q_1 - \Delta q_{II_1} - \left[(q_1 - q_0) - (\Delta q_{II_1} - \Delta q_{II_0})\right] \frac{l_n}{l_1} . \tag{15}$$

The magnitudes q_1, q_0, Δq_{II_1}, and Δq_{II_0} entering equation (15) are expressed in terms of n_Φ and q_m

$$n_\Phi = \frac{q_1}{q_0},$$

whence

$$q_1 = n_\Phi q_0,$$

$$q_m = \frac{q_1 + q_0}{2} = \frac{n_\Phi + 1}{2} q_0,$$

$$q_0 = \frac{2q_{cp}}{n_\Phi + 1}, \tag{16}$$

$$q_1 = \frac{2n_\Phi q_m}{n_\Phi + 1}. \tag{17}$$

There is set

$$\Delta q_{II_1} = a_{II_1} q_m, \tag{18}$$

$$\Delta q_{II_0} = a_{II_0} q_m. \tag{19}$$

The expressions (16), (17), (18), and (19) are substituted in (15).

$$q_n = \frac{2n_\Phi q_m}{n_\Phi + 1} - a_{II_1} q_m - \left[\left(\frac{2n_\Phi q_m}{n_\Phi + 1} - \frac{2q_m}{n_\Phi + 1}\right) - (a_{II_1} q_m - a_{II_0} q_m)\right] \frac{l_n}{l_1},$$

$$q_n = \frac{[2n_\Phi - a_{II_1}(n_\Phi + 1)] - [2n_\Phi - 2 - (a_{II_1} - a_{II_0})(n_\Phi + 1)] \frac{l_n}{l_1}}{n_\Phi + 1} q_m. \tag{20}$$

<u>Determination of a_{II_1} and a_{II_0}.</u>

$$b_{II} = \left[\frac{(\Delta q_{II_1} + \Delta q_{II_0}) l_1}{2} : q_m l_1\right] 100 = \frac{(a_{II_1} + a_{II_0}) q_m l_1}{2q \quad l_1} 100 = \frac{a_{II_1} + a_{II_0}}{2} 100,$$

whence

$$a_{II_1} = \frac{2b_{II}}{100} - a_{II_0}, \tag{21}$$

where b_{II} is the weight of the second longeron without the joints to the center wing in percent of weight of the outer wing.

$$c_{II} = \frac{2\Delta q_{II_0}}{\Delta q_{II_1} + \Delta q_{II_0}} 100 = \frac{2a_{II_0} q_m}{(a_{II_1} + a_{II_0}) q_m} 100 = \frac{2a_{II_0}}{a_{II_1} + a_{II_0}} 100,$$

$$100\, a_{II_0} = \frac{1}{2}(a_{II_1} c_{II} + a_{II_0} c_{II}) = \frac{b_{II}}{100} c_{II} - \frac{1}{2} a_{II_0} c_{II} + \frac{1}{2} a_{II_0} c_{II}.$$

$$a_{II_0} = \frac{b_{II}\, c_{II}}{10\,000}, \tag{22}$$

where c_{II} is the unit weight of the second longeron at the tip of the outer wing in percent of the mean unit weight of the longeron.

The value of q_n according to average statistical data.- From statistical data the weight of the second longeron without the joints constitutes about 15 percent of the weight of the entire outer wing. It may be assumed that the weight of the second longeron along the span is distributed according to the triangle law whence there is obtained

$$a_{II_0} = 0 \tag{23}$$

$$a_{II_1} = \frac{2 \times 15}{100} = 0.30 \tag{24}$$

By substituting these values in formula (20), there is obtained

$$q_n = \frac{[1.7n_\Phi - 0.3] - [1.7n_\Phi - 2.3]\frac{l_n}{l_1}}{n_\Phi + 1} q_m \tag{25}$$

Determination of The Coefficients

Determination of the coefficients Δk.- By formula (9) there is obtained

$$\Delta k = \frac{\Sigma m z^2}{\Delta l_n} : k\frac{q_n c_n^2}{12g} \tag{26}$$

where

$\Sigma m z^2$ moment of inertia of a segment of width Δl_n taking account of its height

$k\frac{q_n c_n^2}{12g}$ area moment at the given section of the wing along the chord

The computation of the moments of inertia with respect to the height of the wing section for a large number of wings showed that for any segment the moment of inertia can with sufficient accuracy be determined by the formula

$$\Sigma p z^2 = \frac{p_n h_n^2}{9} \tag{27}$$

where

p_n weight of the segment

h_n maximum height of profile in the plane of the center of gravity of the segment

If at each section of the wing $k\frac{q_n c_n^2}{12g}$ is taken as unity, the moment of inertia with respect to the height of the profile at each section may be expressed as a fraction of unity and will correspond to the coefficient Δk. Figure 6 shows the variation in the coefficient Δk along the span of two outer wings - the lower curve for an airplane of about five tons weight, the upper curve for an airplane of about twelve tons weight, and between them the curve of the mean value of Δk which should be used in the computation. The value of Δk does not depend on the weight of the airplane but only on the value of h_n expressed as a fraction of the length of the chord c_n.

$$\Delta k = \frac{(q_n + \Delta q_{II_n})h_n^2}{9g} : \frac{kq_n c_n^2}{12g} = \frac{4}{3k}\left(1 + \frac{\Delta q_{II_n}}{q_n}\right)\left(\frac{h_n}{c_n}\right)^2$$

The mean value of the coefficient Δk for the entire outer wing may be taken equal to

$$\Delta k_m = 0.030 \tag{28}$$

<u>Determination of the coefficient k</u>.- From formula (11) there is obtained

$$k = \frac{1}{1 + \Delta k} \tag{29}$$

Substituting for Δk its mean value, there is obtained

$$k_m = 0.970 \tag{30}$$

<u>Determination of the coefficient k_2</u>.- The variation in the value of the coefficient k_2 along the outer wing is shown in figure 7 where four curves are given. From statistical data the center of gravity of the section of the outer wing without longerons at the large end lies at 46.5 percent of the chord and the center of gravity of the wing without longerons at the section lying at distance l_1 from the large end lies at 50 to 54 percent of the chord. Hence all the four curves shown on figure 7 start from one point and diverge somewhat at the tip of the wing. The numbers on figure 7 indicate the percent corresponding to each curve. The mean value of this coefficient for the entire outer wing may be taken equal to

$$k_{2m} = 0.994 \tag{31}$$

Determination of the coefficient k_3.- According to formula (5), it may be written

$$k_3 = 1 - \frac{\Delta q_{\mathrm{I}_n}}{q_n}. \tag{32}$$

In analogy with formulas (14), (18), and (19) there is obtained

$$\Delta q_{\mathrm{I}_n} = \Delta q_{\mathrm{I}_1} - (\Delta q_{\mathrm{I}_1} - \Delta q_{\mathrm{I}_0}) \frac{l_n}{l_1};$$

$$\Delta q_{\mathrm{I}_1} = a_{\mathrm{I}_1} q_m ,$$

$$\Delta q_{\mathrm{I}_0} = a_{\mathrm{I}_0} q_m ,$$

whence there is obtained

$$\Delta q_{\mathrm{I}_n} = \left[a_{\mathrm{I}_1} - (a_{\mathrm{I}_1} - a_{\mathrm{I}_0}) \frac{l_n}{l_1} \right] q_m. \tag{33}$$

In analogy with formulas (21) and (22) it is written

$$a_{\mathrm{I}_0} = \frac{b_1 c_{\mathrm{I}}}{10\,000}; \tag{34}$$

$$a_{\mathrm{I}_1} = \frac{2b_{\mathrm{I}}}{100} - a_{\mathrm{I}_0}, \tag{35}$$

where

b_1 weight of first longeron without the joints to center wing

c_1 unit weight of first longeron at the tip in percent of mean unit weight of the longeron

The weight of first longeron without the joints, according to the wings observed, constituted about 16 percent of the weight of the entire outer wing. By assuming that the distribution of its weight along the wing follows the triangle law, there is obtained

$$a_{\mathrm{I}_0} = 0; \tag{36}$$

$$a_{\mathrm{I}_1} = 0{,}32, \tag{37}$$

whence

$$\Delta q_{\mathrm{I}_n} = \left[0{,}32 - 0{,}32 \frac{l_n}{l_1} \right] q_m. \tag{38}$$

By making use of equation (25), there is obtained

$$k_3 = 1 - \frac{\left[0{,}32 - 0{,}32 \frac{l_n}{l_1} \right] \cdot [n_\Phi + 1]}{[1{,}70\, n_\Phi - 0{,}30] - [1{,}70\, n_\Phi - 2{,}30] \frac{l_n}{l_1}} =$$

$$= 1 - \frac{C}{A - B \frac{l_n}{l_1}}. \tag{39}$$

TABLE I

$(n_\Phi = 4)$

$\frac{l_n}{l_1}$	$B\frac{l_n}{l_1}$	$A-B\frac{l_n}{l_1}$	$0.32\frac{l_n}{l_1}$	C	$\frac{C}{A-B\frac{l_n}{l_1}}$	k_3
0	0	6.50	0	1.600	0.246	0.754
.2	.90	5.60	.064	1.280	.228	.772
.4	1.80	4.70	.128	.960	.204	.796
.6	2.70	3.80	.192	.640	.169	.831
.8	3.60	2.90	.256	.320	.114	.886
1.0	4.50	2.00	.320	0	0	1.000

From table I there is obtained the mean value $k_3 = 0.840$.

TABLE II

$(n_\Phi = 10)$

$\frac{l_n}{l_1}$	$B\frac{l_n}{l_1}$	$A-B\frac{l_n}{l_1}$	$0.32\frac{l_n}{l_1}$	C	$\frac{C}{A-B\frac{l_n}{l_1}}$	k_3
0	0	16.70	0	3.520	0.211	0.789
.2	2.94	13.67	.064	2.820	.205	.795
.4	5.88	10.82	.128	2.110	.195	.805
.6	8.82	7.88	.192	1.410	.179	.821
.8	11.76	4.94	.256	.705	.143	.857
1.0	14.70	2.00	.320	0	0	1.000

From table II there is obtained the mean value $k_3 = 0.844$.

The mean value of n_Φ for all the wings considered is about seven, and therefore for the mean value of the coefficient k_3 there is obtained the value

$$k_{3m} = 0.842 \tag{40}$$

The variation in the mean value of the coefficient k_3 along the span of the outer wing is shown in figure 8.

Determination of Δ_n.- The distance between the center-of-gravity lines of the wing with and without longerons, expressed in fractions of the chord, is not constant along the span but fluctuates between 0.03 and 0.07, and therefore for the entire outer wing may be taken as

$$\Delta_n = 0.050 \tag{41}$$

whence there is obtained

$$12\Delta_n^{2} = 0.030 \tag{42}$$

Determination of the coefficient k_1.- From formula (8) there is obtained

$$k_1 = \frac{k}{(k_2 + 12\Delta_n^{2} - 1)k_3 + 1} \tag{43}$$

By substituting in this formula the corresponding mean values of the coefficients determined previously, equations (30), (31), (40), and (42), there is obtained

$$k_{1m} = \frac{0.970}{[0.994 + 0.030 - 1]\ 0.842 + 1}$$

whence

$$k_{1m} \cong 0.950 \tag{44}$$

The variation in the mean value of k_1 along the span is shown in figure 9.

Derivation of Formulas for The Determination of The Radius of Inertia of any Outer-Wing Section about Its Center of Gravity

$$i_{c.g.} = \sqrt{\frac{(1 + \Delta k)k\frac{q_n c_n^{2}}{12g}}{\frac{q_n + \Delta q_{II_n}}{g}}} = \sqrt{\frac{(1 + \Delta k)k}{12}\ \frac{q_n c_n^{2}}{q_n + \Delta q_{II_n}}}$$

whence

$$i_{c.g.} = 0.288c_n \sqrt{(1 + \Delta k)k \frac{q_n}{q_n + \Delta q_{II_n}}} \qquad (45)$$

For the wings considered

$$(1 + \Delta k)k = 1$$

hence

$$i_{c.g.} = 0.288c_n \sqrt{\frac{q_n}{q_n + \Delta q_{II_n}}} \qquad (46)$$

for

n_Φ = 7.0 (mean statistical value)

b_{II} = 15.0 percent

c_{II} = 0

there is obtained

$$q_n = \frac{[1.7n_\Phi - 0.3] - [1.7n_\Phi - 2.3]\frac{l_n}{l_1}}{n_\Phi + 1} q_m = \frac{11.6 - 9.6\frac{l_n}{l_1}}{8.0} q_m$$

$$\Delta q_{II_n} = \left[0.3 - 0.3\frac{l_n}{l_1}\right] q_m$$

TABLE III

$\frac{l_n}{l_1}$	$9.6\frac{l_n}{l_1}$	$\frac{q_n}{q_m}$	$0.3\frac{l_n}{l_1}$	$\frac{\Delta q_{II_n}}{q_m}$	$\frac{q_n+\Delta q_{II_n}}{q_m}$	$\frac{i_{c.g.}}{c_n}$
0	0	1.450	0	0.300	1.750	0.262
.2	1.92	1.210	.060	.240	1.450	.263
.4	3.84	.970	.120	.180	1.150	.265
.6	5.76	.730	.180	.120	.850	.267
.8	7.68	.490	.240	.060	.550	.273
.9	8.64	.370	.270	.030	.400	.278
1.0	9.60	.250	.300	0	.250	.288

The values of $\frac{i_{c.g.}}{c_n}$ obtained in this table are plotted in figure 15.

COMPARISON OF RESULTS OBTAINED BY DETAILED COMPUTATION WITH THOSE OBTAINED BY THE DERIVED FORMULAS

Mass Moment of Inertia of The Outer-Wing Structure about The Center-of-Gravity Line

By using the detailed method of computation of the mass moments of inertia of the various segments of the outer wing, the broken line shown in figure 10 is obtained. For each wing the broken line was replaced by a smooth curve obtained from the condition that the area bounded by this curve and two ordinates should be equal to the sum of the areas of the columns under the broken line.

For all wings for which the mass moments of inertia were computed in detail from the geometric dimensions and total weight of each outer wing, there were determined the values n_0, n_Φ, and $q_m = P/l_1$ where P did not include the weight of the joints. Eleven sections were taken for each outer wing and in each of the sections, by making use of formula (12), the moment of inertia was determined and the curves shown in figures 11, 12, 13, and 14 were obtained.

From these curves it is seen (figs. 12 and 13) that for the fuselage end of the outer wing there is considerable disagreement between the curves of true moments of inertia and the curve computed by the formula. Remembering, however, that the disagreement in the moments of inertia at the sections near the fuselage is of no special significance for the vibration computation, it must be concluded that the derived formula for determining the moment of inertia at any section of the outer wing gives good agreement with accurate computation, a fact of great importance since the moment of inertia of any section of the outer wing can be determined from a knowledge of only its geometric dimensions, total weight, and the weight of the second longeron.

Radius of Inertia

The wavy curves of figure 15 are those of the radii of inertia of the four outer wings considered, the curves having been obtained by a detailed computation. Notwithstanding the fact that the wings considered differed sharply among each other both as regards their absolute dimensions as well as their taper ratio n_0 with a somewhat different chordwise position of the longerons (the center of gravity of the longerons lay at from 32 to 36 percent of the chord), the scatter of the points of these curves is not large and they can all be replaced by the smooth curve shown in the figure, the curve being drawn from the data of table III.

ILLUSTRATIVE EXAMPLE

The formulas derived previously are valid only for two and three longeron wings of the CAHI type, the position of the longerons and the lines of center of gravity not differing greatly from the position of the longerons shown in figure 4 (the third longeron shown in the figure constitutes only 2 percent of the weight of the outer wing and is not a stiffening member).

The formulas for determining the mass moments and radii of inertia were derived for a cantilever wing the thickness of which at the fuselage end was about 15 to 16 percent of the chord and at the tip about 5 to 7 percent.

By assuming the two-longeron wing shown in figure 16, the data for it being the following:

$$\left.\begin{array}{l} c_2 = 4.43 \text{ meters} \\ h_2 = 0.656 \text{ meter} \end{array}\right\} \text{chord and thickness of center wing at fuselage intersection}$$

$l_2 = 2.35$ meters   length of center wing projecting from fuselage

$$\left.\begin{array}{l} c_1 = 4.10 \text{ meters} \\ h_1 = 0.656 \text{ meter} \end{array}\right\} \text{chord and thickness at juncture of outer wing with center wing}$$

$$\left.\begin{array}{l} c_0 = 1.40 \text{ meters} \\ h_0 = 0.112 \text{ meter} \end{array}\right\} \text{effective chord and thickness at tip of outer wing at distance } l_1 \text{ from large end (fig. 16)}$$

$l_1 = l - \Delta l_1 = 8.85$ meters   effective length of outer wing

l length of outer wing from large end to tip

$\Delta l_1'$ rejected length of tip of wing in reducing area of outer wing to an equivalent trapezoid

G = 363 kilograms weight of outer wing structure including ailerons, bolts, and fillets

Determination of The Center-of-Gravity Line along The Span

Outer-wing structure.- For determining the center-of-gravity line of the outer wing in the preliminary design stage when the weight of the longerons is not known, the approximate method previously given is used

$$n_o = \frac{c_1 h_1}{c_o h_o} = \frac{4.10 \times 0.656}{1.40 \times 0.112} = 17.1$$

From the curve of figure 2 it is found that $n_\Phi = 8$; hence the distance to the center of gravity of the entire outer wing (equation (1)) is

$$l_{c.g.o} = \frac{l_1}{3} \frac{n_\Phi + 2}{n_\Phi + 1} = \frac{8.85}{3} \frac{10}{9} = 3.28 \text{ meters}$$

From statistical data the center of gravity of the outer wing lies at 43.0 percent of the chord at distance $l_{c.g.o}$. The center of gravity at the juncture with the center wing is 43.0 - 0.25 × 3.28 = 42.2 percent. By joining the preceding two points by a straight line and prolonging it to the tip of the wing, the line of centers of gravity of the outer wing along the span is obtained. If the weights of the longerons are known, the center-of-gravity line can be more accurately determined by the method previously given.

Center wing.- From statistical data the center of gravity of the part of the center wing projecting from the fuselage lies at 39 percent of the chord, the distance of which from the root of the center wing is $l_{c.g.c}$. To determine $l_{c.g.c}$ by formula (3), it is necessary first to determine

$$n_c = \frac{c_2 h_2}{c_1 h_1} = \frac{4.43 \times 0.656}{4.10 \times 0.656} = 1.08$$

and then

$$l_{c.g.c} = \frac{l_2}{3} \frac{n_c + 2}{n_c + 1} = \frac{2.35}{3} \times \frac{3.08}{2.08} = 1.16 \text{ meters}$$

By joining the center of gravity of the outer wing at the fuselage end with the general center of gravity of the part of the center wing projecting from the fuselage by a straight line and prolonging it to the fuselage, the line of centers of gravity of the sections of the center wing along the span is obtained. This line does not take into account the weight of the joints at the intersection of the outer wing with the center wing.

Determination of The Mass Moment of Inertia of Outer-Wing Sections about Their Centers of Gravity

At the preliminary design stage when the weight of the second longeron is not known, the mass moment of inertia of any section of the outer wing may be determined by the formula

$$I_{p_{c.g.}} = \frac{q_n c^2{}_n}{118} \text{ kg. sec}^2 \qquad (12)$$

where

q_n weight of a given segment without the second longeron

c_n length of chord of segment in meters

For the two-longeron wing considered in the example, q_n may be determined by the formula

$$q_n = \frac{[1.7n_\Phi - 0.3] - [1.7n_\Phi - 2.3]\frac{l_n}{l_1}}{n_\Phi + 1} q_m$$

This value was obtained on the assumption that the weight of the second longeron without the joints to the center wing on the basis of statistical data on two-longeron wings constitutes on the average 15 percent of the weight of the outer wing and that its weight was distributed along the span according to the triangle law.

In the case where the weight of the second longeron differs considerably from 15 percent and the spanwise weight distribution can not be assumed to follow the triangle law, the value of q_n is determined by formula (20) or graphically.

l_n distance from juncture end of outer wing to the section considered

$$q_m = \frac{G - p_1}{l_1}$$

p_1 weight of attachments of outer wing to center wing

The value of q_m may be determined by the formula

$$q_m = \frac{0.95\ G}{l_1}$$

The factor 0.95 takes into account the concentrated weight of the attachments at the flanges of the longerons plus the weight of the bolts and fillets and was obtained from statistical data on outer wings of longeron construction with a stressed skin. The main attachment of the outer wing with the center wing is along the contour of the skin in which case the skin is generally strengthened at the junction and the distribution of the weight of the outer wing structure along the span is of the type shown in figure 17 where the hatched part represents the weight of the additional strengthening structures at the juncture with the center wing. In this case the attachment of the outer wing with the center wing takes place along the flanges of the longerons, therefore

$$q_m = \frac{0.95 \times 363.0}{8.85} \cong 39.0, \quad n_\Phi = 8$$

whence

$$q_n = \left[13.3 - 11.3\ \frac{l_n}{l_1}\right] 4.33 = 4.33\ \left[A - B\frac{l_n}{l_1}\right]$$

The computation of $I_{p_{c.g.}}$ is conveniently conducted by making use of the following table:

$\frac{l_n}{l_1}$	$B\frac{l_n}{l_1}$	q_n	c_n	$I_{p_{c.g.}}$	$\frac{l_n}{l_1}$	$B\frac{l_n}{l_1}$	q_n	c_n	$I_{p_{c.g.}}$
0	0	57.60	4.10	8.20	0.6	6.78	28.20	2.48	1.48
.1	1.13	52.60	3.81	6.48	.7	7.91	23.30	2.20	.96
.2	2.26	47.80	3.56	5.12	.8	9.04	18.40	1.94	.59
.3	3.39	42.90	3.30	3.96	.9	10.17	13.60	1.66	.32
.4	4.52	37.90	3.02	2.94	1.0	11.30	8.66	1.40	.14
.5	5.65	33.10	2.75	2.05					

The results are plotted and a curve of the mass moments of inertia along the span is obtained.

Determination of The Radii of Inertia of The Outer Wing

If the value of n_Φ is near seven and the weight of the second longeron is near 15 percent of the weight of the outer wing and its weight distribution may be assumed to follow the triangle law, the radius of inertia of the cross sections of the outer wing structure is determined from figure 18. In this example all the preceding conditions are satisfied and therefore the curve of the radii of inertia for the outer wing corresponds to the curve shown on figure 18. If this is not the case, the radii of inertia would have to be determined by the formula

$$i_{c.g.} = 0.288 c_n \sqrt{\frac{q_n}{q_n + \Delta q_{II_n}}} \qquad (46)$$

where

Δq_{II_n} weight of second longeron at section considered

$$\Delta q_{II_n} = \left[a_{II_1} - (a_{II_1} - a_{II_0}) \frac{l_n}{l_1}\right] q_m \qquad (47)$$

The values of the remaining magnitudes are given by formulas (20), (21), and (22).

Translation by S. Reiss,
National Advisory Committee
for Aeronautics.

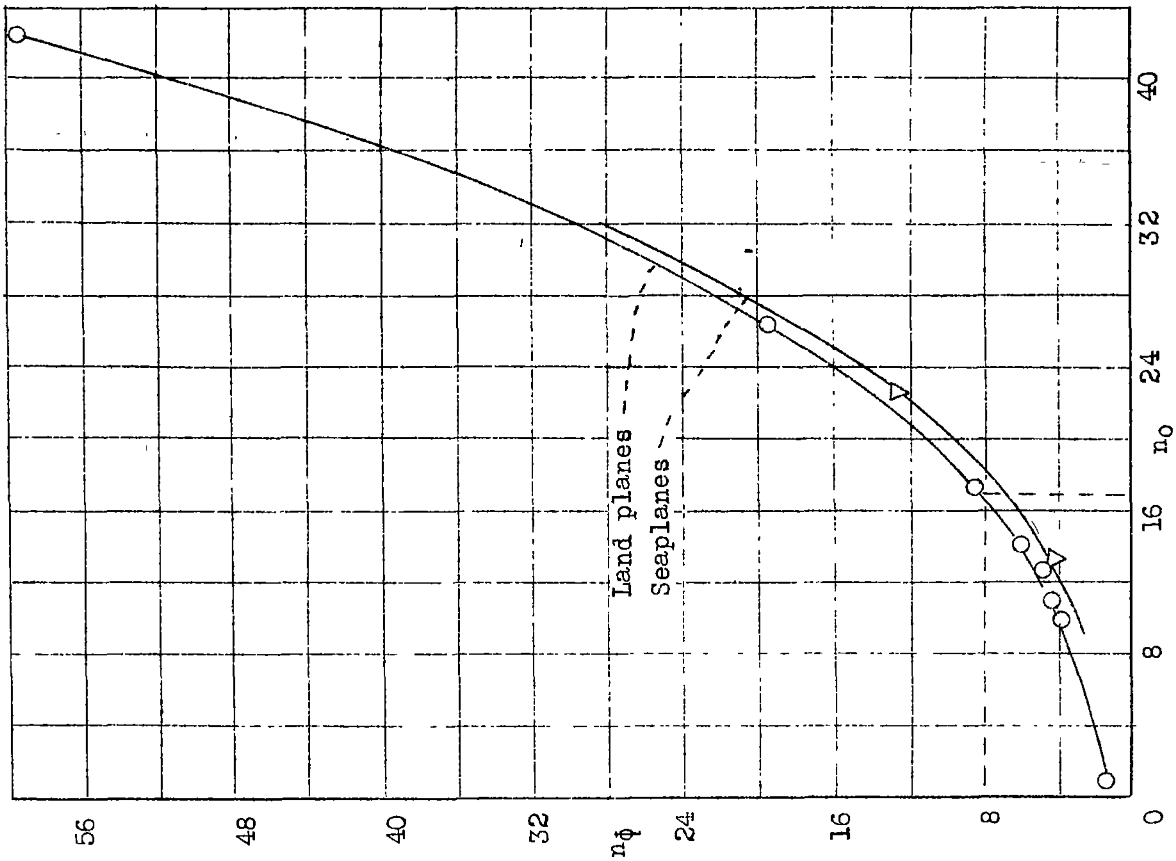

Figure 2

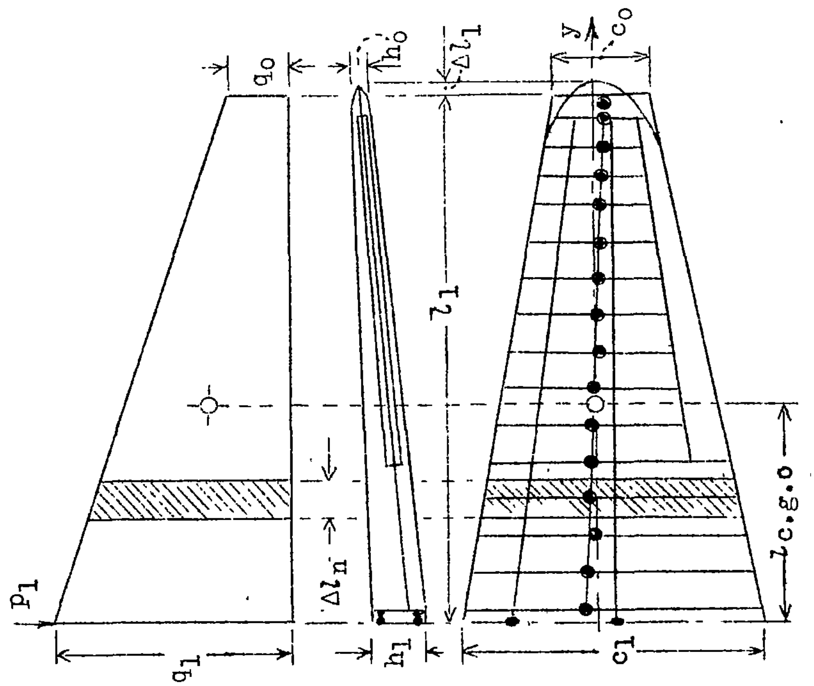

Figure 1

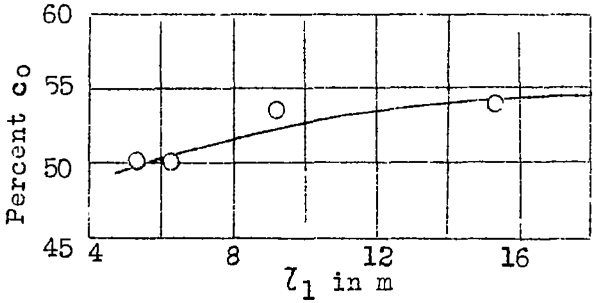

Figure 3

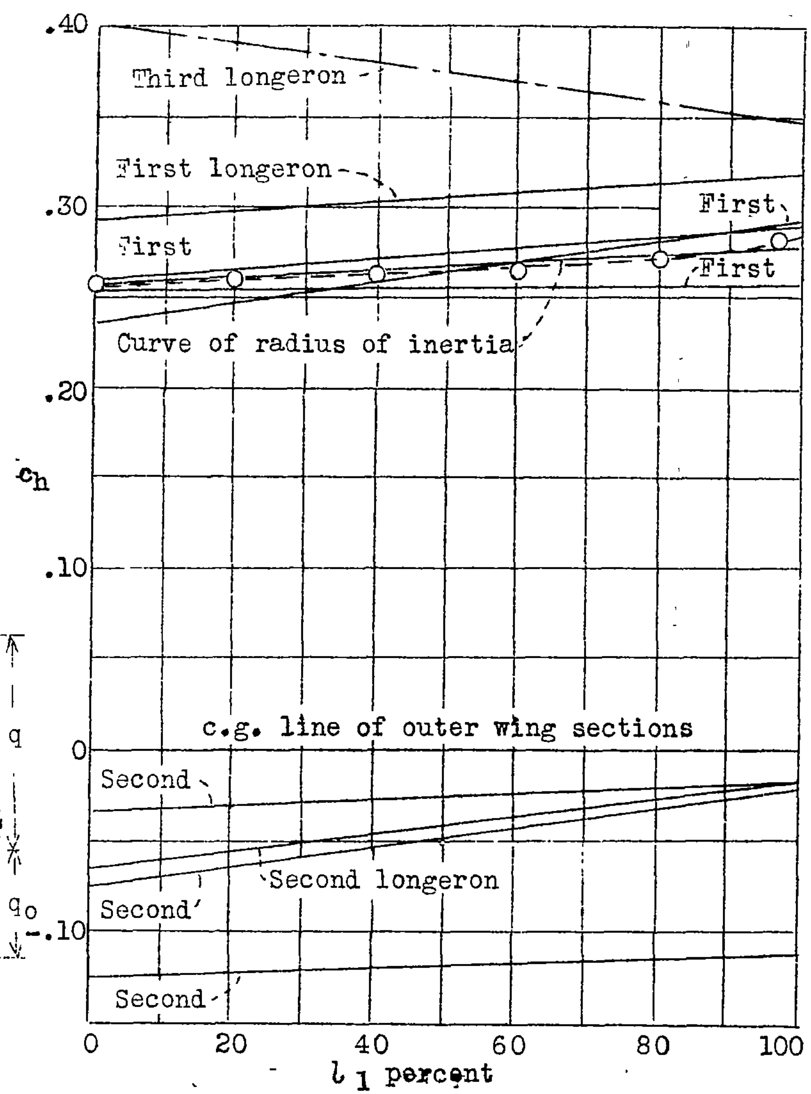

Figure 4

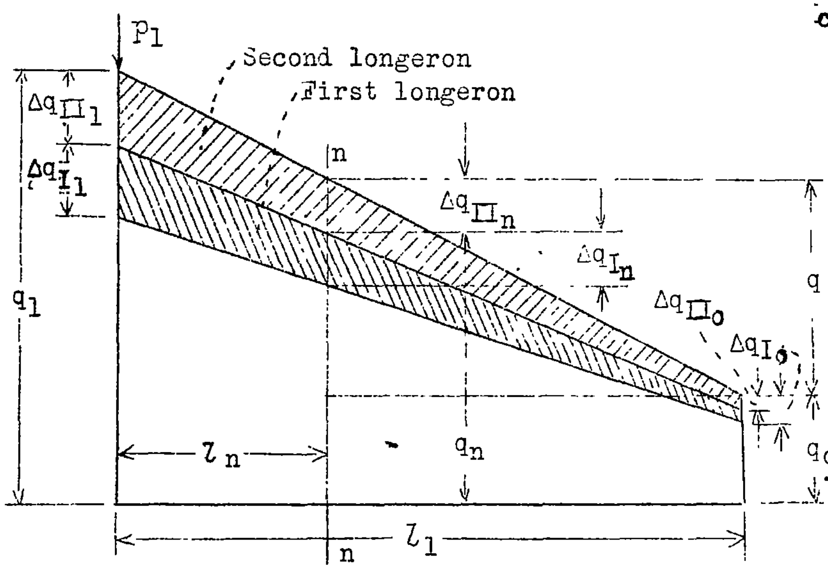

Figure 5

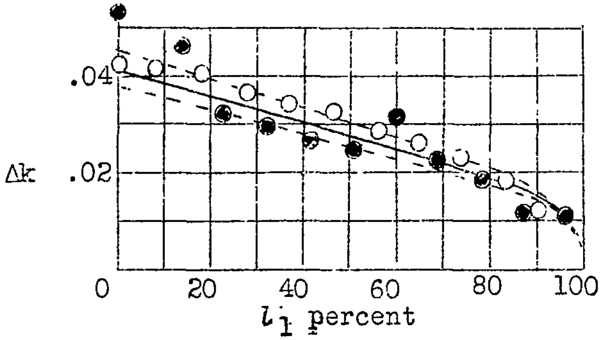

Figure 6

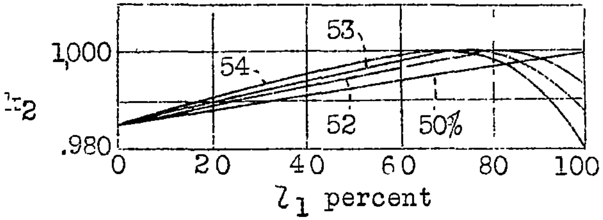

Figure 7

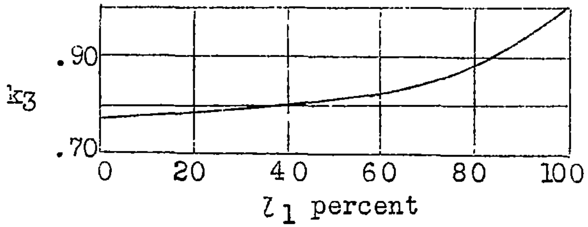

Figure 8

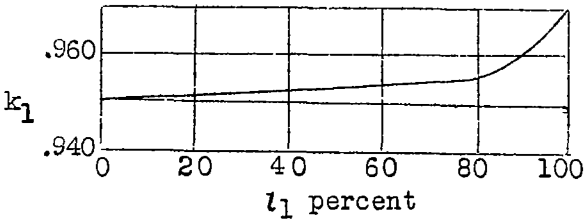

Figure 9

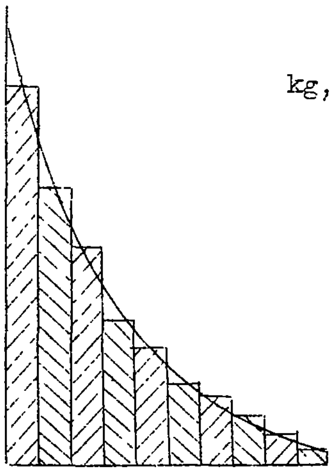
Figure 10

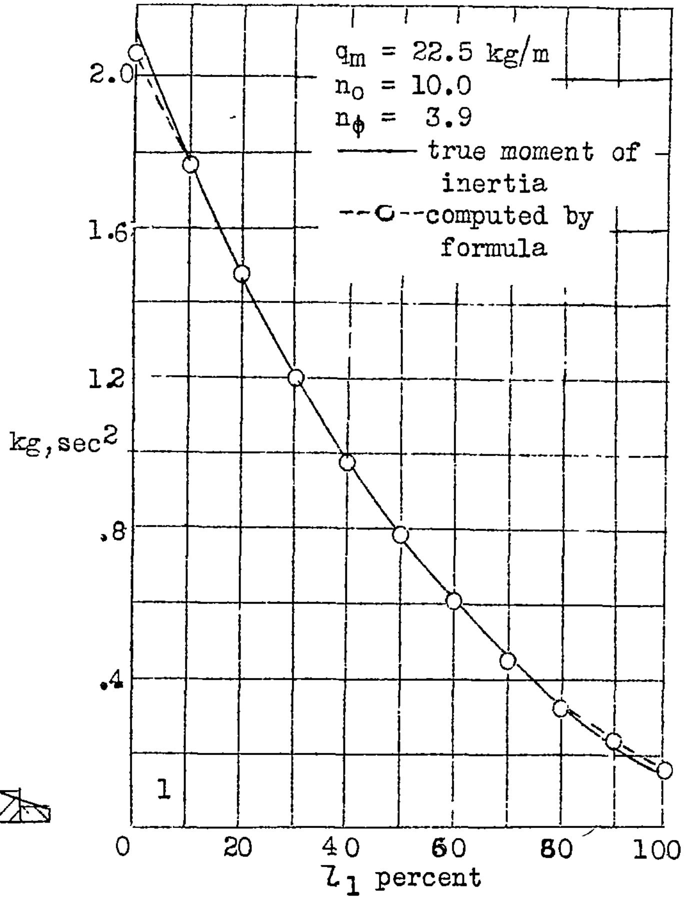

Figure 11

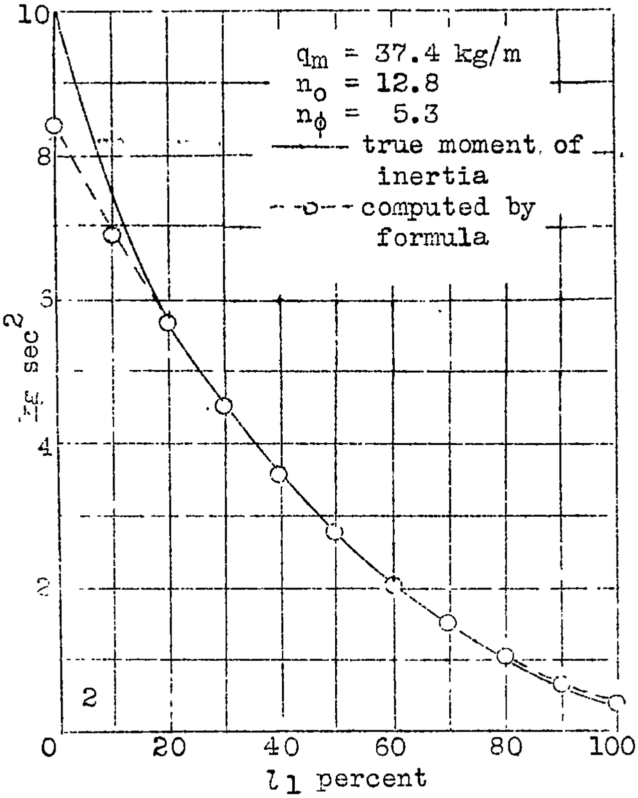

Figure 12

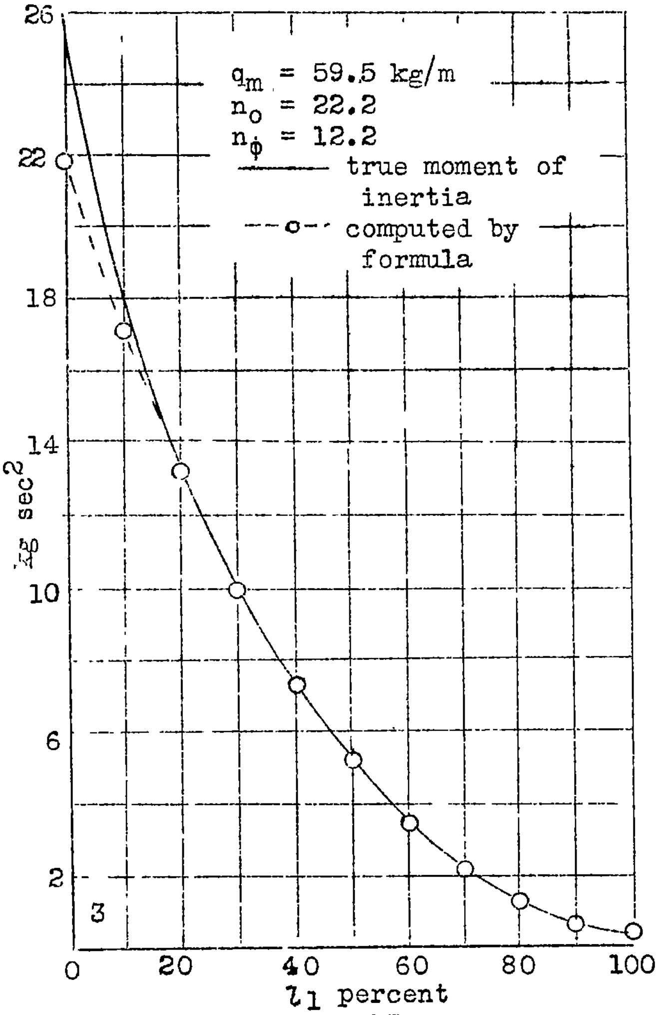

Figure 13

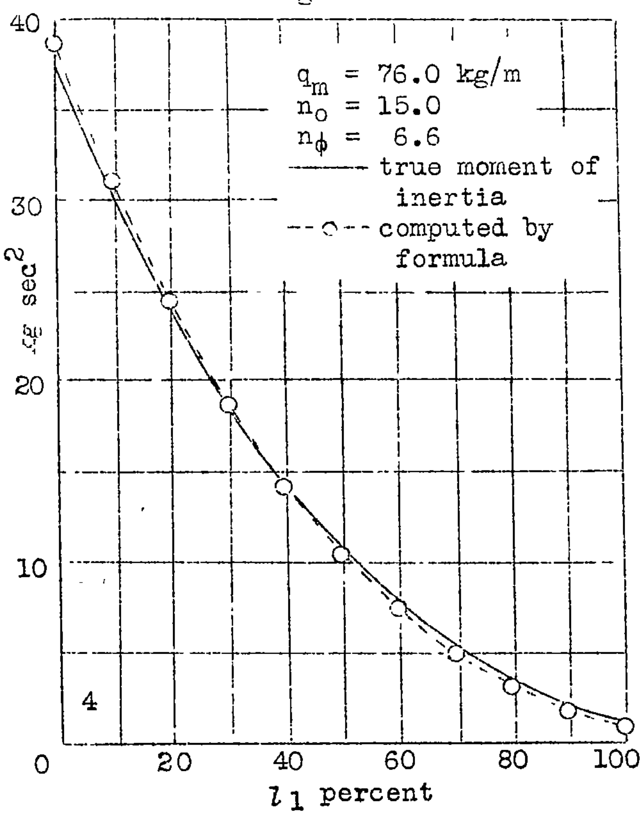

Figure 14

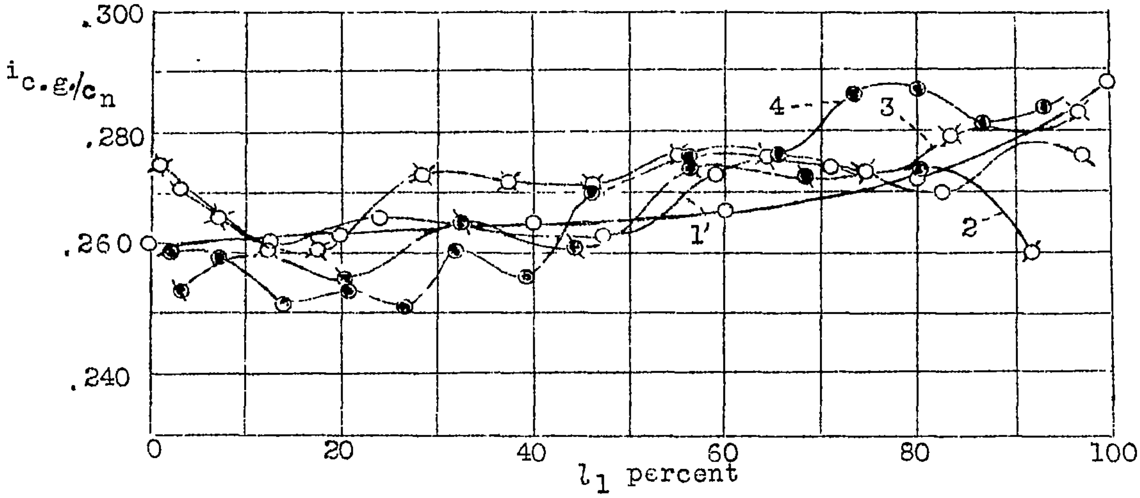

Figure 15

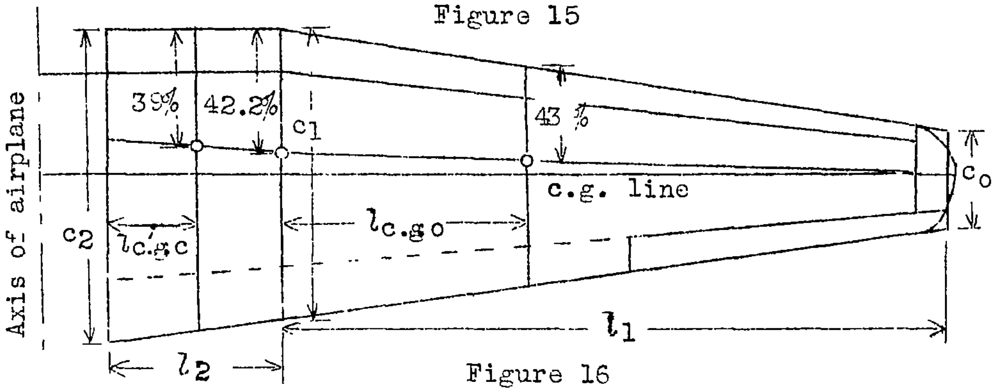

Figure 16

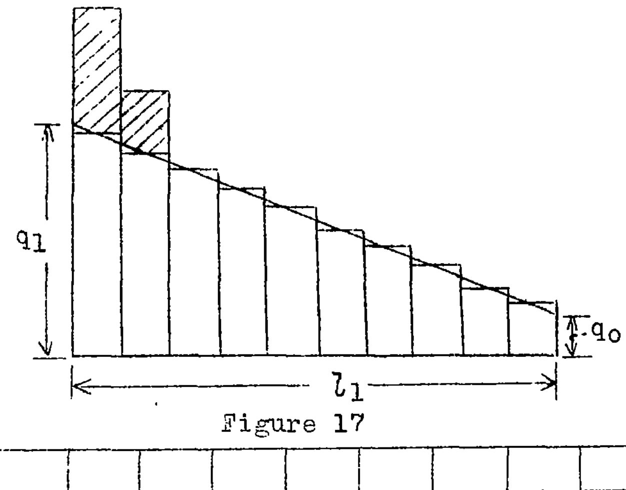

Figure 17

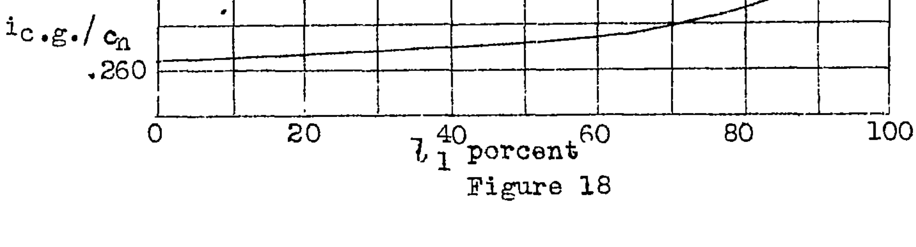

Figure 18