NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM

No. 1074

THE FRICTIONAL FORCE WITH RESPECT TO THE ACTUAL CONTACT SURFACE

By Ragnar Holm

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SUMMARY

Hardy's statement that the frictional force is largely adhesion, and to a lesser extent, deformation energy is proved by a simple experiment.

The actual contact surface of sliding contacts and hence the friction per unit of contact surface was determined in several cases. It was found for contacts in normal atmosphere to be about one-third to one-half as high as the macroscopic tearing strength of the softest contact link, while contacts annealed in vacuum and then tested, disclosed frictional forces which are greater than the macroscopic strength.

INTRODUCTION

The view that the work of friction is largely performed against the adhesion and only to lesser extent against deformation had been expressed by W. B. Hardy (reference 1) in 1920, and proved herewith that the friction does not depend upon whether the sliding surface is smooth or slightly rough. While G. A. Tomlinson (reference 2, p. 905) has supported this concept by a neat comparison between sliding and rolling friction measurements the present report gives a simpler, striking proof for the correctness of this view, and in addition, the calculation of the adhesion per cubic centimeter of actual contact surface corresponding to the friction on several worked-out model problems. It results in adhesion forces which, in

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*"Über die auf die wirkliche Berührungsfläche bezogene Reibungskraft." Wissenschaftliche Veröffentlichungen aus den Siemens-Werken, vol. 17, no. 4, 1938, pp. 38-42.

part are considerably greater than the respective macroscopic tearing strength. While principally concerned with continuous sliding, the motion, unless a favorable lubrication prevails, is probably intermittent (reference 3). For the individual jerks still greater forces (in part elastic) than the mean frictional force are available.

FRICTION TEST WITH MEASURABLE DEFORMATION ENERGY

For the measurement of the coefficient of friction μ, the method of the inclined plane was used. Two metal wires stretched along a plate, formed a track, the inclined plane. The runner also carried on its lower flat surface two identical stretched wires. It was placed with its wires perpendicular to the "rails." Then it was attempted to ascertain the inclination Θ of the track at which the runner remained in motion when it was gently forced from rest. (The runner then usually moved largely without acceleration.) The coefficient of friction for the motion then is

$$\mu = \tan \Theta$$

The new wires sagged a little in one another. At rest microscopically measurable circular impressions prevailed. After one slide the rails had flattened on top in microscopically measurable measure. Through it the work of deformation was computable.

The test was repeated with wires partly new, partly after so many repetitions that a permanent flattening had been reached. This condition was reached after about five slides. The wires, of copper of 1 millimeter thickness, were used after being rubbed clean several hours, or a day, before the test, with cotton moistened with petroleum ether. This ensured a fairly reproducible friction coefficient, which in turn forms an indication of a reproducible surface film. Completely clean metals stick to each other, and where metals slide over each other an external (lubricating) film is invariably responsible for the slidableness. (See references 4 and 5.)

The frictional force A along the path s on the inclined plane is

$$A(s) = P \, s \sin \Theta$$

where P equals constant force.

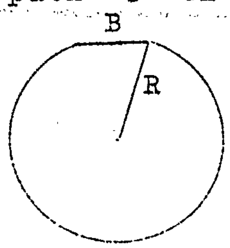

Figure 1.- Flattening of the copper wires.

The work of deformation is computed as follows: Let R denote the wire diameter, B the average flattening along the path s (fig. 1) on the rail wires, and β the corresponding flattening on the runner. The widened area of the runner then has a certain length σ which is greater than B and β, because the runner does not move quite perpendicular to its wires.

The work of deformation A_v is, at the most,* equal to the work performed by the weight P of the runner, when it is first allowed to sink at one point, then - shifted farther for the distance of the average width of impression on runner and rail wire, say by $1/2(B+\beta)$ - allowed to sink in again, and so forth, until the described flattening along the path is secured. Putting $\cos \Theta \approx 1$, the average individual sinking is approximately

$$\frac{B^2 + \frac{\beta^2 \sigma}{s}}{8\,R}$$

while the number of sinking spots is $\frac{2s}{B+\beta}$. Accordingly the work of deformation on path s is, at the most,

$$A_v(s) = P\,\frac{2s}{B+\beta}\;\frac{B^2 + \frac{\beta^2 \sigma}{s}}{8R} = P\,\frac{B^2 s + \beta^2 \sigma}{4R(B+\beta)} \qquad (1)$$

This work disappears after rails and runner are run in. If the adhesion of the surface film has in the meantime experienced no change, the total work of friction must then drop from the amount of A(s) to a lesser amount of about $A(s) - A_v(s)$. In the same proportion the coefficient of friction drops from the initial value μ to the final value μ_v, that is, figuring with $s = 1$:

$$\frac{A_v(1)}{A(1)} = \frac{\mu - \mu_e}{\mu} \qquad (2)$$

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*The resistance to deformation is initially less than P and increases to P during the deformation.

Actually the left side of equation (2) is greater than the right side, partly because A_v was a little too great, as described, partly because the lubricating film has probably changed while running in and the adhesion increased a little. By frequently repeated slides the lubricating film could be damaged so much that μ_e assumed an order of magnitude of 0.4 or more. Table 1 gives two typical measurements, each with new wires.

From the measurements of 1 and 1a:

$$\frac{A_v}{A} = 0.24 \text{ and } \frac{\mu - \mu_e}{\mu} = 0.098$$

From 2 and 2a:

$$\frac{A_v}{A} = 0.20 \text{ and } \frac{\mu - \mu_e}{\mu} = 0.104$$

It is seen that both quantities equal in equation (2) deviate from each other; one is about twice as great as the other. The data indicate that the work of deformation is only of the order of magnitude of 10 to 20 percent of the total work of friction.

Experiments were also made with nickel wires which were so hard that the deformation could not be safely determined after one run; μ was of the same order of magnitude as in the tests with the copper wires, whereas the work of deformation was obviously much less, probably only a few hundredths of the total work of friction.

FRICTIONAL FORCE PER UNIT OF ACTUAL CONTACT SURFACE

The test data compiled in table 1 enable a direct calculation of the actual contact surface. It consists on an average of four equal ellipses with the axes B and β, hence amounts to altogether π, B, β. The normal force is

$$P \cos \Theta = \frac{P}{\sqrt{1 + \mu^2}}$$

Multiplication by μ gives the frictional force and, after further division by π B β, the specific frictional force - that is, the effective frictional force per cubic centimeter of actual contact surface. The thus computed values are shown in table 3.

Further examples give the measurements of the actual contact surface between a graphite brush and a copper ring described in another issue of this journal (reference 5), wherein table 1 also gives the contact pressure p (it averages about 1 t/cm²). The respective specific frictional force, after multiplying by μ, is about equal to 0.3.

Very instructive examples are obtained from friction measurements in air and of the contact resistances (screen resistances) in vacuum after cleaning on copper plates, although the number of contact surfaces could be determined only by comparison with measurements on graphite brushes against copper (reference 6); that is, not direct. The plates for the friction measurements had been previously treated exactly as the crossed wires described earlier in the report. The appended table 2 contains the respective measurements as well as some other data. The actual contact surface is visualized as being divided in n partial surfaces of average magnitude πa^2. The screen resistance R (and contact pressure p, respectively), can be expressed with sufficient accuracy by equations derived elsewhere (reference 6), provided it is borne in mind that a bilateral resistance is involved. The calculation thus proceeds with

$$R = \frac{0.9\ \rho}{2n\ a} = \frac{8 \times 10^{-7}}{n\ a/\text{cm}}\ \Omega \text{ and } \quad p = \frac{0.9\ P}{n\ \pi\ a^2}$$

TABLE 2.- MEASUREMENTS AT THE CONTACT BETWEEN COPPER PLATES

P (kg)	Screen resistance R Ω	Number of partial surfaces, n	Mean radius of part surfaces, a (cm)	Content pressure p (t/cm²)
0.1	1.6×10^{-4}	3.5	0.00143	4.0
2.0	1.7×10^{-5}	20	.00235	5.2

Other interesting examples are found in the measurements of the static friction between pure nickel surfaces in vacuum by Holm and Kirchstein (references 4 and 5) where, of course, the contact pressure p had to be estimated rather than measured. It should be a little lower than the hardness, smaller, say, in about the same proportion as on the contact between copper plates.

Table 3 contains the data of the described measurements, with the yield point or hardness and the tearing strength of the softest contact link as comparison. In the case of the graphite, this tearing strength was, of course, not measured but estimated at one-third of the hardness on the basis of a comparison with other materials.

Consider first cases1 to 4. Here, ψ is always smaller than the tearing strength. A closer connection between the two quantities is not to be expected, as surface foreign films are responsible for ψ. In the case of number 5. involving contact between pure metal surfaces, ψ is much greater than the tearing strength. This may appear absurd at first glance, but presumably ties in with the fact that the macroscopic tearing strength represents no ideal material property, but is dependent upon cracks and other usually occurring defects and is smaller by orders of magnitudes than the respective strength of a body free from defects.

In conclusion a word concerning the elucidation of Coulomb's law of independence of friction coefficient μ from the contact force and the contact surface. This law would apply if p and ψ were material coefficients, whereby the lubricating film itself is counted in with the material to be determined.

Actually p and ψ show a tendency to remain constant in the comparable measurements 1, 3, and 4. The accuracy for its determination is, of course, not as great as for the customary confirmations of Coulomb's law. Thus, while disclaiming a final explanation of this law, it might prove a notable contribution.

Translation by J. Vanier,
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TABLE 1.- FRICTION WITH COMPUTABLE WORK OF DEFORMATION

[Copper wire with $R = 0.005$ cm; $P = 3.24$ kg]

Number	Width of impression		Length of impression σ on runner	Coefficient of friction μ or μ_e	Frictional energy A new wires (g cm)	Deformation A_v	Hardness	Remarks
	B on rails (cm)	β on runner (cm)						
1	0.0135	0.0165	0.023	0.133	425	102	3.6	New
1a	.018	.019	------	.120	----	----	----	Run in
2	.013	.0175	.02	.145	460	93	3.6	New
2a	.018	.020	------	.130	----	----	----	Run in

TABLE 3.- SPECIFIC FRICTIONAL FORCE ψ = FRICTIONAL FORCE PER SQUARE CENTIMETER

Number	Nature of contact	Approximate normal force	Coefficient of friction	Constant pressure	Specific frictional force	Hardness	Tearing strength
1	Crossed Cu-wires	3.2	0.13	4.5*	0.6	3.6	1.5
2	Graphite brush- Cu-ring	1	.3	1	.3	1.4	.46
3	Between Cu-plates	.1	.2	4.0	.8	7.5	3.0
4	Between Cu-plates	2.0	.2	5.2	1.0	7.5	3.0
5	Ni against Ni in vacuum	.004	≈ 4	5	≈ 20	8	2.9

*The fact that p is greater than the original half is a consequence of the hardening due to the deformation.