NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM 1228

BEHAVIOR OF THE LAMINAR BOUNDARY LAYER FOR PERIODICALLY OSCILLATING PRESSURE VARIATION

By August Wilhelm Quick and K. Schröder

Translation of "Verhalten der laminaren Grenzschicht bei periodisch schwankendem Druckverlauf." Ludwig Prandtl zum 70. Geburtstage, Schriften der Deutschen Akademie der Luftfahrtforschung.

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BEHAVIOR OF THE LAMINAR BOUNDARY LAYER FOR PERIODICALLY OSCILLATING PRESSURE VARIATION*

By August Wilhelm Quick and K. Schröder[1]

The calculation of the phenomena within the boundary layer of bodies immersed in a flow underwent a decisive development on the basis of L. Prandtl's trains of thought, stated more than forty years ago, and by numerous later treatises again and again touching upon them. The requirements of the steadily improving aerodynamics of airplanes have greatly increased with the passing of time and recently research became particularly interested in such phenomena in the boundary layer as are caused by small external disturbances. Experimental results suggest that, for instance, slight fluctuations in the free-stream velocities as they occur in wind tunnels or slight wavelike deviations of outer wing contours from the prescribed smooth course as they originate due to construction inaccuracies may exert strong effects on the extent of the laminar boundary layer on the body and thus on the drag. The development of turbulence in the last part of the laminar portion of the boundary layer is, therefore, the main problem, the solution of which explains the behavior of the transition point of the boundary layer. A number of reports in literature deal with this problem, for instance, those of Tollmien, Schlichting, Dryden, and Pretsch. The following discussion of the behavior of the laminar boundary layer for periodically oscillating pressure variation also purports to make a contribution to that subject.

The attempts to calculate such phenomena as undertaken in literature, for instance, by Dryden and Pretsch, were based on the calculation method by Pohlhausen. Very early separation phenomena resulted for very slight variations of the existing pressure distribution; thus there came up for discussion doubts uttered by L. Prandtl among others as to the admissibility of the Pohlhausen method which, due to its using only a single parameter, was possibly not capable of fully embracing all phenomena.

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*"Verhalten der laminaren Grenzschicht bei periodisch schwankendem Druckverlauf." Ludwig Prandtl zum 70. Geburtstage, Schriften der Deutschen Akademie der Luftfahrtforschung, pp. 247-255. (To Ludwig Prandtl upon his 70th birthday, Publications of the German Academy for Aviation Research), Berlin 1945.

[1]The more detailed original report was published under the same title as TM 1257.

Since a method of Schröder recently described elsewhere permits a reliable calculation of the laminar boundary layer, it could be shown with the aid of examples that the boundary layer actually is very sensitive to slight oscillations of pressure and tends easily toward separation.[2]

It was assumed for the calculated examples that the flow concerned is the flow about a plate where the undulation starts only after an initial plane section. After a portion with constant flow velocity $U(s)$ (as customary, let s and n be understood as a system of generally curvilinear tangential and normal coordinates) a sinusoidal $U(s)$-distribution then sets in. The boundary layer calculation was based on dimensionless quantities, with the velocities referred to the constant free stream velocity U_o and the lengths referred to the length of the starting distance L, beginning at the leading edge of the plate.

Figure 1 shows the results of the calculation on an example with the wave length $\lambda = 0.072$ and the maximum fluctuation in the $U(s)$-distribution equalling 1/2 percent. Although the velocity profiles were measured at the position s indicated numerically at each profile a clearer picture was obtained by grouping neighboring profiles according to rising or falling variation of $U(s)$ as indicated by the arrows above each group.

One recognizes from this example that the boundary layer overcomes three waves merely by periodic deformations of the profiles; at the fourth wave, however, a very weak reverse flow begins to appear but subsides again upon increase of $U(s)$, that is, upon pressure drop. However, at the next wave the picture changes completely. A strong reverse flow now appears which subsides only in the profile parts next to the wall, whereas in the central profile parts the reverse flow becomes still stronger.

From this and further fully calculated examples we may conclude that every undulation, even the weakest, in the $U(s)$-distribution finally leads to reverse flow, if only the calculation is carried sufficiently far. Figure 2 shows the corresponding streamline pattern on which one can see particularly clearly the setting-in of the reverse flow and also the formation of a small vortex.

A few remarks concerning the physical interpretation and thus the quantitative evaluation of the calculation results are to follow. The boundary layer actually proves to be extremely sensitive to small periodical oscillations in pressure. Strong variations of the displacement thickness are connected with it. Figure 3 shows the displacement thickness for the example described compared to that for undisturbed pressure distribution; the periodic variation and the considerable amplitudes

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[2]We had the great privilege of several interesting and stimulating discussions with Prandtl on the subject of this report.

of the displacement thickness can be recognized. These periodic variations of the displacement thickness cannot continue without retroaction on the flow since they obviously must be superimposed on the wall undulation and lead to a modification of the external-pressure distribution. This change in the external-pressure distribution can, however, be avoided by amplifying the wall undulation by the amounts of the boundary layer undulation so that for each Reynolds number a new wall results. Thus a boundary layer calculation yields, according to the selected Reynolds number, a series of boundary layer flows along walls of different undulation. In this sense figure 4 illustrates for the present example walls corre-

sponding to Reynolds numbers $R = 10^4$, 10^5, and 10^6. Taken as a basis of comparison, the effective wall, which corresponds to the U(s) distribution for a potential flow, shows an undulation so weak that it is hardly recognizable in the figure. The figure shows further that for small Reynolds numbers a relatively strongly undulated wall is levelled by the boundary layer, whereas for large Reynolds number a relatively weakly undulated wall is equalized by the boundary layer. Hence it follows that for a constant undulation and altered Reynolds number – for instance velocity increase – the boundary layer becomes less effective toward equalizing the wall undulation and thus the effective undulation increases with growing Reynolds number. The starting point of the reverse flow on such a wall should, therefore, travel forward with increasing velocity. The corresponding facts apply to the opposite case of a plane wall with oscillating flow. Here also the starting point of the reverse flow must travel toward the front with increasing velocity when the oscillation of the free stream is held constant. Both behaviors agree with the test experience. In the described manner one succeeds in obtaining data on the behavior of the boundary layer flow on undulated walls, in particular on the setting-in of the reverse flow and thus of a vortex formation in dependence on the Reynolds number.

One obtains a particularly instructive picture of the boundary layer flow by plotting the streamline pattern not in a rectilinear s, n system, but – as in figure 5 – against the wall corresponding to a certain R. The undulation of the streamlines then resulting at the edge of the boundary layer must correspond to the pressure distribution taken as a basis, thus must be practically rectilinear in the present example.

This is being rather well confirmed. Furthermore one can recognize with particular clarity that the flattening of the undulation takes place in the immediate proximity of the wall which is shown by the early smoothed-out course of the streamlines. One can also see that the customary concept of the superimposability of a wall contour on the displacement thickness is admissible, since this flattening of the streamlines has been completed at a distance from the wall in the order of magnitude of the displacement thickness.

In figure 3 a comparison of the drag conditions of the undulated with those of the plane plate has been performed which shows that in this example a local drag reduction of about 25 percent occurs; this reaction is caused by the fact that the regions where the shearing stress is reduced by the undulation compared to the plane plate outweigh the regions where it is increased. However, it is not advisable to utilize this effect in other than regions with pressure drop where there is no danger of a premature boundary layer transition (caused by the undulation) to turbulent state.

The calculations and considerations performed in a more voluminous report and given here in the form of an extract may be summarized in the following conclusions:

1. The laminar boundary layer proves to be actually extremely sensitive to slight variations in the pressure distribution, and tends easily toward separation.

2. The start of the reverse flow may be calculated as a function of the Reynolds number and the pressure oscillation; the retroaction of the undulation of the displacement thickness on the pressure distribution must be taken into consideration.

3. The results of the calculation, in agreement with test results, lead to the interpretation that the transition of the boundary layer from laminar to turbulent is caused by the onset of reversal flow followed by a vortex formation which, in turn, may be produced by fluctuation of the free stream and by wall roughness. If a monotonously increasing pressure rise exists, the point of transition, caused by additional pressure oscillation, will lie generally ahead of the separation point of the laminar boundary layer which results by calculation with the undisturbed pressure distribution.

Translated by Mary L. Mahler
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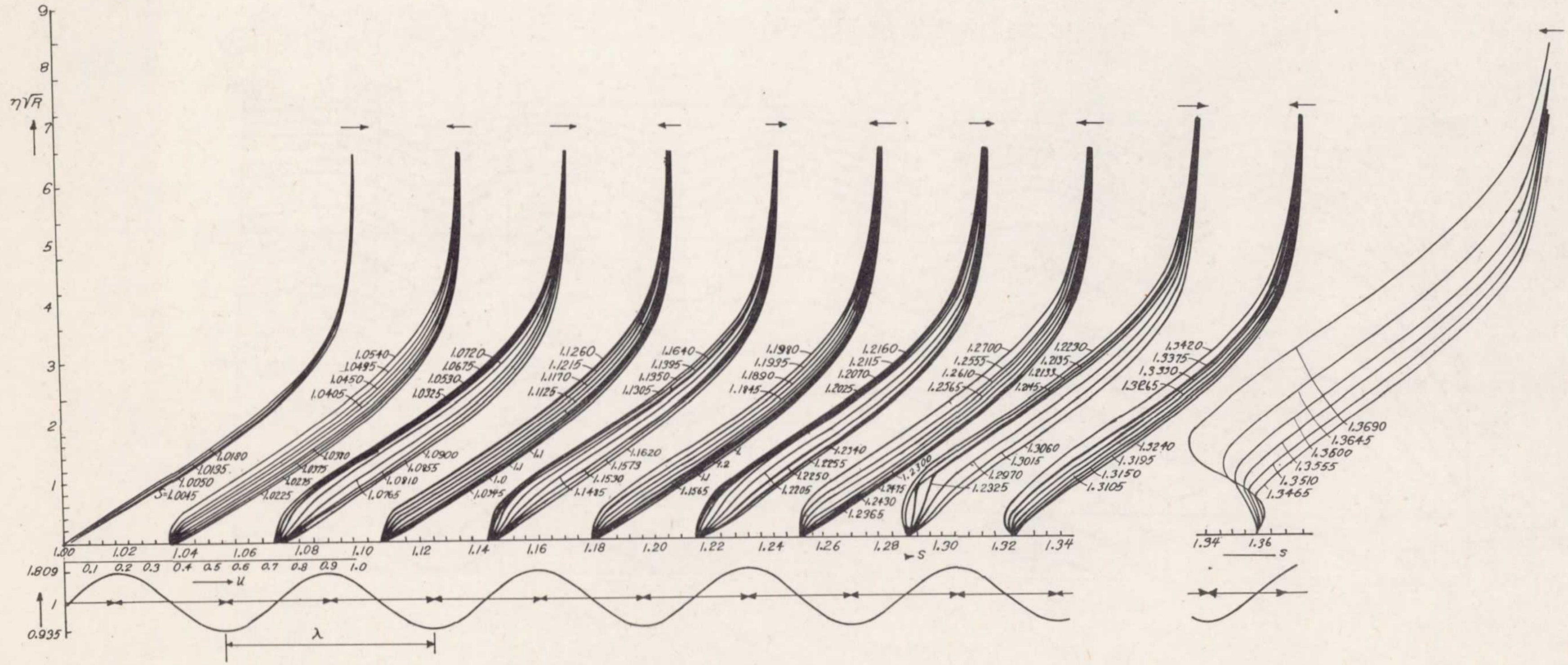

Figure 1.- Distribution of the boundary layer profiles.

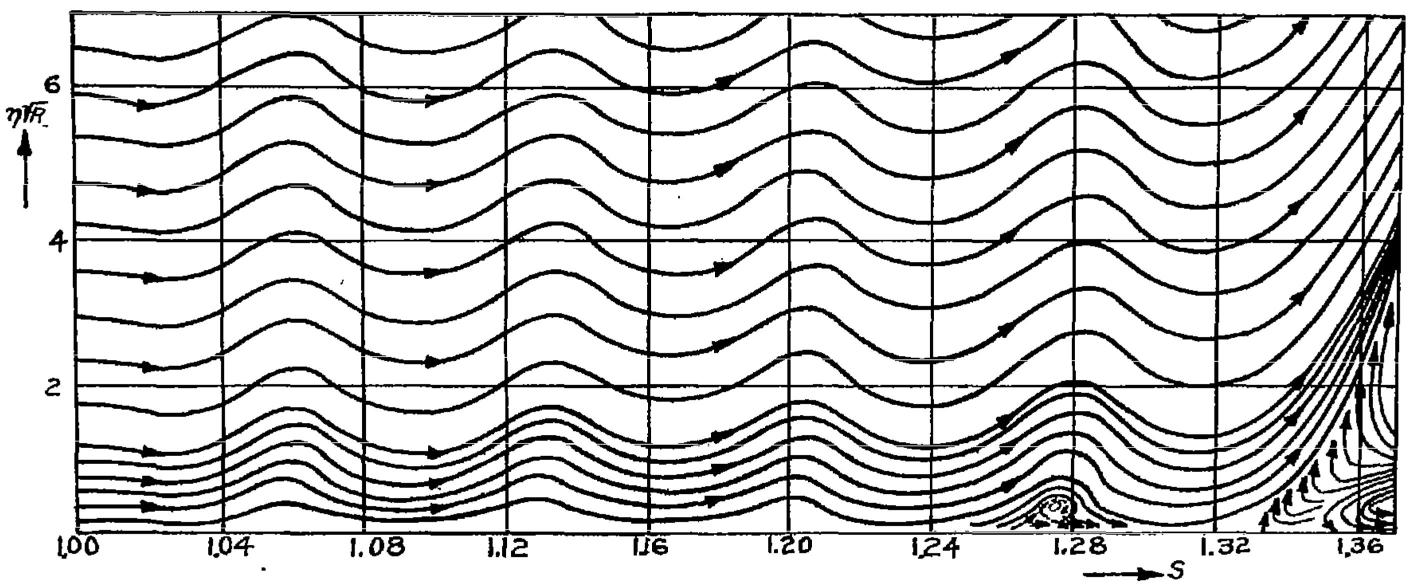

Figure 2.- Distribution of the streamlines.

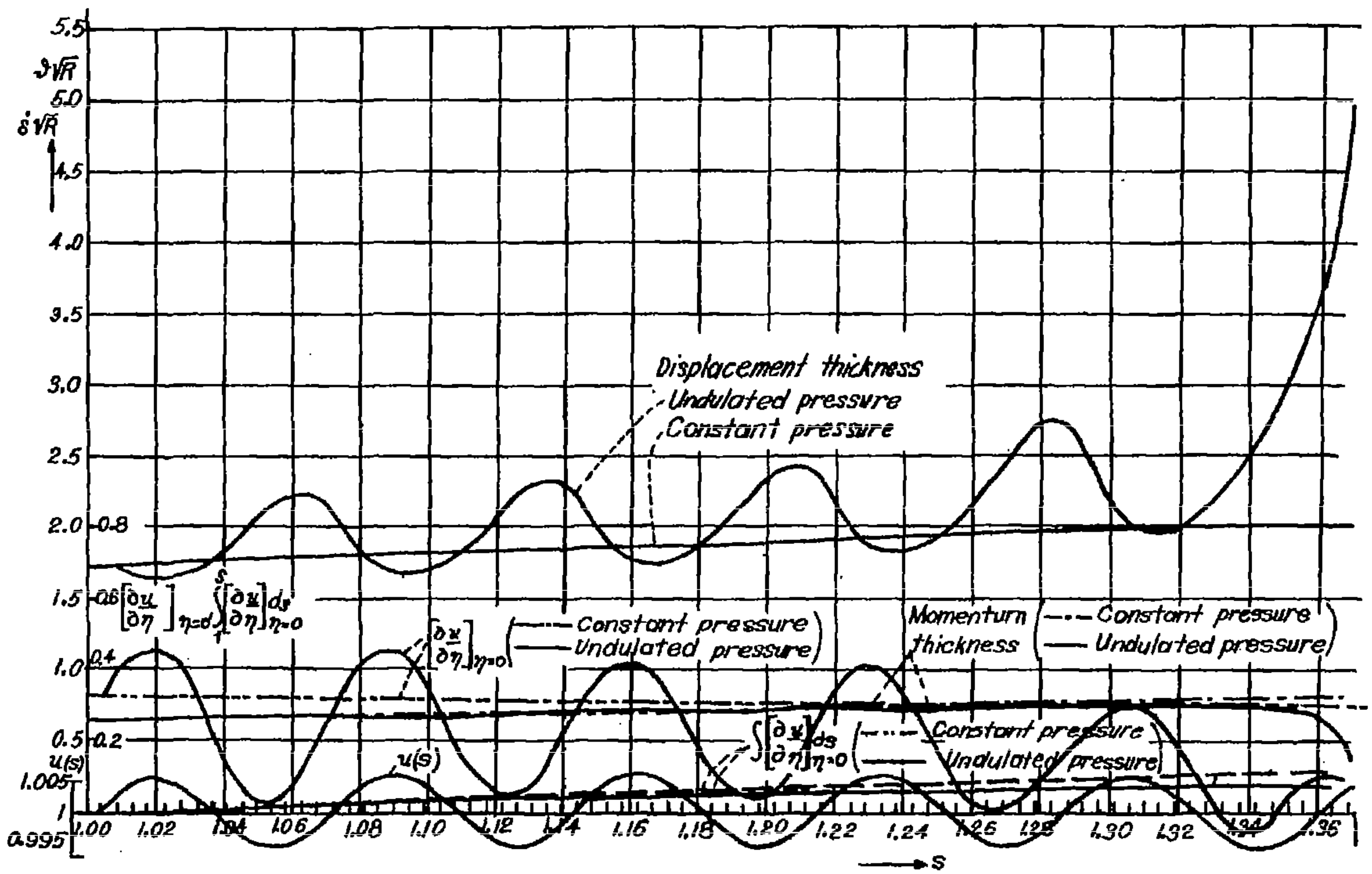

Figure 8.- Distribution of the displacement thickness, the momentum thickness, and the shearing stress.

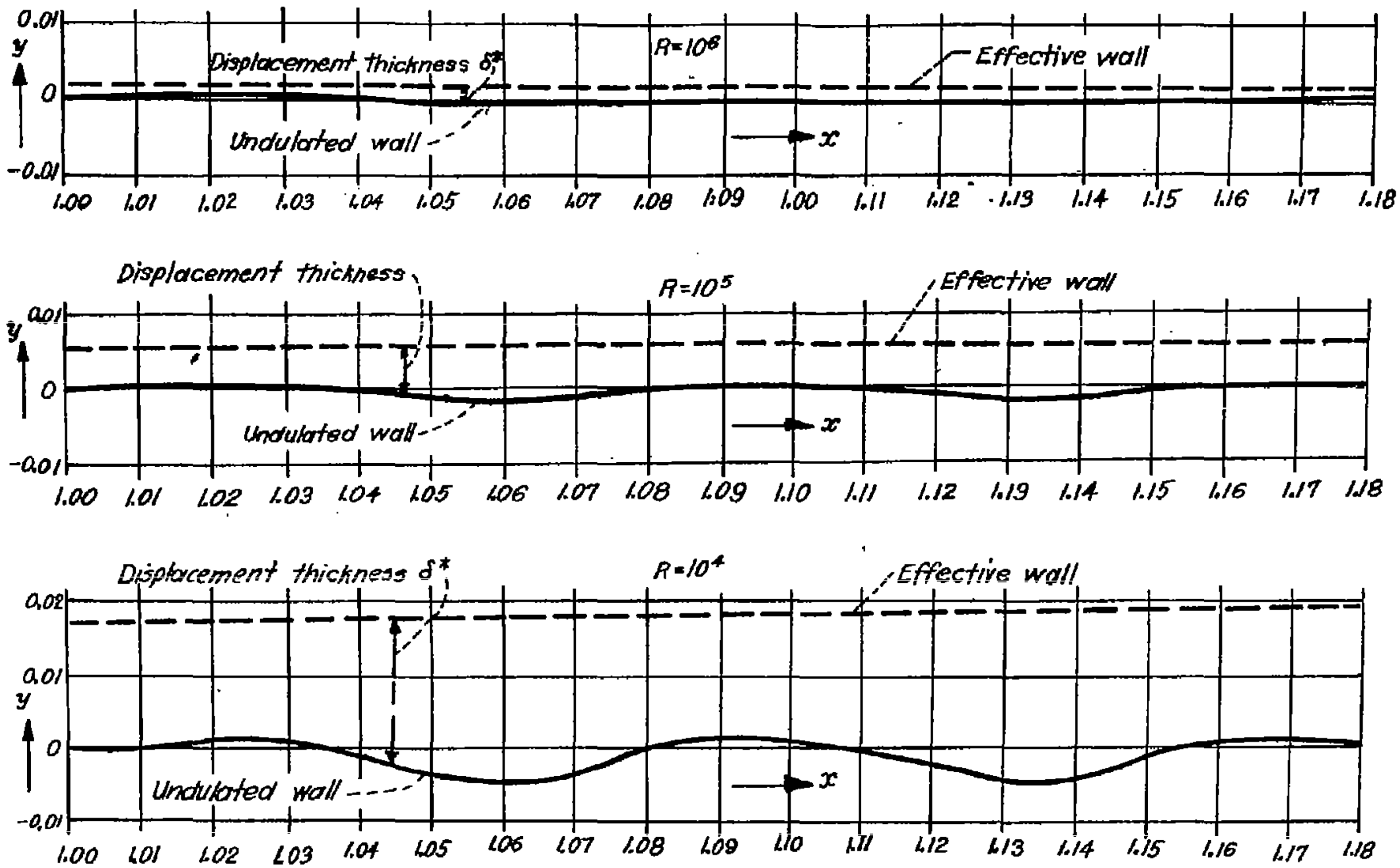

Figure 4.- Distribution of wall undulation and displacement thickness for various Reynolds numbers.

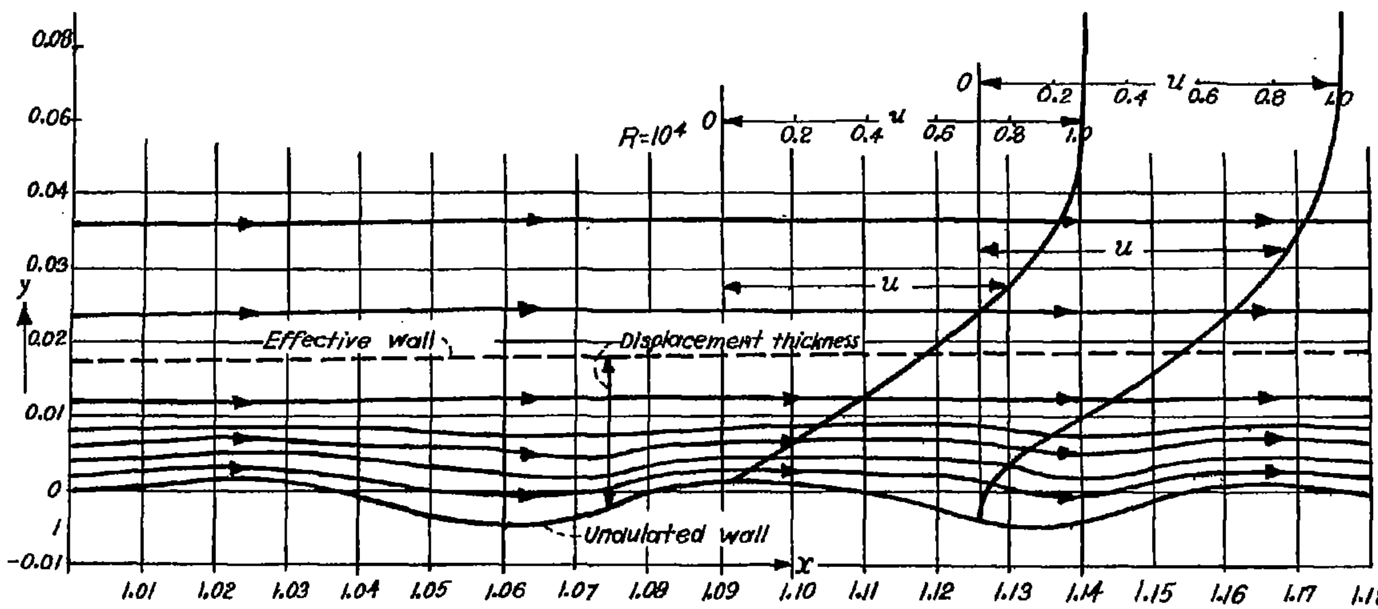

Figure 5.- Distribution of streamlines at the undulated wall.

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