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By H. Pfriem

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SUMMARY

The effect of cyclic gas pressure variations on the periodic heat transfer at a flat wall is theoretically analyzed and the differential equation describing the process and its solution for relatively small pressure fluctuations developed, thus explaining the periodic heat cycle between gas and wall surface.

The processes for pure harmonic pressure and temperature oscillations, respectively, in the gas space are described by means of a constant heat transfer coefficient (α_ω) and the equally constant phase angle (ϵ_α) between the appearance of the maximum values of the pressure and heat flow most conveniently expressed mathematically in the form of a complex heat transfer coefficient (α_c, equations (12),(20)). Any cyclic pressure oscillations, can be reduced by Fourier analysis to harmonic oscillations, which result in specific, mutual relationships of heat-transfer coefficients and phase angles for the different harmonics.

The heat transfer between gas and cylinder wall of reciprocating engines of any type is important for their functioning and dependability in service. The amount of heat transfer depends upon a number of factors, such as the geometric shape, the gas and wall temperatures, the gas velocity, and so forth. The present investigation deals with the effect of a periodic compression of gas on the periodic heat transfer. Proceeding from considerably simplified assumptions, the extent to which the heat of compression developed in the boundary layer immediately adjacent to the wall surface affects the amount of heat passing periodically into the wall is indicated.

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*"Der periodische Wärmeübergang bei kleinen Druckschwankungen." Forschung auf dem Gebiete des Ingenieurwesens, vol. 11, no. 2, March-April 1940, pp. 67-75.

INTRODUCTION

The internal heat transfer between gas and cylinder wall in reciprocating engines especially of the internal combustion type is of primary importance for their functioning and dependability. Owing to the cyclic operating method on all reciprocating engines, the internal heat transfer varies with respect to time, hence, nonuniformly. Two substantially different time intervals must be identified during a complete working cycle: namely, the heat transfer in the cylinder, while pressure balance exists with the outer atmosphere, and the heat transfer with cylinder closed (reference 1). At pressure balance with the outer atmosphere the internal heat transfer is largely conditional upon aerodynamic points of view. During the time interval in which the operating cylinder is closed, the heat transfer is, in addition to the flow processes, considerably affected also by the compression or expansion and the combustion of the fuel.

The following deals primarily with the effect of cyclic compression on the periodically changing and steady heat transfer. In consequence of the compression there occurs in each element of the gas space the heat of compression which in part serves to raise the gas temperature (internal energy) of that space element and in part is carried off through the cylinder walls. Since this heat of compression originates in the same way in the adhering boundary layer directly adjacent to the wall surface also where it is immediately removed by the cooling effect of the wall, and effect of the heat transfer, especially in high-speed reciprocating engines must be definitely expected. The subsequent considerations proceed from simplified assumptions, so that at first only the anticipated effect at low, periodic compression can be computed. A more accurate calculation later on provides the details of the process at higher compression.

The first of the studies dealt with two-dimensional temperature fields, where the boundary layer thicknesses in question are in general small compared to the curvature of the wall surfaces (so far as corners and edges are ignored). Then, if a very thin gas film alongside an isotherm within the boundary layer is considered (see fig. 1), the thermal processes taking place therein can

be described by the first law of thermodynamics. Visualizing this very thin gas film as having such a large section that it momentarily encloses the unit weight of the gas, the first law reads by way of example:

$$dQ = di - Av\,dP \tag{1}$$

where

Q (kcal/kg) heat input introduced per unit weight of thin gas film

i (kcal/kg) heat content (enthalpy)

v (m^3/kg) specific volume

P (kg/m^2) absolute pressure in the gas film

A (kcal/mkg) mechanical equivalent of heat

The heat volume dQ remaining in the thin gas film follows from the difference of the heat volume introduced and removed, respectively, by thermal conduction, if the transmission by radiation can be ignored.

$$dQ = -F\,\Delta q\,d\tau = F\Delta\left(\lambda\frac{\partial\vartheta}{\partial x}\right)d\tau$$

where

F (m^2) cross-sectional area of the gas film

$\Delta q = -\Delta\left(\lambda\frac{\partial\vartheta}{\partial x}\right)\left[\frac{kcal}{m^2h}\right]$ the difference of the heat flow immediately before and behind the gas film

$\lambda\left[\frac{kcal}{mh\,^{\circ}C}\right]$ thermal conductivity of the gas in the film

ϑ (°C) increase of temperature of the gas film against a zero point chosen at random

x (m) position coordinate perpendicular to the cross-sectional area of the gas film

τ (h) time

The cross-sectional area F of a gas film of small thickness Δx is found, with the assumption, that it shall enclose the unit weight of the gas, from: $F \Delta x = v$. At disappearing thickness of this gas film, the amount of heat introduced is

$$dQ = v \frac{\partial\left(\lambda \frac{\partial \vartheta}{\partial x}\right)}{\partial x} d\tau$$

The variation of the heat content of the gas film is on the other hand: $di = c_p \, d\vartheta$

where

c_p(kcal/kg°C) specific heat of the gas at constant pressure

Posting these relations in the equation (1) affords:

$$v \frac{\partial\left(\lambda \frac{\partial \vartheta}{\partial x}\right)}{\partial x} d\tau = c_p d\vartheta - A v dP$$

The above developments retain their validity, even if the gas volume enclosed in the considered gas element is assumed arbitrarily small. Therefore, the above equation remains correct for each smallest gas particle (mass particle), so far as it does not solely enclose individual molecules. Transforming this relation, applicable for the present to the mass point, by means of its velocity $w = dx/d\tau$ in heat flow direction into the usual trilinear coordinates, and assuming equal pressure in all gas particles we get:

$$\frac{c_p}{v}\left[\frac{\partial \vartheta}{\partial \tau} + w \frac{\partial \vartheta}{\partial x}\right] = \frac{\partial\left(\lambda \frac{\partial \vartheta}{\partial x}\right)}{\partial x} + A \frac{dP}{d\tau} \qquad (2)$$

The form of this differential equation of second order is that of the conventional differential equation of heat conduction with convection flow and three dimensionally

divided heat sources. The first term on the left side in this differential equation defines the amount of heat required at instantaneous temperature changes (due to the heat capacity of a gas particle) while the second term represents the heat volume transported by the motion of the gas particles in the direction of the temperature gradient - that is, the heat transfer by convection in the gas. The first term on the right side of the equation represents the difference in heat volume introduced and removed, respectively, of the gas particle by pure heat conduction, while the second term gives the amount of heat released by compression. It is seen that this heat of compression occurring in the space element independent of the quantity of heat removal is solely dependent on the momentary pressure variation.

Subsequently, solutions of this differential equation are developed for the specific case of low cyclic compression in a gas space directly in front of a flat wall.

EFFECT OF LOW CYCLIC COMPRESSION ON THE PERIODIC HEAT TRANSFER

Pressure fluctuations in gas at rest (infinitely thick boundary layer.)- In the simplest case of a low cyclic pressure fluctuation compared to the absolute pressure of gas, the gas properties can be regarded as independent of the pressure and the temperature. Moreover, the velocity w perpendicular to the wall surface can be considered as disappearingly small. The equation (2) can then be simplified to the following linear differential equation of the second order:

$$\frac{\partial \vartheta}{\partial \tau} = a_1 \frac{\partial^2 \vartheta}{\partial x^2} + \frac{A v_1}{c_{p_1}} \frac{dP}{d\tau} \tag{3}$$

where

$a = \frac{\lambda v}{c_p} \left[\frac{m^2}{h}\right]$ thermal conductivity and the subscript 1 indicates the material properties of the gas, the subscript 2 those of the wall

Since arbitrary periodic fluctuations can always be reduced to harmonic variations by series development according to Fourier, the ensuing considerations are restricted to these.

<u>The temperature field in the gas space</u>.- The first solution concerns the following problem: Visualize a large gas space in front of a relatively thick wall with flat surface and in this space harmonic pressure variations. To be found is the periodic heat transfer at the wall surface for any required frequency of pressure fluctuation, when the gas is practically at rest or when a very thick laminar boundary layer exists at the wall surface. The pressure oscillation is to be presented by the following equation*:

$$P_C = P_1 + \Delta P e^{j\omega\tau} \tag{4}$$

where

P_1 (kg/m²) mean time value of the pressure

ΔP (kg/m²) amplitude of the pressure oscillation

ω (1/h) natural frequency of the pressure oscillation

$j = \sqrt{-1}$ fictitious unit

The subscript C signifies that the considered quantity is complex.

With the origin of the place coordinates pointing toward the wall and at right angles to its surface, (see fig. 1): A partial solution of the differential equation (2) for a periodically changing temperature field in the gas space before the wall. (See reference 2 for principal data for the development of this solution.)

$$\vartheta_C = \frac{A\, v_1 \Delta P}{c_{p_1}} e^{j\omega\tau} \left[1 - \frac{1 - \rho_{12}}{2} e^{\psi_1 x}\right]$$

whence with the gas equation $Pv = RT$

$$\vartheta_C = \frac{\kappa - 1}{\kappa} \frac{\Delta P}{P_1} T_1 e^{j\omega\tau}\left[1 - \frac{1 - \rho_{12}}{2} e^{\psi_1 x}\right] \tag{5}$$

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*Of all the complex solutions, the real or imaginary part by itself presents momentarily a physically possible solution. The following calculations are worked out largely complex by utilization of the conventional method (see reference 1); but they can also be worked out real without

T_1 (°K) mean, absolute temperature of the gas

ρ_{12} reflection number of the temperature heat waves

In addition:

$$\kappa = \frac{c_p}{c_v};\ \Delta R = c_p - c_v;\ \psi_1 = \sqrt{\frac{j\omega}{a_1}} = (1+j)\sqrt{\frac{\omega}{2a_1}};\ \rho_{12} = \frac{b_1 - b_2}{b_1 + b_2} \tag{5a}$$

where

$b = \sqrt{\lambda\ c\ \gamma}$, $(\mathrm{kcal/m^2 h^{1/2}\ ^oC})$ heat stress capacity

The still arbitrary zero point of the increase of temperature of the gas is chosen equal to the momentary mean value of the gas temperature at the particular point. The correctness of this solution can be readily checked by introduction into the differential equation (3).

<u>The temperature field in the wall</u>.- Since there are no heat sources within the wall, the temperature field obeys the differential equation of heat conduction for solid bodies:

$$\frac{\partial t}{\partial \tau} = a_2\ \frac{\partial^2 t}{\partial x^2} \tag{6}$$

where

t (°C) increase of temperature of the wall relative to its momentary mean time value

Its solution in the present case is:

$$t_c = \frac{A v_1\ \Delta P}{c_{p_1}}\ e^{j\omega\tau}\frac{1+\rho_{12}}{2}\ e^{-\psi_2 x} = \frac{\kappa - 1}{\kappa}\ \frac{\Delta P}{P_1}\ T_1 e^{j\omega\tau}\ \frac{1+\rho_{12}}{2}\ e^{-\psi_2 x} \tag{7}$$

It is easily proved that the limit conditions at the wall surface are actually fulfilled by the equations (5) and (7), as for $x = 0$; $\vartheta_{c_0} = t_{c_0}$ and $-\lambda_1\left(\frac{\partial \vartheta_c}{\partial x}\right)_0 = -\lambda_2\left(\frac{\partial t_c}{\partial x}\right)_0$

(Continued from page 6) especial difficulties, with some larger calculating effort. For the calculation with complex values see for example B. H. G. Möller. Treatment of oscillation problems. Leipzig 1937.

<u>Heat flow and temperature difference between gas and wall</u>.– Having seen from the foregoing, that the equations (5) and (7) actually represent the solution of the problem, the periodically varying heat flow q any chosen point in the gas space or in the wall can be predicted without difficulties. By way of example, equation (5) affords for the gas space:

$$q_c = -\lambda_1 \frac{\partial \vartheta_c}{\partial x} = \frac{\kappa - 1}{\kappa} \frac{\Delta P}{P_1} T_1 e^{j\omega\tau} \lambda_1 \psi_1 \frac{1 - \rho_{12}}{2} e^{\psi_1 x} \tag{8}$$

with q indicating the heat flow at point x in the gas space.

According to equations (5) and (7) it also affords the momentary difference between the temperature $\vartheta_{c\infty}$ of gas at great distance from the wall and that of the wall surface t_{c_0} $(x = 0)$ at

$$\vartheta_{c\infty} - t_{c_0} = \frac{\kappa - 1}{\kappa} \frac{\Delta P}{P_1} T_1 e^{j\omega\tau} \frac{1 - \rho_{12}}{2} \tag{9}$$

The heat volume q_{c_0} penetrating periodically in the wall surface per unit time per unit surface is according to equation (8):

$$q_{c_0} = \frac{\kappa - 1}{\kappa} \frac{\Delta P}{P_1} T_1 \frac{1 - \rho_{12}}{2} b_1 \sqrt{j\omega}\ e^{j\omega\tau} \tag{10}$$

Thus with equation (9) and $\sqrt{j} = e^{j(\pi/4)}$:

$$q_{c_0} = b_1 \sqrt{\omega}\ e^{j(\pi/4)} (\vartheta_{c\infty} - t_{c_0}) \tag{11}$$

<u>The heat transfer coefficient</u>.– Defining as in steady temperature fields, a heat transfer coefficient for harmonically varying processes according to relations: $q_c = \alpha_c(\vartheta_c - t_c)$, a comparison with equation (11) furnishes the complex heat transfer coefficient α_c describing the periodic heat transfer in the present instance in the form:

$$\alpha_c = b_1 \sqrt{\omega}\ e^{j(\pi/4)} = \alpha_\omega\ e^{j\epsilon_\alpha} \tag{12}$$

from which the absolute value α_ω of this complex heat transfer coefficient α_c for the frequency ω of the pressure oscillation at:

$$\alpha_\omega = b_1 \sqrt{\omega} \tag{12a}$$

and its phase angle

$$\epsilon_\alpha = \pi/4 \tag{12b}$$

It is plain from equation (12a) that the heat volume periodically transferred to the wall during harmonic pressure oscillations in a gas at rest is proportional to the root of the frequency of the pressure oscillation and to the heat stress factor b_1 of the gas. The maximum value of the heat flow hereby lies independent of the frequency of the pressure oscillation, always by the amount of the phase angle $\epsilon_\alpha = \pi/4$ ahead of the maximum value of the temperature difference. The two phenomena are not summarily predictable on the basis of the experience with steady heat transfer. But they find their physically plain explanation in the fact that heat sources exist also under the effect of compression in the gas films adhering directly to the wall surfaces, the heat of which, owing to the great capacity of the wall, is immediately carried off even during formation. The yield of these heat sources, that is, the compression heat produced per unit time, is proportional to the frequency of the pressure oscillation. But, since on the other hand, the rate of diffusion of heat sources is only proportional to the square root of the frequency, an increase in the oscillation frequency number is reflected by a continuously decreasing part of the gas space before the wall subjected to their heat dissipating effect. As a result the amount of heat actually transmitted per unit time under otherwise identical conditions is proportional to the square root of the frequency; correspondingly it affords the same relationship also for the coefficient of heat transfer.

The advance by the phase angle $\pi/4$ of the heat transfer in respect to the temperature in the gas core is lastly due to the fact that the greatest yield of the heat sources in the gas extending directly up to the wall

surface, that is, the momentarily greatest heat of compression per unit time, coincides with the steepest pressure rise, and hence the temperature oscillation in the gas core leads by the phase angle $\pi/2$.

Since gas films even at a certain distance away from the wall surface still contribute to the heat flow and these portions need time to reach the surface, the maximum value of the heat flow must in any case lead by a phase angle ranging between 0 and $\pi/2$ relative to the temperature oscillation; the calculation proves this by the phase angle $\epsilon_\alpha = \pi/4$ independent of the frequency and the nature of the material. To simplify the identification of the periodic heat transfer between a fixed wall and a gas at rest in front of it which manifests harmonically variable, small pressure oscillations the two quantities α_ω and ϵ_α are sufficient. Quantity α_ω multiplied by the greatest value of temperature difference, gives the amplitude of heat flow and ϵ_α is the phase angle, corresponding to the time difference between the maximum value of the harmonically variable temperature difference and that of the heat flow. These two quantities are most conveniently expressed in the form of a complex heat transfer coefficient α_c, which while affording no new physical knowledge provides a suitable representation.

With the application of two real quantities α_ω and ϵ_α for the description of a harmonically variable heat transfer, the ratio between the usually not coincident maximum values of heat flow and the temperature oscillation, as well as their relative temporary displacement $\zeta = \epsilon_\alpha/\omega$ is established. The attempt to express these heat-transfer ratios by the conventional representation with one real value and concurrently existing instantaneous values, results in spite of the continuously finite magnitude of the passing heat volumes and the temperature differences in heat-transfer coefficients, which fluctuate between $+\infty$ and $-\infty$ during a period.

Nonharmonic temperature - heat oscillations can be directly reduced to harmonic ones by Fourier analysis, each upper harmonic affording a new α_ω and ϵ_α which in the present instance of cyclic compression can be solved by means of equations (12), (12a), and (12b), respectively. It is plain, that each upper harmonic

manifests the same phase angle $\epsilon_\alpha = \pi/4$ and has a greater heat-transfer coefficient α_ω corresponding to the root of the multiple of its frequency relative to the fundamental oscillation.

<u>Numerical example and range of validity</u>.- The problem illustrating the orders of magnitude is as follows: Harmonic sound waves with a frequency of 500 Hertz strike a flat wall placed in atmospheric air. Owing to the relatively great wave length a pressure gradient perpendicular to the wall surface is discounted. With a heat stress factor of $b_1 = 0.08$ kcal/m^{2}h$^{1/2}$ °C, according to equation (12a) for air under normal condition the magnitude of the heat-transfer coefficient, is:

$$\alpha_\omega = 0.08 \sqrt{500 \times 2\pi \times 3600} = 8 \times 33.4 = 267 \text{ kcal/m}^2\text{h }^\circ\text{C}$$

This heat-transfer coefficient caused by the compression alone in air at rest is therefore of a magnitude obtainable by steady heat transfer in gas only at very high velocities but not with gas at rest. This fact itself is indicative of the great influence of the compression on the periodic heat transfer in piston compressors and heat engines.

To judge the practical admissibility of the assumption regarding the laminar flow in the gas films directly in front of the wall, the depth effect of the wall influence upon the compressed gas is analyzed. The gas temperature, at a distance $x \geqq \xi$ varies according to equation (5) by less than 1 percent from that at very great distance from the wall, if

$$\frac{1 - \rho_{12}}{2} \left| e^{-\psi_1 \xi} \right| \lesseqgtr 0.01, \text{ that is, when } \xi \gtrapprox \sqrt{\frac{a_1}{\omega}} \ln \frac{1 - \rho_{12}}{0.02} \text{ (m)}$$

However, according to equation (5a) the approximation $\rho_{12} \approx -1 + 2\, b_1/b_2$ is admissible for the reflection factor ρ_{12} at a metallic wall for which always $b_2 \gg b_1$ - thus for an iron wall, for instance $b_2 \approx 200$ kcal/m^{2}h$^{1/2}$ °C for atmospheric air against iron:

$$\rho_{12} \approx -1 + 2\,\frac{0.08}{200} = -0.9992 \approx -1.$$

For atmospheric air the temperature conductivity factor is $a_1 \approx 0.07\ m^2/h$; hence for a frequency of 500 Hertz

$$\xi \gtrapprox \sqrt{\frac{0.07}{500 \times 2\pi \times 3600}}\ \ln 100 \approx 0.365 \times 10^{-3}\ m$$

that is, the wall effect is in many practical cases confined to the still-existing boundary layer, even in larger flows. This proves the practical admissibility of the simple assumptions of these developments for many cases. But, in order to be able to make reliable predictions at substantially smaller oscillation numbers as corresponds to customary rotative speeds in engine design the considerations are extended to include the case of a boundary layer of finite thickness.

Pressure oscillations by turbulent gas core (finite thickness of boundary layer).- In the following calculation the actual boundary layer flow (which in general shows no sharp delimitation against the gas space and also is not always completely laminar) is replaced by a corresponding, idealized boundary layer, manifesting a pure laminar flow parallel to the wall surface and an expressed sharp boundary toward the completely turbulent gas space. The idealization is required only in consideration of a simple calculation; it furnishes no essentially different results from the practical conditions, since similar conditions prevail even by partly turbulent boundary layer. As a result of the assumed complete turbulence in the gas core the temperature in it will always be the same at any point; hence it must always be lower than the adiabatic compression temperature by reason of the heat removal through the boundary layer.

Temperature field and heat flow in boundary layer.- The temperature field within such an ideal boundary layer of constant thickness is visualized as being composed of two portions, as follows. The first portion is produced as the result of the temperature variations in the turbulent gas core, purely on the basis of the heat conductibility of the boundary layer visualized free from heat sources and can be presented by the following equation:

$$\vartheta_c^{(1)} = \Theta e^{j\omega\tau} \frac{e^{-(\delta+x)\psi_1} + \rho_{12}\, e^{-(\delta-x)\psi_1}}{1 + \rho_{12}\, e^{-2\delta\psi_1}} = \Theta e^{j\omega\tau} \frac{e^{-x\psi_1} + \rho_{12}\, e^{+x\psi_1}}{e^{+\delta\psi_1} + \rho_{12}\, e^{-\delta\psi_1}}$$

where

Θ (°C) amplitude of swing of the temperature in the turbulent gas space

δ (m) thickness of the idealized boundary layer

The second portion is the result of the heat sources distributed in the boundary layer with the stipulation that at point $x = -\delta$ the function is always zero and adapts itself at the wall surface $x = 0$ to its reflection conditions.

$$\vartheta_c^{(2)} = \frac{\Theta_0 e^{j\omega\tau}}{1 + \rho_{12} e^{-2\delta\psi_1}} \left[1 + \rho_{12} e^{-2\delta\psi_1} - \frac{1 - \rho_{12}}{2} \left\{ e^{x\psi_1} - e^{-(2\delta + x)\psi_1} \right\} - \left\{ e^{-(\delta + x)\psi_1} + \rho_{12} e^{-(\delta - x)\psi_1} \right\} \right] - \delta \leqq x \leqq 0$$

where

Θ_0 (°C) amplitude of swing of the gas temperature at pure adiabatic compression.

A comparison of this relation with equation (5) gives for $\delta \rightarrow \infty$ the magnitude of the amplitude Θ_0 at

$$\Theta_0 = \frac{\kappa - 1}{\kappa} T_1 \frac{\Delta P}{P_1}$$

The actual temperature field in the boundary layer is obtained as the sum of these two portions, where the momentary time average of the gas temperature is again chosen as zero point of the increase of temperature.

$$\vartheta_c = \frac{\Theta_0 e^{j\omega\tau}}{1 + \rho_{12} e^{-2\delta\psi_1}} \left\{ 1 + \rho_{12} e^{-2\delta\psi_1} - \frac{1 - \rho_{12}}{2} \left[e^{x\psi_1} - e^{-(2\delta + x)\psi_1} \right] - \left(1 - \frac{\Theta}{\Theta_0} \right) \left[e^{-(\delta + x)\psi_1} + \rho_{12} e^{-(\delta - x)\psi_1} \right] \right\};$$

$$-\delta \leqq x \leqq 0 \qquad (13)$$

The temperature oscillation at point $x = -\delta$ follows at:

$$(\vartheta_c)_{-\delta} = \Theta \, e^{j\omega\tau}; \quad x \leqq -\delta \tag{13a}$$

The gas temperature directly at the wall surface $x = 0$ is on the other hand:

$$(\vartheta_c)_0 = \frac{\Theta e^{j\omega\tau}}{1 + \rho_{12} e^{-2\delta\psi_1}} \; \frac{1 + \rho_{12}}{2} \left\{ 1 + e^{-2\delta\psi_1} - \left(1 - \frac{\Theta}{\Theta_0}\right) 2 e^{-\delta\psi_1} \right\} \tag{13b}$$

In addition the heat flow within the boundary layer is represented according to equation (13) by:

$$q_c = -\lambda_1 \frac{\partial \vartheta_c}{\partial x} = \frac{\Theta_0 e^{j\omega\tau} b_1 \sqrt{j\omega}}{1 + \rho_{12} e^{-2\delta\psi_1}} \left\{ \frac{1 - \rho_{12}}{2} \left[e^{+x\psi_1} + e^{-(2\delta + x)\psi_1}\right] - \left(1 - \frac{\Theta}{\Theta_0}\right) \left[e^{-(\delta + x)\psi_1} - \rho_{12} \, e^{-(\delta - x)\psi_1}\right] \right\}; \quad -\delta \leqq x \leqq 0 \tag{14}$$

whence the heat volume transferred to the wall with $x = 0$ is:

$$(q_c)_0 = \frac{\Theta_0 e^{j\omega\tau} b_1 \sqrt{j\omega}}{1 + \rho_{12} e^{-2\delta\psi_1}} \; \frac{1 - \rho_{12}}{2} \left\{ 1 + e^{-2\delta\psi_1} - \left(1 - \frac{\Theta}{\Theta_0}\right) 2 e^{-\delta\psi_1} \right\} \tag{14a}$$

The heat volume removed from the turbulent gas space is with $x = -\delta$ according to equation (14):

$$(q_c)_{-\delta} = \frac{\Theta_0 e^{j\omega\tau} b_1 \sqrt{j\omega}}{1 + \rho_{12} e^{-2\delta\psi_1}} \left\{ (1 - \rho_{12}) \, e^{-\delta\psi_1} - \left(1 - \frac{\Theta}{\Theta_0}\right) \left[1 - \rho_{12} e^{-2\delta\psi_1}\right] \right\} \tag{14b}$$

Temperature field and heat flow in the wall.— Finally in conjunction with equation (13b) the temperature oscillations in the fixed wall can be represented by

$$t_c = \frac{\Theta_0 e^{j\omega\tau}}{1 + \rho_{12} e^{-2\delta\psi_1}} \frac{1 + \rho_{12}}{2} \left\{ 1 + e^{-2\delta\psi_1} - \left(1 - \frac{\Theta}{\Theta_0}\right) 2 e^{-\delta\psi_1} \right\} e^{-x\psi_2}; \quad x \geqq 0 \qquad (15)$$

whence the heat oscillation at the wall surface $x = 0$:

$$(q_c)_0 = -\lambda_2 \left(\frac{\partial t_c}{\partial x}\right)_0 = \frac{\Theta e^{j\omega\tau} b_2 \sqrt{j\omega}}{1 + \rho_{12} e^{-2\delta\psi_1}} \frac{1 + \rho_{12}}{2} \left\{ 1 + e^{-2\delta\psi_1} - \left(1 - \frac{\Theta}{\Theta_0}\right) 2 e^{-\delta\psi_1} \right\} \qquad (15a)$$

It is plain from equations (13) to (15a) that all boundary conditions at points $x = 0$ and $x = -\delta$ are fulfilled. Herewith the equations (13) and (15) respectively, represent the solutions of equations (3) and (6) for the given problem. It merely involves the determination of the temporarily unknown amplitude Θ of the temperature of the turbulent gas space.

The temperature oscillation in the turbulent gas core.— The magnitude of this amplitude of the temperature in the turbulent gas core is found by application of the first law of thermodynamics corresponding to equation (1) to the total turbulent gas space. Hereby is:

$$\frac{dQ}{d\tau} = -(q_c)_{-\delta} \frac{O_t v_1}{V_t} = -\frac{v_1 (q_c)_{-\delta}}{s}; \quad \frac{di}{d\tau} = c_{p_1} \frac{\partial(\vartheta_c)_{-\delta}}{\partial\tau}$$

where

O_t (m^2) total heat-removing surface of the turbulent gas space

V_t (m^3) volume of the turbulent gas space

$s = V_t/O_t$ (m) substitute layer thickness of the turbulent gas space

In conjunction with equations (13a), (14b), and (5) there is thus afforded after a few elementary transformations:

$$\left(1 - \frac{\Theta}{\Theta_0}\right) = \frac{(1 - \rho_{12})e^{-\delta\psi_1}}{(1 - \rho_{12}e^{-2\delta\psi_1}) + s\psi_1(1 + \rho_{12}e^{-2\delta\psi_1})} \qquad (16)$$

From this it is readily apparent that the temperature oscillation of the turbulent gas core is intimately related to $\delta\psi_1 = \delta\sqrt{\frac{j\omega}{a_1}}$ - that is, to the thickness δ and the temperature conductivity factor a_1 of the boundary layer as well as to the natural frequency ω of the oscillation. Moreover, the substitute film thickness s of the turbulent gas space is also of great influence. Equation (16) then affords in general a complex value for certain conditions. This signifies physically that the temperature oscillations in the gas core due to the heat diffusion does not take place in the same phase as the pressure oscillation.

The heat transfer onto metal walls.- At the transmission of heat between gas and metal wall the reflection factor $\rho_{12} = -1$, whence equation (16) gives with sufficient accuracy for technical cases:

$$\left(1 - \frac{\Theta}{\Theta_0}\right) = \frac{2e^{-\delta\psi_1}}{(1 + e^{-2\delta\psi_1} + s\psi_1(1 - e^{-2\delta\psi_1})} \qquad (16a)$$

The temperature oscillation in the turbulent gas space is therefore according to equation (13a)

$$(\vartheta_c)_{-\delta} = \Theta_0 e^{j\omega\tau}\frac{(1 - e^{-\delta\psi_1})^2 + s\psi_1(1 - e^{-2\delta\psi_1})}{(1 + e^{-2\delta\psi_1}) + s\psi_1(1 - e^{-2\delta\psi_1})};\ x \leqq -s;\ \rho_{12} = -1 \qquad (17)$$

Correspondingly we get with $\rho_{12} = -1$ according to equation (13b) the temperature at the wall surface $x = 0$ at:

$$(\vartheta_c)_0 = (t_c)_0 = 0 \tag{18}$$

The heat volume transferred to the wall can be formed according to equations (14a) and (16a) from

$$(q_c)_0 = \frac{1 - e^{-2\delta\psi_1} + s\psi_1(1 + e^{-2\delta\psi_1})}{1 + e^{-2\delta\psi} + s\psi_1(1 - e^{-2\delta\psi_1})} \Theta_0 e^{j\omega\tau} b_1 \sqrt{j\omega} \tag{19}$$

Then the complex heat transfer coefficient is obtained for the periodic heat transfer at low harmonic compression of a turbulent gas with the natural frequency ω before a metal wall from:

$$\alpha_c = \frac{(q_c)_0}{(\vartheta_c)_{-\delta} - (t_c)_0} = \frac{\lambda_1}{\delta} \frac{\delta\psi_1[1 - e^{-2\delta\psi_1} + (s/\delta)\delta\psi_1(1 + e^{-2\delta\psi_1})]}{(1 - e^{-\delta\psi_1})^2 + (s/\delta)\delta\psi_1(1 - e^{-2\delta\psi_1})} \tag{20}$$

For great values of $\delta\psi_1 = \delta\sqrt{\frac{\omega}{a_1}}$ this relation changes as is readily seen to the previously developed equation (12) for infinitely thick boundary layer. On the other hand, for very small values of $\delta\sqrt{\frac{\omega}{a_1}}$

$$\alpha_c \approx \frac{\lambda_1}{\delta} \frac{1 + (\delta/s)}{1 + 1/2\ (\delta/s)} \tag{20a}$$

on the assumption that $\frac{\delta^2\omega}{a_1} << 1$.

For $\psi_1 = 0$ corresponding to a natural frequency of $\omega = 0$, that is, for the limit case of steady heat transfer, equation (20) is exactly correct:

$$\alpha_{c_0} \equiv \alpha_0 = \frac{\lambda_1}{\delta}\left[1 + \frac{\delta}{\delta + 2s}\right] \tag{20b}$$

To this steady limit case there corresponds a uniform distribution of steady heat sources in the entire gas space inclusive of the boundary layer before the fixed wall. The additional term $\delta/\delta + 2s$ in equation (20b) represents the effect of these heat sources existing in the boundary layer. This limit value of the

complex heat transfer coefficient for a disappearingly small frequency is therefore in full accord with the real heat transfer coefficient α_0 to be expected from corresponding stationary tests.

In general $\delta << s$ in technical cases hence the heat transfer coefficient according to equation (20):

$$\alpha_c \approx \frac{\lambda_1}{\delta} \frac{\delta\psi_1(1 + e^{-2\delta\psi_1})}{1 - e^{-2\delta\psi_1}} = \frac{\lambda_1}{\delta} \frac{(\delta\psi_1)}{\tanh.(\delta\psi_1)} \quad ; \quad s >> \delta \qquad (20c)$$

Theoretically the case of $\delta >> s$ is also of importance, in which case according to equation (20):

$$\alpha_c \approx \frac{\lambda_1}{\delta} \frac{\delta\psi_1(1+e^{-\delta\psi_1})}{1-e^{-\delta\psi_1}} = 2\frac{\lambda_1}{\delta} \frac{\left(\frac{\delta\psi_1}{2}\right)}{\tanh\left(\frac{\delta\psi_1}{2}\right)} \quad ; \quad \delta >> s \qquad (20d)$$

Between these limit curves for α_c given by equations (20c) and (20d) lie all practically possible values of the heat transfer coefficients for low cyclic compression of gas for finite thickness of boundary layer.

<u>Graphic representation of the results</u>.- In figure 2 the curves, according to equations (20) to (20d), for finite parameter (δ/s) in Gauss' numerical plane are plotted dimensionless in form affording a complex Nusselt number $Nu_c = \alpha_c\delta/\lambda$ entirely corresponding to the steady heat transfer. The magnitude of this complex number $|Nu_c| = \alpha_\omega\delta/\lambda$ is momentarily given as distance between the origin and the point of the numerical plane, which is determined by the parameter $\delta\sqrt{\frac{\omega}{2a_1}}$ and (δ/s). The phase angle ϵ_α of the complex heat transfer coefficient α_c agrees with that of Nu_c and can therefore be taken direct as geometric angle between this distance and the axis of the real numbers.

In retracing the curve of the complex Nusselt number for a certain ratio (δ/s) under otherwise identical conditions in relation to the frequency, the following obtains: For $\omega = 0$ the complex characteristic agrees

with the corresponding real one for steady heat transfer. At very low, but finite frequencies the portion of the heat volume removed from the gas core still predominates; the amount of the heat-transfer coefficient therefore barely varies, and the phase angle $\epsilon_{\alpha'}$ remains for the time being very small. With increasing amplitude, the heat of compression produced within the boundary layer becomes consistently more important. Since it is partially produced direct at the wall surface and immediately passes to the wall surface even while being formed, the phase angle must ultimately reach greater values and progressively tend toward the previously computed limit value $\pi/4$. In correspondence with the rising importance of the heat of compression within the boundary layer, the value of $\alpha_\omega \delta/\lambda$ itself increases with the frequency. The heat-volume removal from the turbulent gas core on the contrary becomes smaller with increasing amplitude, as is readily apparent from equation (16) where the temperature of the turbulent gas core consistently approaches the adiabatic-compression temperature.

Discussion of the results.- The present calculations manifest good agreement with the physical observation. They enable the numerical prediction of periodic heat volume transferred to a flat wall at low cyclic compression of gas, when a boundary-layer flow of arbitrary, constant thickness exists at the surface of the wall. The assumption of a flat wall is technically fulfilled in almost all cases, since the boundary-layer thickness is almost always very small in comparison to the curvature radius of an uneven wall. The boundary-layer thickness itself can usually be caculated or estimated from known heat-transfer coefficients for steady heat transfer according to the relation $\delta = \lambda_1/\alpha_0$.

The Fourier analysis affords for any periodic pressure variation a sum of pure harmonic pressure oscillations for each of which a complex heat transfer coefficient can be obtained; the dissimilar heat transfer coefficients being arranged methodically. As the thickness of the boundary layer, its material properties and the size of the turbulent gas space are the same for all harmonics it simply results in a relationship with its ordinal number (frequency). In figure 2 the end points of all vectors, which represent the complex heat transfer coefficients for

coefficients for any chosen periodic pressure variation lie therefore on a curve δ/s = constant. Then, if one of these is known δ/s can be determined, and on the basis of this singular relationship all the other values can be obtained. In general the heat-transfer coefficient for the frequency $\omega = 0$, that is, the coefficient existing at normal, steady heat transfer is probably known.

In one point the foregoing assumptions are admittedly not entirely realized physically. For the simplification of the differential equation (2) it had been assumed that the gas particles within the range of a finite temperature gradient execute no movement perpendicular to the wall surface. This is especially untrue of the gas particles of the boundary layer, since they – even in surfaces oblique or parallel to the general direction of the compression – by their restricted freedom of motion due to the viscosity are preponderately compressed perpendicularly to the wall surface.

This defect, while producing no essential change in the existing data of the periodic heat transfer at small pressure oscillations is on the other hand of great importance for the heat volume to be removed in a reciprocating engine. In subsequent developments the effect of the periodic motion of the gas particles immediately before the wall in direction perpendicular to the wall surface is to be explored thoroughly. In another article to be published in the near future, the action of this influence is for the time being computed indirectly from the energy loss required to maintain periodicity.

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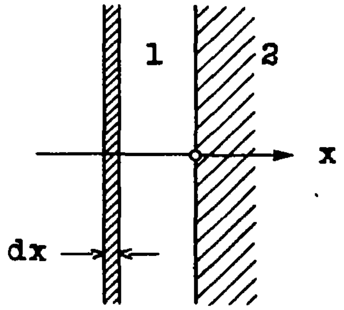

Figure 1.- Elementary layer in the gas space before a flat wall.

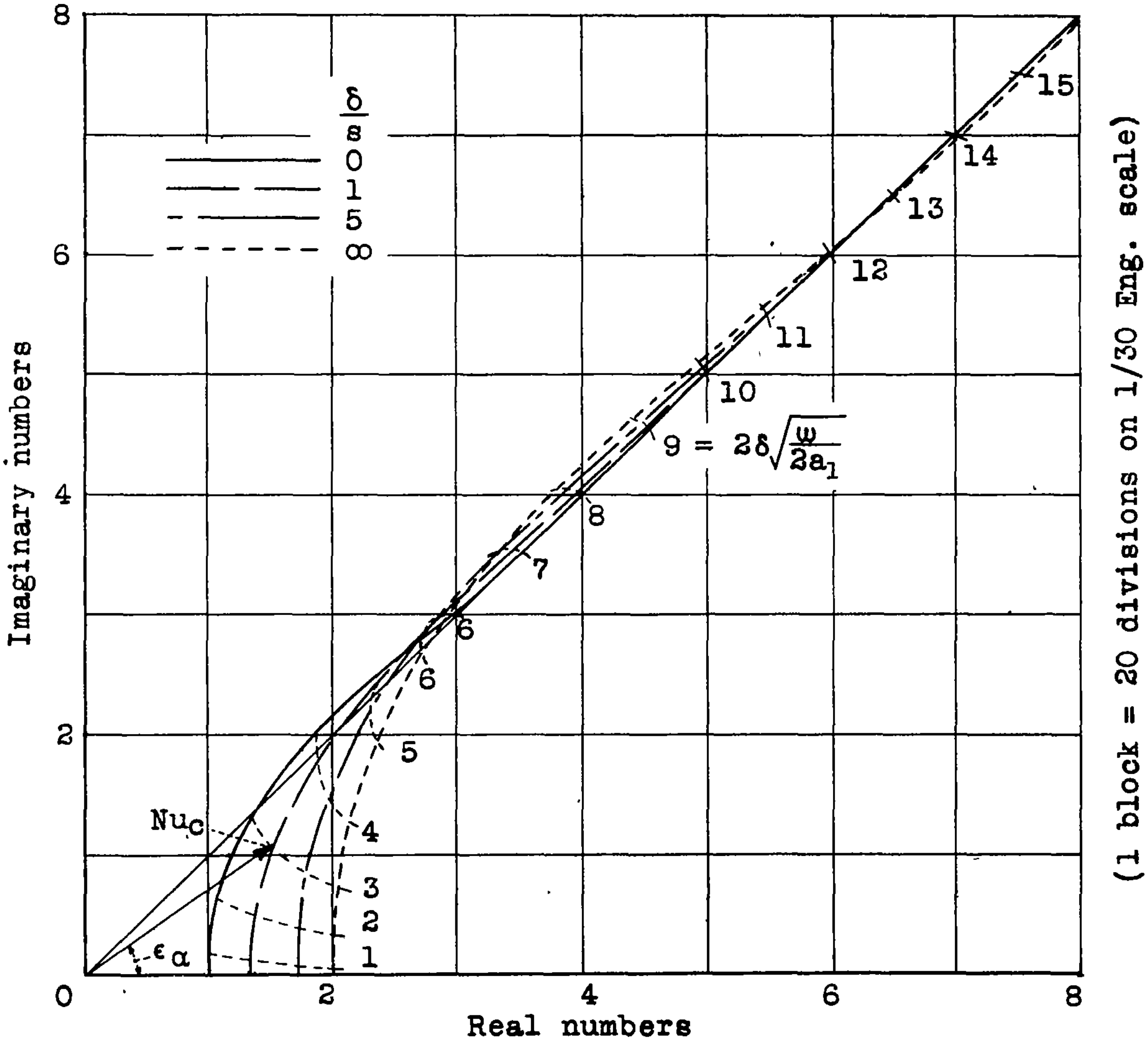

Figure 2.- Representation of the complex Nusselt number, $Nu_c = \alpha_c \delta / \lambda_1$ in the Gauss' numerical plane for small, periodic compression of a gas before a metal wall ($\rho_{12} = -1$) by limited thickness (δ) of boundary layer flow and different value ($s = V_t/O_t$) in the turbulent gas space. The straight line under 45° represents the asymptote of the curve.