STUDIES OF PLANET FORMATION
USING A HYBRID N-BODY + PLANETESIMAL CODE

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The NASA Technical Officer for the grant is Michael Salamon at NASA/HDQ, code SZ.
The goal of our proposal was to use a hybrid multi-annulus planetesimal/n-body code to examine the planetesimal theory, one of the two main theories of planet formation. We developed this code to follow the evolution of numerous 1 m to 1 km planetesimals as they collide, merge, and grow into full-fledged planets. Our goal was to apply the code to several well-posed, topical problems in planet formation and to derive observational consequences of the models. We planned to construct detailed models to address two fundamental issues:

- icy planets: models for icy planet formation will demonstrate how the physical properties of debris disks — including the Kuiper Belt in our solar system — depend on initial conditions and input physics; and

- terrestrial planets: calculations following the evolution of 1–10 km planetesimals into Earth-mass planets and rings of dust will provide a better understanding of how terrestrial planets form and interact with their environment.

During the past year, we made progress on each issue. The next few paragraphs summarize the results of papers published in 2004. We conclude with short summaries of work to be completed during the first half of 2005 and work planned for the second half of 2005.

**Size Distribution of Kuiper Belt Objects**

In the last decade, ground-based and satellite observations have established good orbits and magnitudes for more than 1000 Kuiper belt objects (KBOs) beyond the orbit of Neptune. These data show that the cumulative size distribution of KBOs is roughly a power law, \( N_C \propto r^{-q} \), with \( q_L \approx 3.5 \) for large bodies with radii, \( r \geq 10-100 \) km, and \( q_s \approx 2.5-3 \) for small bodies with radii, \( r \leq 0.1-1 \) km. The transition between the two power laws occurs at a break radius, \( r_b \approx 1-30 \) km.

To understand the origin of the break in the KBO size distribution, we developed an analytic collision model for planetesimals in a collisional cascade. The model adopts an energy-scaling approach to collisions, where the mass ejected in a collision is the ratio of the impact energy \( Q_I \) to the disruption energy \( Q_d \). We write the disruption energy as

\[
Q_d = Q_b r^{\beta_b} + \rho Q_g r^{\beta_g} ,
\]

(1)
where $Q_br^b$ is the bulk (tensile) component of the binding energy and $\rho Q_g r^b$ is the gravity component of the binding energy.

The collision energy $Q_I$ is a function of the mass $m$ and orbital eccentricity $e$ of colliding planetesimals. In the center-of-mass frame, the important parameters are $e$ and the mass ratio, $m_1/m_2$, of the colliding planetesimals. The break radius $r_b$ is then a function of these two parameters and the scaling factors in the expression for $Q_d$.

To test the analytic model, we ran a series of numerical calculations to derive $r_b$ as a function of a wide range of input properties for the planetesimals, the structure of the planetesimal disk, and the formation of Neptune. These results show that the break radius is more sensitive to the initial mass in the Kuiper Belt and the amount of stirring by Neptune than the bulk properties of individual KBOs.

Comparisons with observations indicate that most models can explain the observed sky surface density $\sigma(m)$ of KBOs for red magnitudes $R \approx 22-27$. For $R \leq 22$ and $R \geq 28$, the model $\sigma(m)$ is sensitive to the amount of stirring by Neptune, suggesting that the size distribution of icy planets in the outer solar system provides independent constraints on the formation of Neptune.

![Figure 1 - KBO luminosity functions derived from the planet formation model. The solid line is a model where Neptune forms at 30 AU in 100 Myr; the dashed line is a model without Neptune formation. Observations of KBOs from ground-based and satellite data are shown for comparison.](image-url)
Stellar encounters as the origin of distant solar system objects in highly eccentric orbits

The Kuiper Belt extends from the orbit of Neptune at 30 AU to an abrupt outer edge at ~ 50 AU from the Sun. Beyond the edge is a sparse population of objects with large orbital eccentricities, the so-called scattered disk. Neptune shapes the dynamics of these objects, but the recently discovered planet 2003 VB12 (Sedna) has an eccentric orbit with a perihelion distance, 70 AU, far beyond Neptune’s gravitational influence. Although influences from passing stars could have created the Kuiper Belt’s outer edge and could have scattered objects into large, eccentric orbits, no model currently explains the properties of Sedna.

Figure 2 – Stirring of the eccentricities of planets by the close pass of a Sun-like star. The star is on a marginally bound orbit corotating with the Sun’s disk, with an orbital inclination of $i = 23^\circ$ and argument of perihelion $\omega = 212^\circ$. Each panel lists the distance of closest approach. The histograms show the frequency of the final eccentricity, $e_f$, for 20,000 particles initially in circular orbits at 40–80 AU around the Sun. Magenta histograms: fraction of orbits with $e_f < 0.04$; cyan histograms: fraction of orbits with $e_f < 0.2$; blue histograms: fraction of orbits with Sedna-like orbits ($e_f > 0.5$). The 160 AU encounter places the edge of the Kuiper Belt where observed, as indicated by the arrow in the middle panel.
We used our hybrid planet formation code to investigate the formation of Sedna. In 50–200 Myr, coagulation produces 10–200 Pluto-sized or larger bodies at 40–80 AU. If the Sun formed in a dense cluster, as suggested by radiometric analyses, then another young star probably passed by the young Sun during the planet formation epoch.

We used a suite of n-body simulation to show that a passing star probably scattered Sedna from the Kuiper Belt into its observed orbit. The likelihood that a planet at 60–80 AU can be scattered into Sedna’s orbit is $\sim 50\%$; this estimate depends critically on the geometry of the flyby. Even more interesting, though, is the $\sim 10\%$ chance that Sedna was captured from the outer disk of the passing star. Most captures have very high inclination orbits; detection of these objects would confirm the presence of extrasolar planets in our own Solar System.

Figure 3 - Aftermath of a stellar encounter. The two stars are shown as yellow dots. Planets initially belonging to the top star are shown in magenta; planets initially orbiting the bottom star are shown in cyan. It is clear that both stars have captured planets from the other star.
Plan for 2005

During the first half of 2005, we plan to complete the following projects:

- Hybrid code: we have now finished a suite of calculations testing the hybrid code. The suite covers all previously published n-body calculations for the terrestrial zone, plus some additional calibrations we found useful. Our goal is to establish a set of benchmarks for these types of codes.

- Collision detection: we developed an analytic model to describe the outcome and visibility (observed luminosity) of a binary collision between two planetesimals. We have tested this model with numerical simulations. The calculations confirm the analytic model and show that, in the terrestrial zone, individual collisions between 300 km and large objects are visible. Beyond 3 AU, these collisions are not visible.

![Graph](image.png)

Figure 4 – Time Evolution of the 24 μm excess for planetesimal disks at 3-20 AU and at 30-150 AU.

- Evolution of infrared excess: the paper described above leads to clear predictions regarding the evolution of the 24 μm radiation from dust as a function of time. The figure below shows results for models at 3-20 AU and at 30-150 AU. Recent observations from Spitzer test this picture. With a CfA postdoc, we are working on...
methods to analyze the Spitzer data and to derive more details about the long-term evolution of debris disks and the formation of planets within them.

- Chaotic growth: Goldreich & collaborators have proposed a criterion for the transition from oligarchic growth (where growing protoplanets have roughly circular orbits and accrete from small non-overlapping feeding zones) to what we call 'chaotic growth' (where protoplanets have eccentric orbits and compete to accrete leftover planetesimals). In their view, this transition occurs when the surface density in large protoplanets is comparable to the surface density in leftover planetesimals. In simulations of terrestrial planet formation, we have shown that this criterion is a necessary but not sufficient condition for the formation of planets. We developed a second analytic criterion based on the Hill radius and confirmed with numerical simulations that both criteria provide necessary and sufficient conditions for the growth of oligarchs into Earth-mass planets.

- We have finished including an algorithm for radial migration into the hybrid code. The algorithm is tested and reproduces published analytical and computational results. We have begun to test how migration and gas drag influence the transition to chaotic growth and the distribution of eccentricity and inclination of terrestrial planets.

We expect to submit papers covering the first four projects during the first half of 2005. Throughout 2005, we plan to make additional progress on migration, gas drag, and the evolution of the mid-IR excess during the middle to late stages of terrestrial and icy planet formation.

A website describing our results and plans (including animations of debris disks) is at http://cfa-www.harvard.edu/~kenyon/pf/index.html.

References