Qualitative features extraction from sensor data using short-time Fourier Transform

Abolfazl M. Amini
Department of Electronics Engineering Technology, Southern University, Baton Rouge, LA 70813

Fernando Figueroa
NASA Stennis Space Center, Technology Development and Transfer, HA30, MS 39529

ABSTRACT

The information gathered from sensors is used to determine the health of a sensor. Once a normal mode of operation is established any deviation from the normal behavior indicates a change. This change may be due to a malfunction of the sensor(s) or the system (or process). The step-up and step-down features, as well as sensor disturbances are assumed to be exponential. An RC network is used to model the main process, which is defined by a step-up (charging), drift, and step-down (discharging). The sensor disturbances and spike are added while the system is in drift. The system runs for a period of at least three time-constants of the main process every time a process feature occurs (e.g. step change). The Short-Time Fourier Transform of the Signal is taken using the Hamming window. Three window widths are used. The DC value is removed from the windowed data prior to taking the FFT. The resulting three dimensional spectral plots provide good time frequency resolution. The results indicate distinct shapes corresponding to each process.

Keywords: Feature Extraction, Short-Time Fourier Transform

1. INTRODUCTION

This paper is the continuation of our work for determining the health of a sensor and system by monitoring information gathered from the sensor. In the first paper, we address the application of Fourier Transform along with tapering function. In this paper, we add the time factor. The problem we wish to address is that, which frequency component of the simulated signal arrive at what time. The Short-Time Fourier Transform is used as a first approach. In this approach, a window is multiplied by the signal at different times followed by Fourier Transform. The result is a 2D plot of time and frequency VS amplitude. The Short-Time Fourier Transform works well except that good time resolution (narrow window) is obtained at the cost of poor frequency resolution and vice versa. To obtain good frequency resolution one has to use a wide window.

Once a normal mode of operation is established any deviation from the normal behavior indicates a change. This change may be due to a malfunction of the sensor(s) or the system (or process). For example, if one sensor indicates that the temperature in the tank has experienced a step change, then a pressure sensor associated with the process in the tank should also experience a step change. The step-up and step-down features, as well as sensor disturbances are assumed to be exponential. An RC network is used to model the main process, which is defined by a step-up (charging), drift, and step-down (discharging). A total of 1000 seconds of data are generated with the sampling interval assumed to be 1 second. The time constant for charging and discharging are assumed to be 30 seconds. The system is expected to reach Drift in approximately five time constants. This is in agreement with the RC network in charging mode. The sensor disturbances and spike are added while the system is in drift. The time constant used for the sensor disturbance, which is assumed to be exponential is 4 second. The duration of the sensor disturbance is rather short. The used for the spike, which is assumed to be Gaussian is 0.08. The Gaussian lasts only a few sample points. The system runs for a period of at least three time-constants of the main process every time a process feature occurs (e.g. step change). Then the Short-Time Fourier Transform of the Signal is taken using the Hamming window. Three window widths are used. The window is shifted along the time axis one sample point each time the FFT is taken. The
DC value is removed from the windowed data prior to taking the FFT. The choice of using a tapering function rather than a Gaussian for windowing prior to taking the FFT is to smooth out the side lobes, which would be present, specially, when the focus is on time resolution. The tapering functions eliminate the side lobes and achieve smoothness effectively. This is followed by calculation of spectra for each set of data. The resulting three dimensional spectral plots provide good time frequency resolution. The results indicate distinct shapes corresponding to each process. In the next section the equations used to simulate the distinct shapes in the signal are given along with the tapering function. The section three contains discussion of the results followed by section four, which provides the conclusion. A study plan is laid out in the conclusion section describing the need for expansion of the study to include Gaussian distributed noise, varying feature shapes, and Wavelet Transform for better resolution of time-frequency sensitive signals.

2. MATHEMATICAL BACKGROUND

In this section, we will give the equations used to simulate the results of this paper. We start out by the input signal. The main process is modeled using an RC network. This process has three parts: Step-up, Drift, and Step-down. The equations governing these three parts are given by,

\[
\begin{align*}
    f_{su}(t) &= A(1 - \exp(-(t - t_{su})/\tau_{m})) \\
    f_{sd}(t) &= A\exp(-(t - t_{sd})/\tau_{m}).
\end{align*}
\]

The \(\tau_m\)'s are the Step-up and Step-down time constants. The subscripts \(su\) and \(sd\) signify the Step-up and Step-down features respectively. The exponentials are shifted to accommodate pretest as well as Drift time in the simulations. The pre-factor A is selected arbitrarily. The system is expected to be in Drift mode after approximately five time-constant has elapsed. In order to complete simulation of the input signal, we need to add a Spike and two disturbances while the system is in Drift. These are given by,

\[
\begin{align*}
    f_{sp}(t) &= A + B\exp(-\epsilon(t - t_{sp})^2) \\
    f_{ds}(t) &= A + C\exp(-|t - t_{ds}|/\tau_{ds}).
\end{align*}
\]

The B and C are amplitudes of the spike and the two disturbances respectively. The subscripts \(sp\) and \(ds\) signify the Spike and The Sensor Disturbances features respectively. Again, the exponentials are shifted in time according to time in which they occur. And \(\tau_{ds}\) is the disturbance time constant. The simulated input signal is given in figure 1. The Fourier Transform is given by,

\[
F(\omega) = \int f(t)\exp(-j\omega t)dt.
\]

The Short-Time Fourier Transform is given by,
\[ F_s(\tau, \omega) = \int f(t)W(t - \tau)e^{-j\omega\tau}dt. \]

Where, \( F_s \) is the Short-Time Fourier Transfer and \( W \) is the Hamming window shifted by \( \tau \) each time the FFT is calculated. The Hamming Windows in discrete form is given by

\[ W(m) = 0.54 + 0.46\cos(m\pi / M). \]

Where, \( M \) is the highest harmonic number. The lower case \( m \) ranges from \(-M\) to \(+M\) (2\( M + 1 \) points).

### 3. RESULTS

The simulations are allowed to run for a short period of time designated as pretest. The pretest ends before the Step-up of the main process begin. The simulations are carried out for a total of 1000 seconds. The input signal is sampled at 1 second intervals. The simulated input signal is given in figure 1. In order to better extract the distinctive features of the signal, we found that the DC (drift) has to be removed from the signal prior to taking the windowed FFT of the signal. This in effect eliminates the Drift part of the signal. The effect of this elimination makes extraction of the features easier. Figure 2 shows a three dimensional plot of frequency and time vs. amplitude. The Hamming window used to obtain this plot has only 9 points (9 second). In Short-Time Fourier Transform, the narrower the width of the window the better the time resolution and poorer the frequency resolution becomes. The wider window widths provide very good frequency resolution but the time resolution becomes poorer the wider the width gets. For narrow windows, the signal is multiplied by the window more often, thus, more FFTs are taken. This means better time resolution but poor frequency resolution. For wider windows, the signal is multiplied by the window less often, therefore, less FFTs are taken, which means poor time resolution but better frequency resolution. The width of the window and its relation to the time-frequency resolution is easier to understand in the frequency domain. The convolution theorem makes this relationship (width of the window and time-frequency resolution) very clear. The convolution theorem states that when two signals are multiplied in time domain the Fourier Transform of the respected signals is convolved in the frequency domain. The widths of a signal in time and frequency domains have reciprocal relationship. When the width of the window is narrow its Fourier Transform is wide. Thus, for narrow windows, The Fourier Transform of the signal is convolved with a wide window. This convolution smooths out all frequency features of the signal. That is, frequency resolution becomes very poor. For wider windows, the opposite effect is true since wide windows have narrow Fourier Transform the smoothness effect of the convolution in the frequency domain diminishes. In figures 2 through 4, the width of the window increases from 9 seconds to 101 seconds. For the 9 second window we get excellent time resolution but very poor frequency resolution. This effect is reversed in the 101 second window. This is clearly seen in figures 3 and 4. In these figures, frequency resolution has improved drastically but time resolution has degraded. To better see the effect of the wide window on time axes, we have regenerated figure 4 to obtain figure 5. In figure 5, we have shifted the window along the time axes by 4 sample points (4 second) each time the window is multiplied by the signal. This is done to provide a better view of the time axes by making the plot less dense. The width of the 9 second window is twice the time constant of the sensor disturbance while the 61 second window is twice the time constant of the main process. The most visible feature in figure 2 is the tall and wide shape in the middle that corresponds to the spike. The narrower the window width becomes the more visible the short duration events become. But the wider the window width becomes the more visible the longer duration features become. This is clearly seen in figures 4 and 5 that correspond to the 101 second window. In figure 5, we have shifted the window by 4 sample points to make the effects more visible. In all of the other figures the window is shifted by 1 sample point.
4. CONCLUSION

This paper is continuation of our work for development of a software package, which incorporates signal processing techniques for automated qualitative feature extraction. As a first step, the Fourier Transform was used along with some tapering functions\(^1\). In the first paper, different features of the signal such as Step-up and Step-down, Sensor disturbances, and the spike were easily detected. The work in this paper is intended to address time-frequency issues. It is often desirable to determine which frequency component of a signal arrive at what time. The Short-Time Fourier Transform addresses this problem by giving a 2D plot where time and frequency are plotted VS amplitude. In this approach, time-frequency resolution is a trade off. That is to say, obtaining good time resolution means poor frequency resolution and vice versa. Our results show that the short duration events are best seen when the window used is very narrow while the longer duration events are seen best by wider windows. The optimum width of the window used needs to be investigated further. Intuitively, for a given feature, the optimum window width should be equal to the respected event’s time constant. Our results show that Short-Time Fourier Transform can be very useful for obtaining a general idea on how time frequency relate. The ultimate method for excellent time-frequency resolution is, of course, Wavelet Transform. As with the previous study, the effects of varying amplitudes for each feature as well shapes were not studied. In addition, the simulated input signal studied was noise free. The next step of this work is to add Gaussian distributed additive noise with varying Signal to Noise Ratio (SNR) to the signal. A number of statistically different noisy cases for each SNR should be studied. The statistical analysis of the range of SNRs studied should prove to be valuable. Once the effects of varying amplitudes and shapes for each feature is added with noise the study for the qualitative feature extraction is complete.

ACKNOWLEDGMENTS

This work was supported through a Grant by NASA’s Stennis Space Center.

REFERENCES

Figure 1. Simulated input signal
Figure 2. Short time FT 9s window
Figure 4. short time FT 101s
Figure 5. short time FT 101s window time-shift by 4 samples