Quantitative Analysis of Spectral Interference of Spontaneous Raman Scattering in High-Pressure Fuel-Rich H₂–Air Combustion

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Abstract

We present a theoretical study of the spectral interferences in the spontaneous Raman scattering spectra of major combustion products in 30-atm fuel-rich hydrogen–air flames. An effective methodology is introduced to choose an appropriate line-shape model for simulating Raman spectra in high-pressure combustion environments. The Voigt profile with the additive approximation assumption was found to provide a reasonable model of the spectral line shape for the present analysis. The rotational/vibrational Raman spectra of H₂, N₂, and H₂O were calculated using an anharmonic-oscillator model using the latest collisional broadening coefficients. The calculated spectra were validated with data obtained in a 10-atm fuel-rich H₂–air flame and showed excellent agreement. Our quantitative spectral analysis for equivalence ratios ranging from 1.5 to 5.0 revealed substantial amounts of spectral cross-talk between the rotational H₂ lines and the N₂ O(1/2)-branch; and between the vibrational H₂ O(0,3) line and the vibrational H₂O spectrum. We also address the temperature dependence of the spectral cross-talk and extend our analysis to include a cross-talk compensation technique that removes the interference arising from the H₂ Raman spectra onto the N₂, or H₂O spectra.
1. Introduction

Quantitative measurements of species concentration and temperature in elevated pressure combustion environments are critical to the validation of computer models used in the development of gas turbine and internal combustion engines. Laser-excited Spontaneous Raman scattering (SRS) spectroscopy is perhaps the only practical technique that provides spatially and temporally resolved multi-species information in a simultaneous or single-shot fashion with a high level of accuracy [1-3]. For high pressure combustion studies, SRS has the added benefit of providing stronger signal levels than those performed at atmospheric pressure, because the signal scales linearly with the number density, and hence pressure of the molecules being probed. Furthermore, SRS measurements are relatively simple, less expensive, and more versatile when compared to coherent anti-Stokes Raman spectroscopy (CARS), or stimulated Raman spectroscopy. These facts have motivated a recent renewed interest in fundamental studies that apply SRS diagnostics in high-pressure combustion [4-5].

Although the use of SRS diagnostics for quantitative studies in flame studies is well known, special attention to spectral interferences between Raman scattering from different molecules will still require further study. This requirement is substantiated by the fact that several earlier studies have indicated that spectral interferences can be a significant problem for Raman diagnostics in flames [2-3,6-8]. In fact, the use of a detailed spectral analysis that includes the spectral interferences or ‘cross-talk’ between different vibrational/rotational Raman bands of different molecular species, is the basis for several common SRS techniques including thermometry via Stokes/anti-Stokes ratio [9], and integrated area measurements of vibrational Raman bands for species concentrations [10]. Without considering the spectral interferences among the different Raman bands for multiple species, the accuracy of temperature/species measurements is insufficient for combustion model validation.

The effect of spectral interferences on the SRS signals has been effectively implemented using the well-known cross-talk matrix calibration procedure [2-3,6-8]. The off-diagonal terms of the calibration matrix generally represent the Raman scattering crosstalk of one species onto another. The acquisition and development of an accurate cross-talk calibration matrix is the key factor in obtaining quantitative species
concentration and temperature measurements in flames. Unfortunately, the Raman cross-talk calibration data obtained from these one-of-a-kind setups to date is not easily convertible into a transferable standard that is suitable for use by other researchers, particularly those who deal with high-pressure flames. In order to create a standardized cross-talk calibration matrix with absolute numbers, a comprehensive and accurate study of the spectral interference among Raman scattering in flames of various conditions (fuel-air composition, equivalence ratio, and temperature) over a range of pressure is still required.

The quantitative treatment of the spectral interference in flames is not an easy task since the interference is a molecule-dependent function of temperature, species concentration, and chemical composition. For instance, rotational H$_2$ Raman lines at rotational quantum number, $J = 9$ to $J = 11$ around the 2300 cm$^{-1}$ Raman shift in fuel-rich hydrogen flames may become strong enough to interfere with the N$_2$ Q-branch spectrum (near 2330 cm$^{-1}$). In particular, the rotational $J = 10$ Stokes line of H$_2$ is completely overwhelmed by the N$_2$ Q-branch. In order to facilitate the quantitative analysis of these interferences, a theoretical means of predicting the SRS spectra is essential. The quantitative analysis of the interferences are even further complicated in high-pressure flames due to possible pressure broadening or collisional line-narrowing effects [11].

In this paper, we investigate the spectral interferences between combustion molecules in high-pressure, fuel-rich, hydrogen–air flames using a spectral simulation of the SRS process. We focus our attention in this paper to hydrogen-air flames for the following reasons: chemically, a H$_2$–air flame provides the simplest combination of combustion products in order to facilitate the spectral analysis, yet retains spectral features that are important to more complicated fuel-oxidizer systems that will be studied at a later time; research data on a H$_2$–air flames are important in high-speed propulsion systems such as rocket combustors and pulse detonation engines. Fuel-rich hydrogen flames are studied here because they are typically used in rocket engines for reasons of flame stability and maximum heat-release. Furthermore, the interference between H$_2$ and other molecules becomes important in fuel-rich flames where H$_2$ concentration is high.
We first introduce an analytical methodology to validate an appropriate line-shape model for simulating Raman spectra in high-pressure flames and compare it with experimental data to verify our theoretical model for simulating Raman spectra. We then model the rotational/vibrational Raman spectra of H₂, N₂, and H₂O using an anharmonic-oscillator model with a line-shape model that incorporates the latest collisional broadening coefficient data. Finally, we theoretically investigate the spectral behavior of Raman O-, S-, and Q-branches of the species in hypothetical high-pressure hydrogen flames over a range of equivalence ratios.

2. Modeling of Raman Spectra

2-1. Raman Cross-Section

The theory of Raman scattering in diatomic and polyatomic molecules is well established [12]. According to Placzek’s polarization theory of the Raman effect, the corresponding differential Raman scattering cross-section per molecule at thermal equilibrium is given by [12]

\[
\left( \frac{\partial \sigma}{\partial \Omega} \right)_{\theta}^{\theta} = \left( \frac{\pi}{e_0} \right)^2 (v_0 \pm v_k)^4 g \frac{\exp(-E_i / kT)}{Q(T)} \Phi(\alpha^2, \beta^2, \theta) f(J). \tag{1}
\]

Here, the superscript \( \theta \) indicates the relative angle of the planes of polarization between scattered and incident light; the subscripts of \( i \) and \( f \) represent the initial and final states respectively; \( e_0 \) is the vacuum permittivity; \( v_0 \) is the incident light (laser) frequency; + and – signs refer to the anti-Stokes and Stokes Raman scattering respectively; \( v_k \) is the molecular frequency associated with the rotation-vibration, or the pure rotational transition from \( i \) to \( f \); \( g \) is the total degeneracy; \( E_i \) is the energy of the initial state; \( k \) is the Boltzmann constant; \( T \) is the gas temperature; \( Q(T) \) is the total partition function of the molecule at temperature \( T \); \( \Phi(\alpha^2, \beta^2, \theta) \) is the molecule-dependent temperature-independent invariant that is a function of the mean value of the derived polarizability.
tensor $\alpha^2$ and the anisotropy tensor $\beta^2$. The values for $\Phi(\alpha^2, \beta^2, \theta)$ are tabulated by Long [12] for every branch of vibrational and pure rotational scattering for the molecules of interest; and $f(J)$ is the correction factor arising from the rotation-vibration coupling [13]. Note that Eq. (1) should be multiplied by the factors $(v + 1)b_v^2$ and $vb_v^2$ for vibrational Stokes and anti-Stokes scattering of a diatomic molecule, respectively. Here, $v$ is the vibrational quantum number and $b_v^2$ is defined as $h/8\pi^2c\nu_b$ where $h$ is the Plank constant, and $c$ is the speed of light.

The Raman line frequency is given by $\nu_k = (E_f - E_i)/hc$ where $E_f$ is the energy of the final state. The energy in a rovibrational state $(v, J)$ is expressed as the sum of the vibrational energy term, $G(v)$ and the rotational energy term, $F(v, J)$

$$E(v, J) = \hbar c [G(v) + F(v, J)].$$ (2)

According to the anharmonic oscillator model, the vibrational energy term $G(v)$ of a diatomic molecule is given by [14]

$$G(v) = \omega_v\left(v + \frac{1}{2}\right) - \omega_v \nu_v \left(v + \frac{1}{2}\right)^2 + \omega_v \nu_v \left(v + \frac{1}{2}\right)^3$$ (3)

where $\omega_v$, $\omega_v \nu_v$, and $\omega_v \nu_v$ are the molecular constants and are tabulated by Huber and Hertzberg [15]. According to the non-rigid rotator model that accounts for the influence of the centrifugal distortion, the rotational term for the $^1\Sigma$ (singlet) state of diatomic molecules such as $N_2$ or $H_2$ is given by

$$F(v, J) = B_vJ(J + 1) - D_vJ^2(J + 1)^2 + H_vJ^3(J + 1)^3$$ (4)

where $B_v = B_e - \alpha_v(v + \frac{1}{2}) + \gamma_v(v + \frac{1}{2})^2 - \eta_v(v + \frac{1}{2})^3$, $D_v = D_e + \beta_v(v + \frac{1}{2}) + \xi_v(v + \frac{1}{2})^2$, and $H_v = H_e - \zeta_v(v + \frac{1}{2})$. The terms $B_e$, $D_e$, $H_e$, $\alpha_v$, $\beta_v$, $\gamma_v$, $\eta_v$, $\xi_v$, and $\zeta_v$ are the molecular-
dependent spectroscopic constants, which are tabulated for each molecule of interest by Huber and Hertzberg [15]. Thus for a Stokes branch ($\Delta \nu = \nu_f - \nu_i = +1$), the Raman line frequency is given by

$$
\nu_k(v_i, J_i) = \frac{E(v_i + 1, J_f) - E(v_i, J_i)}{hc}.
$$

The partition function $Q(T)$ for a rovibronic state may be written as the sum of the rotational and the vibrational partition functions

$$
Q(T) = Q_{\text{vib}}(T) + Q_{\text{rot}}(T)
$$

where

$$
Q_{\text{vib}}(T) = \sum_{v=0}^{\infty} \exp \left( -\frac{hcG(v)}{kT} \right),
$$

$$
Q_{\text{rot}}(T) = \sum_{v=0}^{\infty} \sum_{J=0}^{\infty} g_J (2J + 1) \exp \left( -\frac{hcF(v, J)}{kT} \right).
$$

Here $g_J$ is the nuclear spin statistical weight and the term $(2J + 1)$ is the rotational degeneracy.

For calculating a pure rotational Raman spectrum (e.g. rotational H$_2$ S-branch), Eqs. (2), (5), and (6) are replaced by $E(J) = hcF(J)$ where $F(J) = F(0, J)$, $\nu_k(J_i) = [E(J) - E(J_i)]/hc$, and $Q(T) = Q_{\text{rot}}(T)$, respectively.

The correction factor $f(J)$ is given by

$$
f(J) = 1 + 4 \chi^{-1} (B_x/\omega_x)^2 [J_i(J_i + 1) + J_f(J_f + 1)] ,
$$
where $\chi^{-1}$ is the polarizability-related spectroscopic term. The values of $\chi^{-1}$ for N$_2$, and H$_2$ have been found to be 2.5 [13]. The factor $f(J)$ is substantial for N$_2$ at temperature above 1000 K and is significant for H$_2$ because of its large rotational constant [13].

**Polarizability Tensor Components**

According to Placzek-Teller theory, the space-averaged $\Phi(\alpha^2, \beta^2, \theta)$ terms for the Raman $Q$, $O$, and $S$-branches for scattered light that is polarized along the direction of the incident light polarization are given by the following equations, respectively [12,16-17]:

\[
\Phi(\alpha, \beta)_Q = \left| \langle \psi_{v,J} \mid \alpha \mid \psi_{v+1,J} \rangle \right|^2 + \frac{4}{45} S_{J,J} \left| \langle \psi_{v,J} \mid \beta \mid \psi_{v+1,J} \rangle \right|^2,
\]

\[
\Phi(\alpha, \beta)_O = \frac{4}{45} S_{J-2,J} \left| \langle \psi_{v,J} \mid \beta \mid \psi_{v+1,J-2} \rangle \right|^2,
\]

\[
\Phi(\alpha, \beta)_S = \frac{4}{45} S_{J+2,J} \left| \langle \psi_{v,J} \mid \beta \mid \psi_{v+1,J+2} \rangle \right|^2,
\]

where $\psi_{v,J}$ is the vibrational wavefunction for a particular $v$-$J$ state; $\alpha$ is the isotropic part of the polarizability, and $\beta$ is the anisotropic part of the polarizability. $S_{r,r'}$ are the Placzek-Teller coefficients for diatomic molecules given by [1,12]

\[
S_{J,J} = \frac{J(J+1)}{(2J-1)(2J+3)} \quad (Q\text{-branch}),
\]

\[
S_{J,J-2} = \frac{3J(J-1)}{2(2J+1)(2J-1)} \quad (O\text{-branch}),
\]

\[
S_{J+2,J} = \frac{3(J+1)(J+2)}{2(2J+1)(2J+3)} \quad (S\text{-branch}).
\]
Evaluation of Eqs. (10-12) based on the reasonable approximation for vibrational transitions with $\Delta v = 1$ yields [16]

$$\Phi(\alpha, \beta)_Q = \left( \frac{B_v}{\omega_v} \right) \left( \alpha'^2 + \frac{4}{45} S_{J, J} \beta'^2 \right)$$  \hspace{1cm} (16)$$

$$\Phi(\alpha, \beta)_O = \left( \frac{B_v}{\omega_v} \right) \frac{4}{45} S_{J, J-1} \beta'^2$$  \hspace{1cm} (17)$$

$$\Phi(\alpha, \beta)_S = \left( \frac{B_v}{\omega_v} \right) \frac{4}{45} S_{J, J+1} \beta'^2.$$  \hspace{1cm} (18)$$

Substituting Eqs. (16-18) into Eq. (1) and evaluating the degeneracy terms we obtain the following relations:

$$\left( \frac{\partial \sigma}{\partial \Omega} \right)^Z_Z = q(v_0 \pm v_1)^4 (2J + 1) g_s \frac{\exp(-\frac{E_n}{kT})}{Q(T)} \left[ \frac{45}{4} \left( \alpha' \right)^2 + S_{J, J} \right] f(J),$$ \hspace{1cm} (19)$$

$$\left( \frac{\partial \sigma}{\partial \Omega} \right)^Z_O = q(v_0 \pm v_1)^4 (2J + 1) g_s \frac{\exp(-\frac{E_n}{kT})}{Q(T)} S_{J, J-1} f(J),$$ \hspace{1cm} (20)$$

$$\left( \frac{\partial \sigma}{\partial \Omega} \right)^Z_S = q(v_0 \pm v_1)^4 (2J + 1) g_s \frac{\exp(-\frac{E_n}{kT})}{Q(T)} S_{J, J+1} f(J),$$ \hspace{1cm} (21)$$

where the superscript $ZZ$ indicates the scattered light polarized along the direction of the incident light polarization; $q$ is a molecule-independent physical constant; $g_s$ is the nuclear spin statistical weight ($g_s = 1$ for even $J$; $g_s = 3$ for odd $J$); $(2J+1)$ is the rotational degeneracy. The terms $\alpha'$ and $\beta'$ are the isotropic (mean) and anisotropic invariants of the static dipole polarizability derivatives, which are derived from the expansion in the
dimensionless vibrational coordinate \( \xi \equiv (r_r e)/r_e \) (where \( r \) is the internuclear distance and \( r_e \) is the equilibrium internuclear distance) [16-17],

\[
\alpha = \alpha_0 + \alpha' \xi + \frac{1}{2} \alpha'' \xi^2 + \ldots,
\]

(22)

\[
\beta' = \beta_0 + \beta' \xi + \frac{1}{2} \beta'' \xi^2 + \ldots,
\]

(23)

with

\[
\alpha' = \frac{\partial \alpha}{\partial \xi} \bigg|_{\xi=0},
\]

(24)

\[
\beta' = \frac{\partial \beta}{\partial \xi} \bigg|_{\xi=0}.
\]

(25)

**H\textsubscript{2}O Raman Model**

The Raman scattering spectrum of water vapor appears over the range of frequency from 3400 to 4130 cm\(^{-1}\), and results mainly from the \( \nu_1 \) stretch vibrational mode. Generally, the temperatures encountered in combustion environments makes the \( \nu_1 \) band (100)-(000) transition dominant. For the H\textsubscript{2}O Raman spectrum, the classical anharmonic-oscillator model is not suitable for calculating the vibrational energy because of the asymmetric top nature of the water molecule that shows a relatively complicated structure. Fortunately, the vibrational energies in the vibrational ground state (000), and in the various excited states such as the (100), (001) are known from the analysis of a high-resolution infrared spectrum and such database is tabulated in HITRAN [18]. The wavefunctions for those states were also calculated based on the rotational Hamiltonian by Flaud et al [19] that includes the Fermi resonance between the (100) and (020) states, and the Coriolis-like anharmonic coupling between these two states and the (001) state. Based on the wavefunctions and the polarizability derivatives, Avila et al [20] calculated the vibrational Raman cross-sections of H\textsubscript{2}O in the OH-stretching region with great accuracy
up to about 1000 K. In the current report, we then used the results on the Raman line frequency and cross-sections which are tabulated in Ref. 20 to simulate the H2O Raman spectra in our high-pressure flames.

The partition function of H2O was calculated using the well known polynomial expression [18,21]

$$ Q(T) = a + bT + cT^2 + dT^3, $$

(26)

where coefficients $a$, $b$, $c$, and $d$ are tabulated over the range of temperature up to 3005 K by Gamache [21].

It should be noted that the resulting calculated H2O Raman spectrum does not include the hot bands of the (110)-(010) and (011)-(010) transitions that become important over 1500 K as well as the additional hot bands from the (020)-(000), (100)-(000), and (001)-(000), which would appear at higher temperature as Avila et al [20] give caution. These hot bands generally "stretch" a H2O spectrum out to the shorter wavelength, thus they form a longer blue-side tail. According to Avila et al [20], the intensities of these hot bands amount to over 10% of that of the main bands so that the present calculation for H2O, likely, also has errors of this level at higher temperature conditions.

2-2. Line-Shape

**Pressure-Broadening Coefficient**

Collisional broadening coefficients of molecules are a dominant factor for the line-shape modeling. Recently collisional broadening coefficients for H2 with perturber molecules have been proposed by advanced line-shape calculations and those numbers over a range of temperature and pressure have been successfully applied to CARS thermometry in high-pressure flames [22-24]. Bergmann et al [22] obtained $J$-dependent H2 collisional broadening coefficients for higher temperature from 1800 to 2200 K for $J = 0$ to 7 in a fuel-rich CH4–air flame up to 40 bar. Hussong et al [23] provided an effective, $J$-
dependent polynomial equation for the $\text{H}_2$ collisional broadening coefficient including the inhomogeneous contribution (nonlinear dependence on the perturber concentration) in the $\text{H}_2$–N$_2$ binary system:

$$\bar{\gamma}^0_j (T, C) = (\gamma^\text{self}_0)_{,j} + (\bar{\gamma}^\text{self})_{,j} T + (a_1)_{,j} + (a_2)_{,j} C + (a_3)_{,j} C^2 + (a_4)_{,j} C^2 T,$$

(27)

where $\bar{\gamma}^0_j$ [cm$^{-1}$/atm] is the total broadening coefficient as a function of temperature, $T$ and concentration of perturber N$_2$, $C$. Here $C$ is defined by the N$_2$ fraction relative to H$_2$ concentration under the assumption of the H$_2$–N$_2$ binary system. All other parameters, $(\gamma^\text{self}_0)_{,j}$, $(\bar{\gamma}^\text{self})_{,j}$, $(a_i)_{,j}$ ($i = 1$ to 4) are tabulated for $J = 0$ to 5 in Ref. 23.

Since the influence of H$_2$O as an additional collider plays an important role at higher temperature in terms of speed-inhomogeneous effects as Hussong et al [23-24] mentioned in their reports, a ternary system H$_2$-N$_2$-H$_2$O should be considered for better accuracy over the range of flame temperature. Unfortunately, the collisional broadening coefficients of ternary mixture including H$_2$O at high temperature have not yet been reported in the literature. We thus calculated the H$_2$ broadening coefficient for each flame condition using Eq. (27) in this study. Furthermore, in an effort to simplify the effects of temperature and $J$-number dependence on the collisional broadening for hydrogen, we use the broadening coefficient of $J = 3$ for all lines in the H$_2$ spectrum. Note that the above-mentioned assumption regarding the binary system and the single-value coefficient has been previously found to be both appropriate and practical from a diagnostics standpoint [25].

There is an alternative expression for the collisional broadening coefficient, which has been proposed in the HITRAN database [18]. Molecular spectroscopic information (including the collisional broadening coefficients) of major combustion-related species such as N$_2$ or H$_2$O are available in the database while there is no information regarding H$_2$, due to the fact that it is not an infrared active molecule. We then used HITRAN [18] to find spectroscopic information for the collisional broadening coefficients of N$_2$ and H$_2$O. In the database, the collisional broadened halfwidth is written as
\[
\tilde{\gamma}_J^0 (T, C)^{\text{HITRAN}} = \left( \frac{T_0}{T} \right)^n \left[ \gamma_{\text{air}}^0 (P_0, T_0) C + \gamma_{\text{self}}^0 (P_0, T_0) (1 - C) \right],
\]

where \( \gamma_{\text{air}}^0 (P_0, T_0) \) and \( \gamma_{\text{self}}^0 (P_0, T_0) \) are the air-broadened, and the self-broadened width at the reference temperature, \( T_0 = 296 \text{ K} \) and the reference pressure, \( P_0 = 1 \text{ atm} \), and \( n \) is the coefficient of temperature dependence. As for \( \text{N}_2 \), only \( \gamma_{\text{self}}^0 (P_0, T_0) \) was considered and it was assumed to a single value of 0.048 cm\(^{-1}\)/atm. This value was derived from HITRAN by calculating an intensity-weighted, averaged halfwidth of all \( \text{N}_2 \) \( Q \)-branch lines from \( J = 1 \) to 40 in the vibrational 1-0 transition, which is the dominant transition at room temperature. For the \( \text{H}_2\text{O} \) lines, we used the widths given by \( \gamma_{\text{air}}^0 (P_0, T_0) = 0.0095 \) and \( \gamma_{\text{self}}^0 (P_0, T_0) = 0.0923 \), which were also derived from the HITRAN database by calculating an average halfwidth of all lines in the vibrational transition (000)-(100) between 3400 to 3710 cm\(^{-1}\) at room temperature. A value of \( n = 0.5 \) was used for \( \text{N}_2 \) according to Ref. 18 and \( n = 0.67 \) was found reasonable for \( \text{H}_2\text{O} \) after calculating an averaged \( n \) value for the (000)-(100) transition.

**Process of Line-Shape Model Selection**

Spectral line shapes of Raman scattering spectra at atmospheric pressure are typically modeled by a Gaussian profile, and those at higher pressures are usually modeled by a Voigt profile which is the convolution of Gaussian and Lorentzian collisional broadening distributions. However, the Voigt profile is based on the impact approximation, so it may not be valid for modeling spectral shapes at very high pressures due to the break down of the approximations in the far wings of spectral lines [27]. In some cases that require higher accuracy, such as in a laser absorption technique, motionally-narrowed Galatry profiles [26] could be also employed. These line-shape models were developed for a single isolated transition. In practice however, multiple overlapping transitions are common and the effect of their overlap may become important at high pressures. Thus, consideration of the line-narrowing due to the interaction between different transitions, the so-called line-mixing effect, may be required to properly model the Raman spectra for
quantitative measurements. The line-mixing effect in nonlinear Raman processes of pure gases at high pressure has been previously studied [11,28] but not in flames.

Based on the above points, we see the need to develop a methodology to choose an appropriate high-pressure line-shape model for a particular species, and to evaluate the limitations of these models. Nagali [29] discussed the applicability of the major line-shape models and introduced an effective approach for choosing an appropriate model for neutral species within the temperature and pressure range of interest. In this study, we follow the approach described by Nagali [29] but include, for the first time, $N_2$ concentration-dependent collisional broadening coefficients to the analysis. This permits the determination of an appropriate collisional broadening model for simulating Raman spectra in high-pressure flames.

We consider three criteria for modeling the spectral lineshapes of Raman scattering spectra: 1) Motional narrowing (in which the Galatry profile is considered); 2) Impact approximation (in which the Voigt profile is valid); and 3) Line mixing (in which the additive approximation is not valid). To define the effectiveness and limitation of each criterion in terms of the range of temperature and pressure, we first define a collisional scale factor, i.e. the molecular collision duration, $t_{\text{coll}}$ [s]. When the interaction length is set to the radius of the collision-broadening cross-section, $t_{\text{coll}}$ can be expressed as

$$t_{\text{coll}}(T, C) = \frac{1}{v_{\text{mol}}} \frac{c \cdot \Delta \nu_c}{\sqrt{v_{\text{mol}} N_f}}$$

(29)

where $\Delta \nu_c$ [cm$^{-1}$] is the collisional broadening fullwidth which is twofold of the total broadening coefficient $\tilde{\gamma}_f^0$ at room temperature and atmospheric pressure, $c$ [m/s] is the speed of light, $N_f$ [cm$^{-3}$] is the number density of the molecule of species $f$; and $v_{\text{mol}}$ [m/s] is the mean speed of the gas molecule which is approximately equal to another measure of the molecular speed, root-mean-square speed [30]
\[ v_{\text{mol}}(T) = \sqrt{\frac{3RT}{M}} , \]

(30)

where \( R \) [J K\(^{-1}\) mol\(^{-1}\)] is the gas constant, \( M \) [g mol\(^{-1}\)] is the molecular mass.

The wavelength region over which the impact approximation (collisional broadening due to intermolecular collisions which can be described by Lorentzian profile) is valid can be determined using the expression [31]

\[ 2\pi \nu_{\text{coll}} \Delta \nu_{\text{imp}} \ll 1 , \]

(31)

where \( \Delta \nu_{\text{imp}} \) is the spectral region in which the impact approximation is accurate. Thus, in the impact approximation region, \( \Delta \nu_{\text{imp}} \) can be written as

\[ \Delta \nu_{\text{imp}}(T, C) = \frac{0.1}{2\pi \nu_{\text{coll}}} . \]

(32)

Here we use a factor of 0.1, which is a smaller value than that suggested by Nagli (0.2) [29]. However, the exact value for this quantity is still open to interpretation for future studies. Following Nagali [29], if we require that at least 95 % of the area under the line shape be accurately described by the impact approximation, the criterion that the impact approximation is valid is then given by

\[ \frac{\Delta \nu}{2} \leq 0.079 \Delta \nu_{\text{imp}} . \]

(33)

Here, \( \Delta \nu_{c} \) [cm\(^{-1}\)] is the collisional broadening fullwidth given by

\[ \Delta \nu_{c}(T, C, P) = 2\gamma^{0}_{c} P , \]

(34)
where $P$ [atm] is the total gas pressure. In a regime above the boundary defined by Eq. (33), a line shape may still be modeled by Voigt function, but a breakdown at the far wings will likely be observed.

The so-called line-mixing effects caused by the interaction between different transitions become important when the frequency separation of collisionally-coupled transitions, relative to consecutive rotational levels, is smaller than, or of the same magnitude as the collisional width [31]. That is, if neighboring, interacting transitions $v_{k1}$ and $v_{k2}$ are such that

$$
|v_{k1} - v_{k2}| \leq \frac{\Delta v_c}{2},
$$

then line-mixing effects should be considered. Here, $|v_{k1} - v_{k2}|$ is the separation distance in frequency between the interacting transitions, and $\Delta v_c$ is the averaged collisional fullwidth of the two different transitions of interest. Since we have to consider multiple lines in Raman spectra we used the averaged line separation, $|v_{k1} - v_{k2}| = v_{sep}$ between each consecutive rotational/rovibrational line for practical convenience. Thus, the boundary of the line-mixing can be written as

$$
v_{sep} \leq \frac{\Delta v_c}{2}.
$$

Here, values for $v_{sep}$ were found to be 200.0, 19.8, 3.6, and 0.73 cm$^{-1}$ for the rot-H$_2$ ($J = 0$ to 10), vib-H$_2$ ($J = 0$ to 17 in vibrational 1-0 transition), H$_2$O (000-100 transition), and N$_2$ ($Q$-branch, $J = 1$ to 40 in vibrational 1-0 transition), respectively.

Because of interrupted motions caused by the collisional scattering of molecules, a velocity-changing collision results in a line shape with a sub-Doppler linewidth, which involves line-narrowing effects [26]. If velocity-perturbing collisions are assumed to be
independent of state-perturbing collisions, then the Galatry profile ("soft" collision model) can be used to model the motional narrowing [29]. When the relationship given by

\[ \Delta \nu_c \geq \Delta \nu_D, \]  

(37)
is true, here \( \Delta \nu_D \) [cm\(^{-1}\)] is the full-width-half-maximum (FWHM) Doppler width, collisional broadening effects dominate the Doppler broadening so that the refinement of the motional narrowing effect is not necessary [29]. This is the third criterion for the line-shape model. Here, the Doppler width is given by

\[ \Delta \nu_D (T) = \frac{2 \nu_0}{c} \sqrt{2 \ln(2) k T / M}, \]  

(38)

where \( \nu_0 \) is the transition center frequency. Note that the motional narrowing is different from the collisional narrowing in its overall shape due to the line-mixing effect at high pressure.

From Eqs. (29-34 & 36-38), the following three boundaries are formed and they provide a way to choose the appropriate line-shape model for H\(_2\), N\(_2\), and H\(_2\)O gases over a range of pressure, temperature, and species concentration:

\[ P = \frac{\Delta \nu_D/2}{\overline{\nu}_0 (C, T)}: \text{Galatry boundary - A} \]  

(39)

\[ P = \frac{0.079 \Delta \nu_{\text{imp}}}{\overline{\nu}_0 (C, T)}: \text{Voigt boundary - B} \]  

(40)
\[ P = \frac{v_{sep}}{\tilde{\gamma}_j^0(C,T)} : \text{Line-mixing boundary - C} \quad (41) \]

With these boundaries, the region with \( P\tilde{\gamma}_j^0 \leq \Delta \nu_D / 2 \) indicates the necessity of considering the motional narrowing effect, i.e., use of the Galatry profile; that with \( \Delta \nu_D / 2 < P\tilde{\gamma}_j^0 \leq 0.07\Delta \nu_{imp} \) indicates the use of the Voigt profile for entire line (additive approximation is valid); that with \( 0.07\Delta \nu_{imp} < P\tilde{\gamma}_j^0 < v_{sep} \) supports the use of the Voigt profile, with the caveat, that there is a possible breakdown in the far-wing (additive approximation is likely valid); and that with \( P\tilde{\gamma}_j^0 \geq v_{sep} \) indicates the necessity of considering the line mixing effect.

2-3. Target Flames

In order to analyze spectral interference in Raman scattering in high-pressure combustion environments, we generate thermodynamic properties for five different flames under the assumption of complete combustion and chemical equilibrium. Chemical equilibrium compositions of several different hydrogen-air flames, which hereafter we call "target flames", were then calculated by the CEA code [32] for assigned equivalence ratios, pressure (30 atm), and an initial gas temperature (300 K). Table 1 shows the calculated mole fractions of H₂, N₂, and H₂O and the adiabatic temperatures. The equivalence ratios were varied from 1.5 to 5.0, with the resulting temperatures varying from 1402 K to 2262 K. First, the calculated temperatures, pressure, and N₂ mole fractions as perturber concentrations were used as input parameters to calculate the collisional broadening coefficients and the line-shape diagrams. The calculated temperatures, pressure, and number densities of H₂, N₂, and H₂O (derived from the mole fractions and temperatures) were then used for the Raman spectral simulations.

3. Results and Discussion

3-1. Choice of Line-Shape
To choose the best line-shape function to model Raman spectra in our target flames over a range of pressure, temperature, and perturber concentration we introduce a three-dimensional diagram that is consists of the boundaries defined in Section 2-2. Figure 1 shows the diagram designed for the S(3) line of the pure rotational H₂ (hereafter indicated rot-H₂) spectrum (ν₀ = 1035 cm⁻¹). This diagram directs one to choose an appropriate line-shape model for spontaneous Raman spectra simulation in any flame of interest with a prior knowledge of pressure, temperature, and N₂ concentration. The locations of the five target flames (labeled case # 1 to # 5 in Table 1) are marked in the diagram to determine which line-shape model is the most appropriate. All five cases here are between boundary A and B as shown in Fig. 1, which indicates the entire line shape can be modeled by the Voigt profile with the additive approximation. Note that the same is true for other major rot-H₂ lines of higher J number while the Galatry boundary (boundary A) slightly moves toward higher pressure depending on the transition center frequency ν₀.

Figure 2 shows a three-dimensional diagram for the rovibrational H₂ (hereafter indicated as vib-H₂) spectrum with points representing the target flames. Here the fundamental frequency of the vib-H₂ spectrum (4160 cm⁻¹) was assigned for the value of ν₀ in the calculation of the diagram. In this figure, the line-mixing boundary (boundary C) appears because the averaged line separation (νₚₑₚ) of the vib-H₂ spectrum is 19.8 cm⁻¹, which is much smaller than rotational separation of the rot-H₂ spectrum by 180 cm⁻¹. All the target flames in Fig. 2 are also between boundary A and B, the Voigt profile regime.

Figure 3a and 3b show the line-shape diagrams for the vibrational Q-branches of N₂ and H₂O spectra with points representing the target flames. The fundamental vibrational Raman frequencies of N₂ and H₂O (2331 and 3657 cm⁻¹, respectively) were assigned to ν₀ in the calculations of the diagrams. Here, only the self-broadening is considered for N₂, and the N₂ fraction relative to H₂O concentration is constant, i.e. C = 0.65. Accordingly, the diagrams have only two dimensions without a dimension of perturber N₂ concentration. As shown in Fig. 3a, all the target flames are located between boundary B and C, so the line shape for the N₂ Q-branch can be appropriately modeled by the Voigt function with the assumption of the additive approximation in the near-wing
region, but their line shape could break down in the far-wing regions. The impact and
additive approximation are, for the most part, appropriate for the entire region of the H₂O
spectrum in the target flames according to Fig. 3b.

In general, the Voigt profile was found to be a reasonable model for simulating the
Raman spectrum over the range of pressure up to 30 atm, and for temperatures from
approximately 1500 K to 2500 K. While there may be a slight effect of breakdown at the
far wings of the N₂ Q-branch, the effect is small and is ignored in favor of the simplicity
of using the Voigt function for the entire region of interest. This consequence is very
useful for reducing the computational requirements of the spectral simulation and
analysis. Hence, we used the Voigt profile for modeling the line shape of Raman spectra
in the target flames. In the sections below, we will empirically validate the use of the
Voigt profile for simulating Raman scattering spectra via comparisons between
calculated spectra and experimental data.

In calculations of the Voigt-function a and x parameter [33] we used the overall
convolution Gaussian width, Δν₀ [cm⁻¹] given by

\[ Δν₀(v, J) = \sqrt{Δν_D^2 + Δν_{spec}^2}. \]  (42)

Here Δν_{spec} is the instrumental function, which is the spectral resolution of a
spectrograph. Finally, the profile of the Raman spectrum at temperature (T), and pressure
(P) was calculated by the integration (via summation) of all Voigt-shaped lines based on
the additive approximation assumption.

Regarding the temperature dependence of the collisional broadening coefficient there
are at least two alternative expressions (CARS-, or HITRAN-expression) as we discussed
above. We are not aware of a published report that indicates whether or not these two
expressions make any difference to the line-shape diagram. In Fig. 4, we compare the
line-shape diagrams derived from the two different expressions for the broadening
coefficient. As shown in Fig. 4, the boundary A, and B based on the two expressions,
which have been validated by the different spectroscopic experiments and calculations,
agree very well when the HITRAN-based temperature dependence coefficient, n was set
to 0.15. This agreement indicates the both expressions are comparable, and that they can be applied to spontaneous Raman scattering applications, even though we used a CARS-derived expression for H\textsubscript{2}, and a HITRAN-derived expression for N\textsubscript{2}, and H\textsubscript{2}O in this paper.

3-2. Validation of the Raman Model

We validate our theoretical models for the Raman scattering intensity and line shapes in high-pressure environments by comparison with experimental data measured in high-pressure flames. We used a specially designed high-pressure gaseous burner that provides a precise and stable source of hot combustion products in a high-pressure environment. The burner provides a homogeneous stream of combustion products via a tightly-spaced array of multiple fuel-rich H\textsubscript{2}-air premixed flamelets (10 atm). In this particular premixed burner design, the 7 x 7 array of premixed flame tubes (1.1 mm dia.) had a center-to-center spacing of 2.60 mm. Spontaneous Raman scattering from H\textsubscript{2}, N\textsubscript{2}, and H\textsubscript{2}O were then measured at a reduced-temperature, post-flame zone using our SRS diagnostic system which consists of a pulsed Nd:YAG laser, a laser pulse-stretcher, a high-speed electro-mechanical shutter, and a spectrometer fitted with a non-image-intensified CCD camera. For the simulations, we used an instrumental resolution of $\nu_{\text{spec}} = 35$ cm\textsuperscript{-1} which corresponds to the spectral resolution of the spectrometer used in the measurement. It should be noted that our previous measurements with this high-pressure burner indicated that a condition close to the chemical equilibrium was reached over the range of equivalence ratio at the measurement location, however, the measured temperatures were below the adiabatic equilibrium temperatures by more than 100 K [34]. Further details on the burner and diagnostics have been described in our previous reports [25, 34-35].

For a quantitative simulation of a Raman spectrum, we empirically determine the factor $(\alpha'/\beta')^2$, the squared ratio of the isotropic term to the anisotropic term of the polarizability derivative, and we verify the relative intensity of the $Q$-branch to the $S/O$-branches in vibrational N\textsubscript{2} or H\textsubscript{2} Raman spectrum. Note that calculated $S/O$-branches of N\textsubscript{2}, or H\textsubscript{2} Raman spectrum using theoretically determined values of $\alpha'$ and $\beta'$ [16-17] do
not agree with our experimental data, thus we determined these values empirically as follows. We measured the factor \((\alpha'/\beta')^2\) for \(\text{N}_2\), or \(\text{H}_2\) vibrational Raman spectrum in ambient-temperature air and compare it with a calculated spectrum at the same condition. As shown in Fig. 5, we found the best agreement in terms of the relative intensity of the \(Q\)-branch to the \(O\)-, and \(S\)-branch between data for the room air and a calculated \(\text{N}_2\) Raman spectrum at 300 K when using the factor for \((\alpha'/\beta')^2\) set at a value of 0.25. The suitability of this value for \(\text{N}_2\) was also substantiated by the good agreement between measured data and calculated values for a \(\text{N}_2\) Raman spectrum in a 10-atm \(\text{H}_2\)-air flames \((\phi = 2.06)\) as shown in Fig. 6. We found that the factor \((\alpha'/\beta')^2\) for \(\text{H}_2\), when set at 0.35, provided the best agreement between a calculated \(O\)-branch intensity of the \(\text{vib-}\text{H}_2\) spectrum and a measured spectrum from a 10-atm \(\text{H}_2\)-air flame \((\phi = 3.0)\) as shown in Fig. 7.

We then compare the experimental Raman spectra measured in 10-atm fuel-rich \(\text{H}_2\)-air flames with theoretical Raman spectra calculated at the temperatures determined experimentally by the line-intensity distribution of the \(\text{rot-}\text{H}_2\) Raman spectrum [13,25]. Figure 6 shows the measured \((\phi = 2.06)\) and calculated \((T = 1425 \text{ K})\) results of the \(\text{rot-}\text{H}_2\) and the vibrational \(\text{N}_2\) Raman spectra. The transition frequencies (wavelength) and line shapes of calculated spectra show excellent agreement with experimental data for both species. As indicated in this Fig. 6, the \(\text{N}_2\) \(O\)-branch is a possible spectral interference (cross-talk) with the \(S(9)\) and \(S(8)\) lines of the \(\text{rot-}\text{H}_2\) band. Also, the \(\text{rot-}\text{H}_2\) \(S(10)\) line, which is barely seen in this figure, is completely overwhelmed by the \(\text{N}_2\) \(O\)-branch. We will examine these spectral interferences in detail in the section below.

Figure 7 shows the measured \((\phi = 3.0)\) and calculated \((T = 1133 \text{ K})\) results of the vibrational \(\text{H}_2\text{O}\) and the \(\text{vib-}\text{H}_2\) Raman spectra. The calculated spectra for both species show excellent agreement with the experimental data in terms of the transition frequency, intensity and line shape. Note that we can clearly observe the spectral interference between the \(\text{H}_2\text{O}\) spectrum and the \(\text{vib-}\text{H}_2\) \(O(0,3)\) line around 657 nm. This interference is important because it affects the determination of the \(\text{H}_2\text{O}\) number density in fuel-rich high-pressure flames, thus we will examine it in detail in the following section.

Based on the comparisons between the spectral simulation and the experimental data, we have shown that the current theoretical models for Raman scattering line position,
intensity, and shape, are of sufficient accuracy to examine the effects of Raman spectral interferences amongst the major species of interest in high-pressure H\textsubscript{2}-air flames.

**Quantitative Comparison between Different Species**

In order to quantitatively analyze the contribution of spectral interferences in the overall Raman spectra for all species of interest, the relative scattering intensities of each different species has to be realized and properly implemented in the calculations. To do this, we define an empirical scaling factor for the scattering intensity of a rotational or vibrational Raman spectrum of species \( j \) relative to the N\textsubscript{2} Raman scattering intensity as follows:

\[
\Gamma_j = \frac{\sum_{n=a}^{b} m_{n} I^{k,p}_j / N_j(\phi, P, T_{ms})}{\sum_{n=c}^{d} m_{n} I^{k,p}_{N_2} / N_{N_2}(\phi, P, T_{ms})},
\]

(43)

where \( m_{n} I^{k,p}_j \) is the pixel intensity (CCD count per pixel) of a measured rotational or vibrational Raman spectrum of species \( j \) at equivalent ratio of \( \phi \) and pressure of \( P \); \( n \) is the pixel number and \( a, b, c, \) or \( d \) indicate a corresponding pixel range for the summation of the spectrum of species \( j \); \( N_j(\phi, P, T_{ms}) \) is the calculated chemical-equilibrium number density of species \( j \) for \( \phi, P, \) and the measured temperature \( T_{ms} \) via the rot-H\textsubscript{2} spectrum. The subscript, ‘N2’ indicates the molecular nitrogen species used as the reference.

Based on the scaling factor \( \Gamma_j \) and the intensity-normalized calculated Raman spectrum, the scale-corrected intensity of a theoretical Raman spectrum for species \( j \) at pressure of \( P \) and temperature of \( T \) can be obtained by

\[
I_{j}^{\text{cal}}(P, T) = \Gamma_j N_j(\phi, P) \int I_{j}^{\text{cal}}(P, T) d\lambda
\]

(44)
where \( N_j(\phi, P) \) is the adiabatic number density of species \( j \) for \( \phi \) and \( P \); and \( \text{cal} I_j(P, T) \) is the intensity of the calculated Raman spectrum at \( P \) and \( T \).

Based on Eq. (43) and the measured Raman scattering data obtained in 10-atm H\(_2\)-air flames (Fig. 6, and Fig. 7), we were then able to determine the scaling factor for each species. Note that the instrumental function did not affect the resulting scale-corrected spectra, since the spectral intensity response of the entire optical system used for the measurements are calibrated in both intensity and wavelength using standard reference lamps. We then make the assumption that the combustion products have reached chemical equilibrium, and also with use of the scaling factors described above, and by using the parameters found in Table 1, we calculate the scale-corrected rotational or vibrational Raman spectra of N\(_2\), H\(_2\), and H\(_2\)O in 30-atm fuel-rich H\(_2\)-air flames. This finally enables a quantitative comparison of the spectral interferences between the different species of interest.

3-3. Spectral Interference

According to the experimental data shown in Figs. 6 and 7, there are two kinds of noticeable spectral interferences in Raman scattering in fuel-rich hydrogen flames: interference between the higher \( J \)-number (8 \( \leq J \leq 11 \)) \textit{rot}-H\(_2\) lines and the N\(_2\) spectrum; and between the \textit{vib}-H\(_2\) \( O(0,3) \) line and the vibrational H\(_2\)O spectrum. The behavior of Raman spectral interference is typically nonlinear since the rate of spectral interference is a function of both temperature and number density of at least two species in a spectral range of interest. For instance, in fuel-\textit{rich} flames, rotational H\(_2\) line intensities of transitions with higher \( J \) states are determined by a trade-off between the Boltzmann rotational distribution and the number density which is governed by the local stoichiometry. Thus, one of our questions is in which case does the Raman interference become more important: H\(_2\)-rich, lower-temperature flames, or nearly stoichiometric, higher-temperature flames? In order to examine this question in a quantitative fashion, we simulated rotational and/or vibrational Raman spectra of H\(_2\), N\(_2\), and H\(_2\)O in high-pressure hydrogen-air flames at different equivalence ratios using the theoretical formalism presented above.
Interference between rot-$H_2$ and vib-$N_2$

Figure 8 shows the calculated Raman spectra of the rot-$H_2$ S-branch and the vibrational $N_2$ spectra in 30-atm $H_2$–air flames from $\phi = 1.5$ to 5.0. Occurrences of spectral interference among these spectra can be clearly seen in this series of spectral simulations. Our simulation indicates that it is particularly important to quantitatively analyze the interference that the rot-$H_2$ $S(10)$ line gives to the $N_2$ $Q$-branch since the $S(10)$ line is completely overlapped by the $N_2$ line. Figure 8 also confirms that there is an interference between the $N_2$ $O$-branch (the blue-side tail) and the lines from the rot-$H_2$ $S(8)$, or $S(9)$ lines; and that there is an interference between the $N_2$ $S$-branch (the red-side tail) and the rot-$H_2$ $S(11)$ line. Note that the higher $J$-number ($\geq 9$) line intensities of the rot-$H_2$ do not necessarily become stronger with increase of the equivalence ratio, while the lower $J$-number ($\leq 7$) line intensities apparently increase with the equivalence ratio. The interference between the rot-$H_2$ and the $N_2$ spectrum indicates that further attention should be paid to the spectral shape of the $N_2$ $Q$-branch in hydrogen-rich environments. This effect is further exacerbated by the fact that the $N_2$ $Q$-branch gradually shrinks as temperature lowers due to the less population of higher vibrational states.

In order to quantify and clarify the flame conditions in which the interference is significant, we define the cross-talk rate as the overlap rate between one spectrum and another. The cross-talk rate, $K_t$, is the ratio of the integrated spectral area of one spectrum (always smaller one) to that of another spectrum, which interferes with the former spectrum, for a certain range of wavelength. The spectral range of the integration for each case is shown in the figure captions. Figure 9 shows the cross-talk rate between the $N_2$ $O$-branch and the rot-$H_2$ $S(8)$ line (hereafter indicated as $K_t^{N-H_8}$); that between the $N_2$ $O$-branch and the rot-$H_2$ $S(9)$ line (indicated as $K_t^{N-H_9}$); and that between the rot-$H_2$ $S(10)$ line and the $N_2$ $Q$-branch (indicated as $K_t^{H_810-N}$) over the range of equivalence ratios used in this study. A high amount of the cross-talk, $K_t^{N-H_8} = 0.48$ (48% overlap within the spectral range) is observed at $\phi = 1.5$. $K_t^{N-H_8}$ becomes smaller as $\phi$ increases, yet $K_t^{N-H_8}$ is more than 0.08 for $\phi$ over 4.0. Similarly, the cross-talk $K_t^{N-H_9}$ reaches more than 0.65 (65% overlap) at $\phi = 1.5$. The higher amount of cross-talk arising from the “hot bands”
of the N$_2$ Q-branch also contributes to the interference. $K_{r N_2H_5}^{N_2H_9}$ has a minimum value of 0.3 (30% overlap) at $\phi = 3.0$ and stays over this value over the range of $\phi$ examined here. It is evident from Fig. 9 that the $S(8)$ and $S(9)$ lines should not be included in the temperature determination process via the rot-H$_2$ distribution in fuel-rich flames because of the higher level of interference.

In Fig. 9, the cross-talk rate $K_{r N_2H_5}^{N_2H_9}$ shows a different behavior than the previous cases for the $S(8)$ and $S(9)$ lines: it varies from 0.023 (2.3% overlap) to its maximum value around 0.036 (3.6% overlap) at $\phi \sim 2.5$. A cross-talk level of several percent is not insignificant, and may affect the accurate measurement of N$_2$ number density and/or N$_2$ vibrational temperature, if this effect is not accounted for in the subsequent analysis.

As an aid in the interpretation of Raman spectra with such an interference, we propose a practical method to compensate the cross-talk effect of $K_{r N_2H_5}^{N_2H_9}$ using a theoretical calculation with a priori knowledge of the temperature and an observed rot-H$_2$ line intensity data. Since the intensity distribution of the rot-H$_2$ Raman transitions is governed only by temperature under assumption of the thermal equilibrium, the relative intensity ratio of any pair of lines in the rot-H$_2$ Raman spectrum can be theoretically derived based on a known temperature. Hence one can estimate the line intensity of the rot-H$_2$ $S(10)$ from a measured intensity of, for instance, the rot-H$_2$ $S(6)$ line, which is generally strong enough to be measured from background noise and free from any major spectral inference. The theoretical Raman intensity ratio of the rot-H$_2$ $S(10)$ line to the $S(6)$ line, $R_{hit}^{s10-s6}$, was calculated with Eq. (21) over a range of temperature. The polynomial coefficients of the curve fitted to the theoretical plots of $R_{hit}^{s10-s6}$ are shown in Table 2. The thermodynamic temperature can be measured by the intensity distribution of the rot-H$_2$ Raman spectrum [13] within a few percent accuracy [25] and thus, a corresponding intensity ratio between the two lines at the temperature can be obtained via the polynomial function in Table 2. The "hidden" contribution of the rot-H$_2$ $S(10)$ to the N$_2$ Q-branch would be then revealed by multiplying the obtained intensity ratio to the measured the $S(6)$ line intensity in a Raman spectrum of interest. Finally, the obtained $S(10)$ line intensity would then be subtracted from a integrated intensity of the N$_2$ spectrum to find a corrected N$_2$ number density.
Note that the rot-H\textsubscript{2} S(7) shown in Fig. 8 is not suitable for use in this cross-talk compensation process because it coincides with the common atomic Na emission line (3s–3p transition) near 589 nm, which can be quite intense in industrial flames. For the other important cross-talk contributors to the vibrational N\textsubscript{2} spectrum resulting from the S(9), or S(11) lines in H\textsubscript{2}, a cross-talk calibration matrix formalism as previously mentioned in the Introduction section would have to be implemented in order to compensate for these effects.

**Interference between vib-H\textsubscript{2} and vib-H\textsubscript{2}O**

Figure 10 shows the calculated vibrational Raman spectra of H\textsubscript{2} and H\textsubscript{2}O in 30-atm H\textsubscript{2}–air flames over the range of equivalence ratio ranging from 1.5 to 5.0. As the equivalence ratio increases (thus, the number density of hydrogen molecule increases), the intensity of the vib-H\textsubscript{2} spectrum becomes stronger while the intensity of the H\textsubscript{2}O spectrum do not change much. As clearly seen in Fig. 11, the vib-H\textsubscript{2} O(0,3) line significantly interferes with H\textsubscript{2}O, particularly in highly H\textsubscript{2}-rich flames. There is also interference around 661 nm by higher rovibrational transitions of the vib-H\textsubscript{2} Q-branch. In order to clarify these interferences, we obtain the cross-talk rate between the vib-H\textsubscript{2} and the H\textsubscript{2}O spectra ($K_{r}^{\text{vib-H\textsubscript{2}}} \text{H\textsubscript{2}O}$) following the above-mentioned manner. The result over the range of equivalence ratios is shown in Fig. 11. The curve in Fig. 11 indicates that the cross-talk rate $K_{r}^{\text{vib-H\textsubscript{2}}} \text{H\textsubscript{2}O}$ has a monotonic and positive relation with equivalence ratio. Notice that the cross-talk rate $K_{r}^{\text{vib-H\textsubscript{2}}} \text{H\textsubscript{2}O}$ has a value of 0.08 (8\% overlap) at $\phi = 1.5$ and reaches 0.18 (18\% overlap) at $\phi = 5.0$. This level of cross-talk causes major errors in the measurement of H\textsubscript{2}O number density if not properly accounted for.

Since the vib-H\textsubscript{2} O(0,3) line are mostly overwhelmed by the stronger H\textsubscript{2}O spectrum, it is generally hard to realize this interference. We then propose the same analytical compensation technique previously used for the cross-talk rate $K_{r}^{\text{N-H\textsubscript{2}O10}}$ to remove the contribution by the vib-H\textsubscript{2} O(0,3) line to the H\textsubscript{2}O spectrum. Using the a priori knowledge of temperature determined by the rot-H\textsubscript{2} Raman transitions and a measured vib-H\textsubscript{2} line intensity, a corresponding vib-H\textsubscript{2} O(0,3) line intensity can be obtained and subtracted from the H\textsubscript{2}O spectral intensity data. Here, we suggest use of the vib-H\textsubscript{2} O(0,5) line for
the compensation process because it is the strongest line amongst the vib-H$_2$ O-branches and is relatively free from major spectral inferences. The theoretical Raman intensity ratio of the vib-H$_2$ O(0,3) line to the O(0,5) line, $R_{\text{H}_2}^{03-05}$, was calculated over a range of temperature. The polynomial coefficient of the fitting curve to the theoretical plots of $R_{\text{H}_2}^{03-05}$ is shown in Table 2. The value calculated from the polynomial function in Table 2 with a known temperature and a vib-H$_2$ O(0,5) intensity can serve to compensate the H$_2$O spectral intensity in fuel-rich flames.

Another source of interference that arises from the blue-side tail of the vib-H$_2$ Q-branch around 661 nm, can cause a minor overestimation in measurements of H$_2$O number density by less than ~1.5% (e.g. at $\phi = 5.0$) after compensating for the vib-H$_2$ O(0,3) contribution. There is not a simple, straightforward analytical expression for this interference because this portion of the vib-H$_2$ Q-branch contains multiple lines that originate from different rovibrational transitions, and its dependence on the spectral band factor (spectral resolution and/or pressure broadening) is complicated. Thus, the cross-talk matrix calibration method needs to be applied to take this interference into account.

**Temperature dependence of spectral interferences**

It is well known that the off-diagonal terms of the cross-talk calibration matrix represent spectral interference of one species onto another and may be expressed as a function of temperature [2-3,6-7]. Although the temperature dependence of the cross-talk matrix has been typically assumed to be a polynomial relation [6-7], we are not aware of any studies that can substantiate this common assumption for high-pressure combustion environments. Figure 12 shows the cross-talk rate normalized by the number density of hydrogen ($K_i/N_{\text{H}_2}$) as a function of temperature regarding the N$_2$ Q-branch and H$_2$O Raman spectra. By normalizing by the number density of H$_2$, which is the only species whose Raman spectrum (either rotational or vibrational) interferes with the N$_2$ or H$_2$O Raman spectrum in H$_2$-Air flames, a pure dependence of the cross-talk on temperature can be extracted. From Fig. 12 we confirm that the temperature dependence of the H$_2$-normalized cross-talk for nitrogen and water is well approximated by a monotonic, second-order (or less) polynomial function for the hydrogen flames in this study.
Note that the spectral database for the H$_2$O Raman scattering used here does not include a treatment of the hot bands, which is significant on the blue-side of the H$_2$O spectrum above a temperature of 2000 K. As a result, the database tends to underestimate the spectral intensity by no less than 10% in high temperature regions. Thus, we have to be aware that an experimentally observed H$_2$O Raman spectrum will be more spread out towards the blue-side than predicted by our calculation. This hot-band behavior of H$_2$O could interfere with the vib-H$_2$ O(0,4) line in the equivalence ratio range from about 1.0 < φ < 2.0, in high-temperature flames. According to our experimental data (with the same spectral resolution of the present simulation) in 10-atm H$_2$–air flames over the wide range of equivalence ratio (φ = 0.2 – 5.0) [34], the blue-side tail end of H$_2$O spectrum touches the vib-H$_2$ O(0,4) line but barely interferes even at φ = 1.5. Hence it is not necessary to include the vib-H$_2$ O(0,4) in the cross-talk calibration process for H$_2$O Raman spectrum.

4. Conclusions

In order to improve and make spontaneous Raman scattering diagnostics more quantitative and accurate for the measurement of temperature and species in high-pressure combustion, we theoretically investigated the spectral interference of Raman spectra in fuel-rich, H$_2$–air flames at pressures up to 30 atm. The rotational/vibrational Raman scattering of H$_2$, N$_2$, and H$_2$O were modeled with particular attention paid to the O-/S-branches as well as the common Q-branch using an anharmonic oscillator model and the most recent spectral database available in literature. We applied an analytical methodology, which is based on a collision-scale analysis and molecular vibration, to validate an appropriate line-shape model for simulating the Raman spectrum in high-pressure flames. The line-shape diagrams for the species showed that the use of the Voigt profile with the additive approximation was appropriate and practical for the Raman spectral analysis at the target flame conditions used in this study.

Our model was validated by direct comparisons between the calculated Raman spectra and measured data in 10-atm H$_2$–air fuel-rich flames, which showed excellent agreement in every detail, including the O-, and S-branch intensities. The spectral
interferences amongst the Raman spectra of the species of interest were then calculated for the target high-pressure H₂-air flames, and were analyzed over the range an equivalence ratio range of \( \phi = 1.5 \) to 5.0, and temperature. Below is a summary of the findings of the quantitative analysis on the Raman spectral interference in 30-atm fuel-rich H₂-air flames:

(i) The spectral overlap (cross-talk rate) between the pure rotational H₂ \( S(8) \) line and the N₂ \( O \)-branch was as high as 48\% at \( \phi = 1.5 \).

(ii) The spectral overlap between the pure rotational H₂ \( S(9) \) line and the N₂ \( O \)-branch was 65\% at \( \phi = 1.5 \) and about 30\% at \( \phi = 3.0 \).

(iii) The spectral overlap between the pure rotational H₂ \( S(10) \) line and the N₂ \( Q \)-branch was 3.6\% at \( \phi = 2.5 \).

(iv) The spectral overlap between the vibrational H₂ \( O(0,3) \) line and the H₂O spectrum was 18\% at \( \phi = 5.0 \).

In addition, we introduce an analytical technique using the Raman intensity ratio for a neighboring pair of the H₂ lines, to compensate for the cross-talk between the \( \text{rot-H}_2 \) \( S(10) \) line and the N₂ \( Q \)-branch; and that between the \( \text{vib-H}_2 \) \( O(0,3) \) line and the H₂O spectrum. We also provide numerical coefficients for implementing this compensation technique. Finally, we demonstrated that the Raman cross-talk between \( \text{rot-H}_2 \) and N₂, or \( \text{vib-H}_2 \) and H₂O has a second-order (or less) polynomial relation with flame temperature; this information can then be used as an aid in selecting an appropriate functional form for calibrations in future studies.

Since the collisional broadening effects on spectral line-shape are relatively minor at the modest spectral resolution considered here, the findings and techniques discussed in this report are applicable to both atmospheric and high-pressure (at least up to 30 bar) flames.

**Acknowledgement**

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References


(19) J. M. Flaud, C. Camy-Peyret, “Vibration-Rotation Intensities in H$_2$O-Type Molecules Application to the 2$\nu_2$, $\nu_1$ and $\nu_3$ Bands of H$_2^{16}$O,” *J. Mol. Spectrosc.* 55, 278-310 (1975).


Table 1  Adiabatic temperatures and species compositions calculated for hydrogen-air combustion system using the CEA chemical equilibrium code using assigned pressures and equivalence ratios.

<table>
<thead>
<tr>
<th>Case #</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<tbody>
<tr>
<td>$P$ [atm]</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>$\phi$</td>
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<td>2.0</td>
<td>3.0</td>
<td>4.0</td>
<td>5.0</td>
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<tr>
<td>$T$ [K]</td>
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<td>2065</td>
<td>1770</td>
<td>1560</td>
<td>1402</td>
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<tr>
<td>$\text{H}_2$ [Mole frac.]</td>
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<td>0.257</td>
<td>0.409</td>
<td>0.509</td>
<td>0.58</td>
</tr>
<tr>
<td>$\text{H}_2\text{O}$ [Mole frac.]</td>
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<td>0.257</td>
<td>0.205</td>
<td>0.17</td>
<td>0.145</td>
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<tr>
<td>$\text{N}_2$ [Mole frac.]</td>
<td>0.55</td>
<td>0.479</td>
<td>0.381</td>
<td>0.317</td>
<td>0.271</td>
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</table>
Table 2: Polynomial coefficients of curves fitted to the theoretically calculated intensity ratios of lines for the rotational hydrogen $S$-branch and vibrational hydrogen $O$-branch as a function of temperature (K). The polynomial coefficients have the form of 

$$R(T) = aT^4 + bT^3 + cT^2 + dT + e.$$ 

<table>
<thead>
<tr>
<th>Coeff.</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$d$</th>
<th>$e$</th>
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<td>$R_{HI}^{10-96}$</td>
<td>...</td>
<td>$-1.813 \times 10^{-11}$</td>
<td>$1.362 \times 10^{-7}$</td>
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<td>$-3.354 \times 10^{-9}$</td>
<td>$1.047 \times 10^{-5}$</td>
<td>$-1.508 \times 10^{-2}$</td>
<td>$9.504$</td>
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</table>
Fig. 1 Three-dimensional diagram used for choosing the proper line-shape model in high-pressure combustion environments for the $S(3)$ line of pure rotational $H_2$ Raman spectrum. Pressure is in log scale. White circles indicate the locations of the five target flames in shown in Table 1.
Fig. 2 Three-dimensional diagram used for choosing a proper line-shape model in high-pressure combustion environments for vibrational H$_2$ Raman spectrum. Pressure is in log scale. White circles indicate the locations of the five target flames shown in Table 1.
Fig. 3 Two-dimensional diagrams used for choosing a proper line-shape model in high-pressure combustion environments for (a) vibrational N₂ Raman Q-branch (self-broadening), and (b) vibrational H₂O Raman (C = 0.65). White circles indicate the locations of the five target flames shown in Table 1.

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Fig. 4 Comparison of the boundaries in the diagram for the rotational $\text{H}_2 S(3)$ line with different pressure broadening coefficients based on the CARS-expression and the HITRAN-expression. *Solid line*: CARS-expression; *dashed line*: HITRAN-expression with temperature coefficient $n = 0.15$. 
Fig. 5 Measured and calculated Raman spectra of vibrational $N_2$ in room-temperature air (300 K) at 10 atm used to determine the ratio of mean/anisotropic polarizability derivatives, $\alpha'/\beta'$ of nitrogen. Experiment (points): 200-shot averaged, 0.1 mm spectrometer slit. Calculation (line): $V_{\text{spec}} = 35 \text{ cm}^{-1}$. 
Fig. 6 Measured and calculated vibrational N\textsubscript{2} and rotational H\textsubscript{2} Raman spectra in a 10-atm H\textsubscript{2}-air flame. The spectra were calculated using the experimentally determined temperature via the rot-H\textsubscript{2} transitions, and was found to be 1425 K ($\nu_{\text{spec}} = 35$ cm\textsuperscript{-1}). The experimental data is represented by the points ($\phi = 2.06$, 200-shot averaged, 0.1 mm spectrometer slit). Solid line: rotational H\textsubscript{2}. Dashed line: vibrational N\textsubscript{2}. Two calculated lines are shown separately but not superimposed.
Fig. 7 Measured and calculated Raman spectra of H$_2$O, and vibrational H$_2$ in a 10-atm H$_2$-air flame. The spectra were calculated at an experimentally determined temperature obtained via the rot-H$_2$ transitions (not shown in this figure), and was found to be 1133 K ($\nu_{spec} = 35$ cm$^{-1}$). The experimental data is represented by points ($\phi = 3.0$, 200-shot averaged, 0.1 mm spectrometer slit). Solid line: vibrational H$_2$. Dashed line: vibrational H$_2$O. Two theoretical lines are shown separately but not superimposed.
Fig. 8 Calculated vibrational Raman spectra of H$_2$O and H$_2$ in 30-atm H$_2$-air combustion at different equivalence ratios for the five flame cases shown in Table 1 with $\nu_{\text{spec}} = 35$ cm$^{-1}$. Dotted line is rot-H$_2$, Dashed line is N$_2$, and Solid line are superimposed (summation of the two spectra). Intensity is in arbitrary units but in the same scale for all equivalence ratios.

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Fig. 9 Cross-talk rate \( (K_r) \) between \( N_2 \) \( O \)-branch and \( \text{rot-} H_2 \) lines in 30-atm \( H_2 \)-air combustion at different equivalence ratios, calculated from the spectra shown in Fig. 8. **Triangles** are \( K_r^{N-H_2^8} \), **Squares** are \( K_r^{N-H_2^9} \), and **Circles** are \( K_r^{N-H_2^{10}} \). The spectral ranges for the determination of \( K_r \) for \( S(8), S(9), \) and \( S(10) \) are 592.7 ~ 596.3, 598.1 ~ 602.0, 602.3 ~ 609.2 nm, respectively.
Fig. 10 Calculated Raman spectra of vibrational H₂ and H₂O in 30-atm H₂-air combustion at different equivalence ratios for the five target flames in Table 1 with ν_{spec} = 35 cm⁻¹. Dotted line is vib-H₂, Dashed line is H₂O, and Solid line is superimposed line (summation of the two spectra). Intensity is in arbitrary units but in the same scale for all equivalence ratios. Maximum values of vib-H₂ for the truncated peaks are included.
Fig. 11 Cross-talk rate ($K_p$) between $vib$-H$_2$ and H$_2$O spectrum in 30-atm H$_2$-air combustion at different equivalence ratios, calculated from spectra shown in Fig. 10. The spectral range for the determination of $K_p$ is from 650.0 to 664.0 nm.
Fig. 12 Temperature dependence of the cross-talk rate \( (K_r) \) in 30-atm H\(_2\)-air combustion. Here the cross-talk rate is normalized by the adiabatic number density of H\(_2\) for each condition to highlight the true temperature dependence. Circles are \( K_r^{N-Hs} \); Triangles are \( K_r^{VH-W} \). Lines are second-order polynomial fit to the points.