On the Relation Between Spotless Days and the Sunspot Cycle

Robert M. Wilson and David H. Hathaway
Marshall Space Flight Center, Marshall Space Flight Center, Alabama

January 2005
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<th>Description</th>
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<tr>
<td>AM</td>
<td>ascent-maximum amplitude (class)</td>
</tr>
<tr>
<td>AP</td>
<td>ascent-period (class)</td>
</tr>
<tr>
<td>ASC</td>
<td>ascent duration</td>
</tr>
<tr>
<td>E(FSD)</td>
<td>epoch of first spotless day</td>
</tr>
<tr>
<td>E(LSD)</td>
<td>epoch of last spotless day</td>
</tr>
<tr>
<td>EM</td>
<td>epoch of maximum amplitude</td>
</tr>
<tr>
<td>Em</td>
<td>epoch of minimum amplitude</td>
</tr>
<tr>
<td>FL</td>
<td>fast rise-large amplitude</td>
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<td>fl</td>
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<td>fast rise-short period</td>
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<td>FSD</td>
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<td>number of spotless days</td>
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<td>SS</td>
<td>slow rise-small amplitude</td>
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<tr>
<td>ss</td>
<td>slow rise-short period</td>
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</table>
NOMENCLATURE

cl  confidence level
n  number
P  probability
r  correlation coefficient
$r^2$  coefficient of determination
RM  maximum amplitude
Rm  minimum amplitude
$(Rm)_{n+1}$  $R_m$ for cycle $n+1$
se  standard error of estimate
t  elapsed time in months from $E_m$
t$_1$  elapsed time in months from $E(FSD)$ to $Em$
$(t_1)_{n+1}$  $t_1$ for cycle $n+1$
t$_2$  elapsed time in months from $E(LSD)$ to $EM$
t$_3$  elapsed time in months from $E(FSD)$ to $E(LSD)$
t$_4$  elapsed time in months from $E(LSD)$ for cycle $n$ to $E(FSD)$ for cycle $n+1$
$(t_4)_n$  $t_4$ for cycle $n$
x  independent variable
y  dependent variable
1. INTRODUCTION

In the mid-19th century (1826–1868), a German apothecary and amateur astronomer, Samuel Heinrich Schwabe, studied the Sun’s annual variation of the number of clusters of spots and the number of days when no spots were observed. From his observations, he found that the Sun’s spottedness varies over an interval of ≈10 yr, with the peak in the number of clusters of spots indicating a maximum in solar activity and the peak in the number of days when no spots were observed indicating a minimum in solar activity. Thus, he asserted that solar activity, as evinced by sunspots, varies somewhat regularly over time in cyclic fashion.1–5

Following this startling declaration, the professional Swiss astronomer Rudolf Wolf devised a simple formula for describing the solar cycle, one that combines the number of groups—similar to Schwabe’s clusters of spots—and the actual number of individual spots that can be seen on the Sun each day. On the basis of his relative sunspot number, he was able to approximate the past record of solar activity from the days of Galileo Galilei (1610) and initiate an international collaboration for monitoring sunspots that continues through the present. In contrast to the 10-yr length found by Schwabe, Wolf found the average length of the solar cycle to be ≈11 yr, although, strictly speaking, individual cycles were seen to vary in length up to several years either side of the 11-yr average.5–8

More recently, on the basis of another proxy of solar activity—the group sunspot number, arguments have been raised that, while the two proxies of solar activity differ only slightly since about 1882, their differences are more substantial during earlier years. Hence, because the group sunspot number is based on a greater number of observers than was used by Wolf, it has been suggested that Wolf’s reconstruction—both in terms of timing and amplitude—might be in error, especially for the earliest cycles—those prior to the mid-1800s.9–16

About 10 yr ago, Wilson showed that the first spotless day (FSD) prior to the onset of the new cycle, which occurs during the declining phase of the old cycle, can be used for the prediction of the occurrence of solar minimum of the new cycle.17 In this Technical Publication, the use of spotless days in relation to the timing and size of the solar cycle is again examined, but in much greater detail, as part of a continuing study of the characteristics of sunspot cycles.18–55
2. RESULTS AND DISCUSSION

2.1 Characteristics of Spotless Days

Figure 1 (bottom) displays the variation of smoothed monthly mean sunspot number (the 12-mo moving average of monthly mean sunspot number) for the interval of January 1973 through January 2004, plotted as the dotted line. Individual sunspot cycle numbers (21–23) are identified along the bottom, as are the epochs of cycle minimum (Em) and maximum (EM). Conventionally, a sunspot cycle is reckoned using smoothed monthly mean sunspot number for determining its minimum and maximum, ascent and descent durations, and period (or cycle length). In the upper portion of figure 1, the number of spotless days (NSD) is plotted, which allows for easy identification of the occurrences of the FSD and the last spotless day (LSD) for each of cycles 21–23. The actual number of spotless days that were observed in each of the cycles is also shown. Depicted between the two timelines are various descriptors based on the occurrence of spotless days. These include \( t_1 \), \( t_2 \), \( t_3 \), and \( t_4 \), where \( t_1 \) is the elapsed time from the epoch of FSD to Em, \( t_2 \) is the elapsed time from the epoch of LSD to EM, \( t_3 \) is the elapsed time from the epoch of FSD to the epoch of LSD, and \( t_4 \) is the elapsed time from the epoch of LSD for cycle \( n \) to the epoch of FSD for cycle \( n+1 \).

Figure 1. Variation of smoothed monthly mean sunspot number and spotless days for January 1973 through January 2004.
Table 1 summarizes selected parameters for the modern era sunspot cycles 10–24, including some information on cycle 9. Identified in the table are the cycle number, the epochs of FSD, minimum, LSD, and maximum—E(FSD), Em, E(LSD), and EM; \( t_1, t_2, t_3, \) and \( t_4 \); ascent duration (ASC) and period (PER); the number of spotless days (NSD) during \( t_1, t_3, \) and ASC; the minimum and maximum amplitudes \( Rm \) and \( RM \); and the ascent-period (AP) and ascent-maximum amplitude (AM) classifications of the cycles based on their median values, where sl, fl, ss, and fs mean, respectively, slow rise-long period, fast rise-long period, slow rise-short period, and fast rise-short period for the AP class, and SL, FL, SS, and FS mean, respectively, slow rise-large amplitude, fast rise-large amplitude, slow rise-small amplitude, and fast rise-small amplitude for the AM class.

<table>
<thead>
<tr>
<th>Cycle</th>
<th>E(FSD)</th>
<th>Em</th>
<th>E(LSD)</th>
<th>EM</th>
<th>( t_1 ) (mo)</th>
<th>( t_2 ) (mo)</th>
<th>( t_3 ) (mo)</th>
<th>( t_4 ) (mo)</th>
<th>ASC (mo)</th>
<th>PER (mo)</th>
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<th>Class</th>
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<td>–</td>
<td>Jul 1843</td>
<td>–</td>
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<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>55</td>
<td>149</td>
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<tr>
<td>10</td>
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<td>Dec 1855</td>
<td>Apr 1858</td>
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<td>22</td>
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<td>135</td>
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<td>Mar 1867</td>
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<td>Oct 1916</td>
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<td>Aug 1923</td>
<td>Jul 1926</td>
<td>Apr 1928</td>
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<td>Jul 1935</td>
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<td>122</td>
<td>114</td>
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<tr>
<td>24</td>
<td>Jan 2004</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
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<td>–</td>
<td>–</td>
<td>–</td>
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</tr>
</tbody>
</table>

\( t_1 \): E(FSD) → Em
\( t_2 \): E(LSD) → EM
\( t_3 \): E(FSD) → E(LSD)
\( t_4 \): E(LSD) → E(FSD)\(_{n-1}\)

Table 1. Selected sunspot cycle parameters.

Figures 2 and 3, respectively, show the 2 × 2 contingency tables for the AP and AM classes. Clearly, of the two classification schemes, the AM classification is presently the only statistically significant grouping, having a probability of obtaining the observed result, or one more suggestive of a departure from independence, based on Fisher’s exact test,\(^56\) \( P = 0.9 \) percent. Thus, there appears to exist a real correlation between the length of rise of a cycle and its maximum amplitude, associating fast-rising cycles with larger than average sized cycles and slow-rising cycles with smaller sized cycles. This correlation is often referred to as the “Waldmeier effect,” named in honor of Max Waldmeier, a former director of the Swiss Federal Observatory in Zürich, Switzerland, who first suggested such a relationship. (Interestingly, if cycle 23 turns out to be a cycle of shorter length, then it would be classified as fs and the probability would be reduced from 14.3 percent to ≈10 percent, which indicates a marginally significant result for the AP classification scheme.)
2.2 Cyclic Variations

Figure 4 depicts the cyclic variation of $Rm$, $RM$, ASC, and PER for the modern era cycles. In figure 4 and succeeding figures, except figs. 13 and 15, filled triangles indicate cycles of shorter length and filled circles indicate cycles of longer length. The filled box, associated with cycle 23, merely indicates that its period class is presently unknown.
On average, the elapsed time from cycle minimum to succeeding cycle minimum is \(\approx 131\) mo, having a standard deviation of \(\approx 10.3\) mo. Close inspection of PER, however, reveals that seven of the last eight fully described sunspot cycles have been cycles of shorter period, averaging \(\approx 121.9\) mo and having a standard deviation of 3.3 mo. Thus, statistically speaking, if cycle 23 turns out to be a cycle of shorter period, then Em for cycle 24 would be expected to occur about August 2006\(\pm 8\) mo. On the other hand, if cycle 23 bucks this recent trend and turns out to be a cycle of longer period, averaging \(\approx 138.7\) mo with a standard deviation of 3 mo, then Em for cycle 24 would be delayed until about late 2007 to early 2008.

Interestingly, if cycle 23 turns out to be a cycle of shorter period, then both \(Rm\) and \(RM\) would be expected to be larger than average in size, since shorter period cycles often are followed by cycles of larger than average minimum and maximum amplitudes. From figure 4, it can be shown that there exists a statistically significant (at the 0.5-percent level of significance or 99.5-percent level of confidence) upward trend in \(Rm\), one that can be described linearly as \(y = -4.416 + 0.622 \times\), where \(y\) is \(Rm\) and \(x\) is the cycle number. The inferred regression has a coefficient of correlation \(r = 0.726\) and standard error of estimate (se) of 2.6 units of sunspot number. Thus, presuming that the upward trend in \(Rm\) continues, cycle 24 would be expected to have an \(Rm = 10.5 \pm 4.6\)—the 90-percent prediction interval.

On the basis of figure 4, evidence for a strong upward linear trend in \(RM\) is lacking. However, as shown in a previous study using group sunspot number,\(^{16}\) the trend is quite noticeable, suggesting an \(RM\) that measures \(\approx 136.5 \pm 41.3\)—the 90-percent prediction interval—for cycle 24. Also, because of the Waldmeier effect, a larger than average sized \(RM\) for cycle 24 implies that its ASC will be shorter than average, meaning that cycle 24 would be expected to be a fast-rising cycle, one that peaks in <4 yr after Em, or probably sometime in 2010.

Figure 5 displays the variation of \(t_1, t_2, t_3,\) and \(t_4\). As before for \(Rm\) and \(RM\), variations of these parameters also appear to vary systematically, with a substantial decrease being seen in \(t_1\) and \(t_3\) and a substantial increase being seen in \(t_2\) and \(t_4\).

For \(t_1\), eight of the last eight cycles (16–23) have had a value that has ranged between 25 and 40 mo, averaging \(\approx 34\) mo with a standard deviation of 5.5 mo. Thus, presuming cycle 24 to behave similarly to cycles 16–23, Em for cycle 24 should occur about November 2006\(\pm 10\) mo, using January 2004 as E(FSD), for cycle 24, a date in close agreement found previously based on the assumption that cycle 23 is a cycle of shorter period.

For \(t_2\), eight of the last eight cycles (16–23) have had a value that has ranged between 20 and 29 mo, averaging \(\approx 25\) mo with a standard deviation of 3.7 mo. Thus, presuming cycle 24 to behave similarly to cycles 16–23, EM for cycle 24 should follow E(LSD) by 2 yr. E(LSD) for cycle 24 has not yet been observed.

For \(t_3\), eight of the last eight cycles (16–23) have had a value that has ranged between 44 and 75 mo, averaging \(\approx 54\) mo with a standard deviation of 10 mo. Thus, presuming cycle 24 to behave similarly to cycles 16–23, the interval between E(FSD) and E(LSD) for cycle 24 should extend \(\approx 4.5\) yr, inferring that E(LSD) for cycle 24 should occur sometime in 2008.
For $t_4$, eight of the last eight cycles (16–23) have had a value that has ranged between 50 and 83 mo, averaging ≈72 mo with a standard deviation of 11 mo. Thus, presuming cycle 24 to behave similarly to cycles 16–23, the interval between E(LSD) of cycle 24 and E(FSD) of cycle 25 should extend ≈6 yr, inferring that E(FSD) for cycle 25 should occur sometime around 2014.

Figure 6 shows the variation of the elapsed time from EM for cycle $n$ to E(FSD) for cycle $n+1$, mathematically equal to $t_4 - t_2$. For the last eight cycles (16–23), this difference has spanned 29 to 57 mo, averaging ≈47 mo with a standard deviation of 9 mo. Thus, presuming cycle 24 to behave similarly to cycles 16–23, the time from EM for cycle 24 to E(FSD) for cycle 25 should be ≈4 yr. During this interval, there will be zero spotless days; this interval corresponds to about 2010–2013.
Figure 6. Variation of $t_4 - t_2$ for cycles 10–23.

Figure 7 displays the variation of $t_3 + t_4$, which is the elapsed time from E(FSD) for cycle $n$ to E(FSD) for cycle $n+1$. Notice that a statistically significant downward trend is hinted, with the largest discrepancy being associated with cycle 15, the cycle marking the start of a string of short-period cycles. For cycle 24, the regression predicts the value for $t_3 + t_4$ to be $119.5 \pm 20.5$ mo—the 90-percent prediction interval.
Figures 8 and 9 depict the NSDs for three intervals of time: $t_1$, $t_3$, and ASC. For $t_1$, eight of the last eight cycles (16–23) have had a value of NSD that spans 114 to 316 days, averaging ≈198 days with a standard deviation of 78 days. For $t_3$, eight of the last eight cycles (16–23) have had a value of NSD that spans 226 to 568 days, averaging ≈361 days with a standard deviation of 133 days. More interestingly, for ASC, there appears to exist a statistically significant downward trend in NSD ($r = -0.7$ and $se = 63.8$), such that during the rising portion of cycle 24, one should expect <215 spotless days ($100 \pm 114$ days).

![Figure 8. Variation of NSD during $t_1$ for cycles 10–23.](image)

2.3 Parametric Correlations

Figure 10 shows scatterplots of $t_2$, $t_3$, and $t_4$ against $t_1$. Expressed as linear regressions, all are found to be statistically significant; expressed as 2×2 contingency tables, only the latter two scatterplots are statistically meaningful. Thus, knowledge of $t_1$ allows for determination of the later occurring parameters $t_2$, $t_3$, and $t_4$. Furthermore, knowledge of $t_1$ seems to provide a strong indication for the period class of a sunspot cycle, with shorter period usually being associated with shorter $t_1$ ($\leq 40$ mo) and longer period usually being associated with longer $t_1$ ($\geq 62$ mo). The recent behavior of $t_1$ (cycles 16–23) has
Figure 9. Variation of NSD during $t_3$ and ASC for cycles 10–23.

consistently been of shorter length and six of seven of the fully described cycles have been of shorter period. (Recall that, as yet, cycle 23’s period remains unknown.) Presuming cycle 24 to have the average length of 34 mo for $t_1$, based on cycles 16–23, then $t_2$, $t_3$, and $t_4$ equals, respectively, 24 ± 11 mo, 54 ± 18 mo, and 70 ± 24 mo using the inferred regressions—the 90-percent prediction intervals.

Figure 11 displays the scatterplots of $t_2$ and $t_4$ against $t_3$. Expressed as linear regressions, both are found to be statistically significant; expressed as $2 \times 2$ contingency tables, only the latter scatterplot is statistically meaningful. Thus, knowledge of $t_3$ allows for determination of the later occurring parameters $t_2$ and $t_4$. Presuming cycle 24 to have the average length of 54 mo for $t_3$, based on cycles 16–23, then $t_2$ and $t_4$ equals, respectively, 25 ± 9 mo and 71 ± 17 mo—the 90-percent prediction intervals.

Figure 12 depicts the scatterplot of $t_4$ against $t_2$. Expressed as a linear regression, it is found to be statistically significant; expressed as a $2 \times 2$ contingency table, it is not. Thus, knowledge of $t_2$ allows for determination of the later occurring parameter $t_4$. Presuming cycle 24 to have the average length of 25 mo for $t_2$, based on cycles 16–23, then $t_4$ equals 68 ± 22 mo—the 90-percent prediction interval.
Figure 10. Scatterplots of (a) \( t_2 \), (b) \( t_3 \), and (c) \( t_4 \) versus \( t_1 \).
Figure 11. Scatterplots of (a) $t_2$ and (b) $t_4$ versus $t_3$.

Figure 13 shows the scatterplot of $t_1$ for cycle $n+1$ against $t_4$ for cycle $n$. The correlation between the parameters is statistically important, whether it is expressed linearly or as a $2 \times 2$ contingency table. Furthermore, it is a valuable correlation that is directly applicable for predicting $t_1$ for cycle 24, since $t_4$ for cycle 23 has been determined (equal to 72 mo, on the basis of January 2004 being the E(FSD) for cycle 24; see table 1), indicated by the downward-pointing arrow along the $x$ axis. Based on the linear fit, $t_1$ for cycle 24 is expected to be $\approx 36 \pm 21$ mo—the 90-percent prediction interval. Based on the $2 \times 2$ contingency table, because $t_4$ for cycle 23 is to the right of the median (=50 mo, the vertical line), the expected value for $t_1$ for cycle 24 is expected to fall in the lower right quadrant of the $2 \times 2$ contingency table, indicating that its value should be $< 40$ mo (the horizontal line), probably somewhere between 25 and 36 mo—the observed range of values for the lower right quadrant. Using January 2004 as
E(FSD) for cycle 24 and expecting $t_1$ for cycle 24 to measure about 25–36 mo, based on the $2 \times 2$ contingency table, it follows that Em for cycle 24 should occur sometime in 2006 and implies that cycle 23 is a cycle of shorter period.

Figure 14 displays the scatterplot of $Rm$ against $t_1$. The correlation, whether expressed linearly or as a $2 \times 2$ contingency table, is statistically important. Based on the linear fit and using $t_1$ equal to 34 mo (the average of $t_1$ values for cycles 16–23), $Rm$ for cycle 24 is expected to be $\approx 7.9 \pm 4.6$—the 90-percent prediction interval. Based on the $2 \times 2$ contingency table, because $t_1$ for cycle 24 is expected to be $< 40$ mo, $Rm$ for cycle 24 should be in the upper left quadrant of the table, inferring that its value will be larger than $\approx 5.4$.

Figure 15 depicts the scatterplot of $Rm$ for cycle $n + 1$ against $t_4$ for cycle $n$. Expressed as a linear fit, the correlation is statistically important. Since $t_4$ has been determined to be $\approx 72$ mo for cycle 23, denoted by the downward-pointing arrow along the $x$ axis, $Rm$ for cycle 24 should measure $\approx 8.1 \pm 4.7$—the 90-percent prediction interval.
Figure 13. Scatterplot of $t_1$ for cycle $n+1$ versus $t_4$ for cycle $n$.

Figure 14. Scatterplot of $Rm$ versus $t_1$. 
Figure 15. Scatterplot of $Rm$ for cycle $n+1$ versus $t_4$ for cycle $n$.

Figure 16 shows the scatterplot of $Rm$ against NSD during $t_1$. The correlation is statistically important, whether expressed as a linear fit or in terms of a $2 \times 2$ contingency table. As noted earlier, NSD during $t_1$ has had a much narrower range during cycles 16–23 than during previous cycles (see fig. 8), averaging $\approx 198$ days with a standard deviation of 78 days. Presuming that NSD during $t_1$ for cycle 24 will also be near 200 days, then, according to the linear fit, $Rm$ for cycle 24 should be equal to $\approx 7.3 \pm 4.8$—the 90-percent prediction interval. Based on the $2 \times 2$ contingency table, $Rm$ should fall within the upper left quadrant, meaning that it should be larger than 5.1.

Figure 16. Scatterplot of $Rm$ versus NSD during $t_1$. 

Figure 17 displays scatterplots of $RM$ against $t_1$, $t_2$, and $t_3$. Expressed as linear fits, all correlations are found to be statistically important. Expressed as $2 \times 2$ contingency tables, only the correlation between $RM$ and $t_3$ is found to be statistically important; the correlation between $RM$ and $t_1$ is marginally significant. As yet, none of these parameters is strictly known, although an estimate for the expected value of $t_1$ for cycle 24 has been given above (from fig. 13 on the basis of the value of $t_4$ for cycle 23) and values for all the parameters can be estimated from their cyclic variations (fig. 5). Because $t_1$ for cycle 24 is expected to be <40 mo, probably in the range of 25 to 36 mo, using the average value of $t_1$ for cycles 16–23 (equal to 34 mo), implies that $RM$ for cycle 24 is expected to be equal to $\approx 136.4 \pm 61.8$—the 90-percent prediction interval. For $t_2$, using the average value found for cycles 16–23 (equal to 25 mo) suggests an $RM$ equal to $\approx 133.6 \pm 59.9$—the 90-percent prediction interval. Similarly, for $t_3$, using its average value for cycles 16–23 (equal to 54 mo) suggests an $RM$ equal to $\approx 139.500 \pm 54.5$—the 90-percent prediction interval. Together, these predictions seem to support the view that $RM$ for cycle 24 likely will be larger than average in size, hence, located within the upper left quadrant of each of the subfigures.

Figure 18 depicts scatterplots of $RM$ against NSD during $t_1$ and NSD during ASC. For the $RM$ against NSD scatterplots, the correlation is statistically significant, whether plotted linearly or as a $2 \times 2$ contingency table. Using a value of $\approx 200$ for NSD ($t_1$), $RM$ for cycle 24 is expected to be $\approx 135.3 \pm 55.6$—the 90-percent prediction interval, based on the linear fit, and greater than or equal to $\approx 115$, based on the $2 \times 2$ contingency table. For the latter, neither the linear fit nor the $2 \times 2$ contingency table is found to be statistically important, only of marginal or near marginal significance.

Figure 19 shows the scatterplot of PER versus $t_3 + t_4$. On the basis of the $2 \times 2$ contingency table, the distribution appears to be of marginal statistical significance. The importance of the plot is that a value of 117 (see table 1) for $t_3 + t_4$ has been determined for cycle 23, denoted by the downward-pointing arrow along the x axis, inferring that cycle 23 likely will be a cycle of shorter period. Hence, its period should fall in the lower left portion of the figure.
Figure 17. Scatterplots of $RM$ versus (a) $t_1$, (b) $t_2$, and (c) $t_3$. 
Figure 18. Scatterplots of $RM$ versus (a) NSD during $t_1$ and (b) NSD during ASC.

Figure 19. Scatterplot of PER versus $t_3 + t_4$. 
3. CONCLUSION

Section 2 has shown that spotless days can be used to characterize the sunspot cycle, in particular, the timing and size of its minimum and maximum amplitudes, and perhaps the length of the cycle. Variations of possibly systematic behavior are seen in the time between first spotless day occurrence and minimum amplitude occurrence \(t_1\), the time between last spotless day occurrence and maximum amplitude occurrence \(t_2\), the time between first and last spotless day occurrences \(t_3\), the time between last spotless day occurrence for cycle \(n\) and first spotless day occurrence for cycle \(n+1\) \(t_4\), the time between maximum amplitude occurrence for cycle \(n\) and first spotless day occurrence for cycle \(n+1\) (mathematically equivalent to \(t_4-t_2\)) and the time between first spotless day for cycle’s \(n\) and \(n+1\) (or \(t_3+t_4\)). For cycles 16–23, \(t_1\) has averaged \(\approx34\) mo, ranging between 25 and 40 mo; \(t_2\) has averaged \(\approx25\) mo, ranging between 20 and 29 mo; \(t_3\) has averaged \(\approx54\) mo, ranging between 44 and 75 mo; and \(t_4\) has averaged \(\approx72\) mo, ranging between 50 and 83 mo. Also, a statistically significant, linear downward trend \((r=-0.7\) and \(se=63.8\) days) in actual number of spotless days occurring in the ascending portion of the cycle is suggested, such that during the rise of cycle 24 there is only a 5-percent chance that more than 215 spotless days are expected. Likewise, there appears to exist a statistically significant, linear upward trend \((r=0.726\) and \(se=2.6\) units of sunspot number) in \(Rm\), such that for cycle 24 there is only a 5-percent chance that it will measure \(<5.9\).

Parameters \(t_2\), \(t_3\), and \(t_4\) are each correlated with \(t_1\) and \(t_1\) for cycle \(n+1\) correlates strongly with \(t_4\) for cycle \(n\). Because \(E(FSD)\) for cycle 24 occurred in January 2004, \(t_4\) for cycle 23 measures \(\approx72\) mo. Such a value suggests that \(t_1\) for cycle 24 will measure about 25–36 mo, implying that minimum amplitude for the next cycle will occur sometime in 2006 and that cycle 23 is a cycle of shorter period (also true from the PER versus \(t_3+t_4\) plot; see fig. 19).

The maximum amplitude \(RM\) is found to correlate strongly with \(t_1\). Presuming that \(t_1\) for cycle 24 will be about 25–36 mo, \(RM\) for cycle 24 is expected to be \(\approx136\), which agrees closely with its expected value based on the inferred linear upward trend of maximum amplitude of group sunspot number.

Figure 20 shows the variation relative to sunspot minimum occurrence of the average number of spotless days and the number of cycles having spotless days for cycles 16–23. Clearly, the nearer to \(Em\), the larger the average number of spotless days per month, with more than half of cycles 16–23 having a spotless day within 2 yr of sunspot minimum. If cycle 24’s minimum is to occur in the latter half of 2006, then plainly the number of spotless days will become more pronounced during 2005 and 2006.
Figure 20. Variation of the (a) average number of spotless days and (b) number of cycles having spotless days for cycles 16–23 between $t$ equal to −48 and 6 mo relative to $Em$. 
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On the Relation Between Spotless Days and the Sunspot Cycle

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Spotless days are examined as a predictor for the size and timing of a sunspot cycle. For cycles 16–23 the first spotless day for a new cycle, which occurs during the decline of the old cycle, is found to precede minimum amplitude for the new cycle by about ≈34 mo, having a range of 25–40 mo. Reports indicate that the first spotless day for cycle 24 occurred in January 2004, suggesting that minimum amplitude for cycle 24 should be expected before April 2007, probably sometime during the latter half of 2006. If true, then cycle 23 will be classified as a cycle of shorter period, inferring further that cycle 24 likely will be a cycle of larger than average minimum and maximum amplitudes and faster than average rise, peaking sometime in 2010.

Sun, sunspot cycle, solar cycle, solar cycle prediction, spotless days