Constitutive Theory Developed for Monolithic Ceramic Materials

With the increasing use of advanced ceramic materials in high-temperature structural applications such as advanced heat engine components, the need arises to accurately predict thermomechanical behavior that is inherently time-dependent and that is hereditary in the sense that the current behavior depends not only on current conditions but also on the material's thermomechanical history. Most current analytical life prediction methods for both subcritical crack growth and creep models use elastic stress fields to predict the time-dependent reliability response of components subjected to elevated service temperatures. Inelastic response at high temperatures has been well documented in the materials science literature for these material systems, but this issue has been ignored by the engineering design community. From a design engineer's perspective, it is imperative to emphasize that accurate predictions of time-dependent reliability demand accurate stress field information.

Ceramic materials exhibit different time-dependent behavior in tension and compression. Thus, inelastic deformation models for ceramics must be constructed in a fashion that admits both sensitivity to hydrostatic stress and differing behavior in tension and compression. A number of constitutive theories for materials that exhibit sensitivity to the hydrostatic component of stress have been proposed that characterize deformation using time-independent classical plasticity as a foundation. However, none of these theories allow different behavior in tension and compression. In addition, these theories are somewhat lacking in that they are unable to capture the creep, relaxation, and ratesensitive phenomena exhibited by ceramic materials at high temperatures.

The objective of this effort at the NASA Lewis Research Center has been to formulate a macroscopic continuum theory that captures these time-dependent phenomena. Specifically, the effort has focused on inelastic deformation behavior associated with these service conditions by developing a multiaxial viscoplastic constitutive model that accounts for time-dependent hereditary material deformation (such as creep and stress relaxation) in monolithic structural ceramics. Using continuum principles of engineering mechanics, we derived the complete viscoplastic theory from a scalar dissipative potential function. Constitutive equations for the flow law (strain rate) and evolutionary law were formulated on the basis of a threshold function, identified here as $F$ (see the figure), that is sensitive to hydrostatic stress and allows different behavior in tension and compression. For illustration, a set of threshold flow stress values has been adopted that roughly corresponds to values anticipated for isotropic monolithic ceramics. Specifically, the compressive uniaxial threshold stress value $\sigma_c$ is 2.00 MPa, and the tensile uniaxial threshold stress value $\sigma_t$ is 0.20 MPa. Furthermore, inelastic deformation is treated as inherently time dependent. A rate of inelastic strain is associated with every state of stress. As a result, creep, stress relaxation, and rate sensitivity are phenomena resulting from applied boundary conditions and are not treated separately in an ad hoc fashion.
Threshold function projected onto the $\sigma_{11}$-$\sigma_{22}$ stress plane.

Complete details of the model and its attending geometrical implications have been developed, but a quantitative assessment has yet to be conducted since the material constants have not been suitably characterized for a specific material. Incorporating this model into a nonlinear finite element code would provide industry a means to numerically simulate the inherently time-dependent and hereditary phenomena exhibited by these materials in service. Utilization of this approach has the potential to improve the accuracy of life prediction results for structural ceramics in high-temperature power and propulsion applications.

**Bibliography**


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