

# Study of Electromagnetic Scattering From Material Object Doped Randomly With Thin Metallic Wires Using Finite Element Method

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## Abstract

A new numerical simulation method using the finite element methodology (FEM) is presented to study electromagnetic scattering due to an arbitrarily shaped material body doped randomly with thin and short metallic wires. The FEM approach described in many standard text books [1,2] is appropriately modified to account for the presence of thin and short metallic wires distributed randomly inside an arbitrarily shaped material body. Using this modified FEM approach, the electromagnetic scattering due to cylindrical, spherical material body doped randomly with thin metallic wires is studied.

## I. Introduction

Electromagnetic (EM) scattering properties such as mono-static/bistatic radar cross section of a homogeneous material object can be controlled or modified by embedding metallic/non-metallic inclusions in the object. These inclusions can be in the form of short metallic wires, small thin metallic plates, or small metallic particles of various shapes [3]. The EM scattering properties can be controlled by selecting proper size, shape of these inclusions. Recent advances in fabrication technology have allowed for the inclusion of nano-scale metallic wires in these objects. However, current fabrication techniques used have no control over the spacing and arrangement of these small sized wires in a binding medium. As a result, these short wires are arranged in an arbitrary fashion. It is important and economically advantageous to know the EM scattering properties of these objects populated randomly with these wires/particles prior to its actual fabrication. In this work we describe a modified FEM approach to determine the EM scattering properties of randomly dispersed thin and short metallic wires in an arbitrarily shaped material object.

## II. Numerical Modeling

In this section, first, we present a brief outline of the FEM approach to estimate the EM scattering from an arbitrarily shaped material object without any metallic wires or other inclusions. Then, we explain the steps to modify this simple FEM approach to account for presence of randomly populated thin and short metallic wires in the object. Figure 1 shows an arbitrary shaped material object of permittivity  $\epsilon_r$  and permeability  $\mu_r$  populated randomly with small/nano size metallic wires. To estimate the EM scattering from the object shown in Figure 1 without any metallic wires, the entire scattering volume is split into two regions by enclosing the object by a fictitious surface  $S_1$  as shown in Figure 1. The electric field inside the surface  $S_1$  satisfy the vector wave equation

$$\nabla \times (1/\mu_r) \nabla \times \vec{E}^I - k_0^2 \epsilon_r \vec{E}^I = 0 \quad (1)$$

Following the usual steps involved in the FEM formulation, the equation (1) can be reduced to [4]

$$\int_V ((1/\mu_r) \nabla \times \vec{T} \cdot \nabla \times \vec{E}^I - k_0^2 \epsilon_r \vec{E}^I \cdot \vec{T}) dv = - \int_{S_1} \vec{T} \cdot \hat{n} \times ((1/\mu_r) \nabla \times \vec{E}^I) ds_1 \quad (2)$$

where  $\vec{T}$  is a vector testing function and  $\hat{n}$  is the unit outward normal to the surface  $S_1$ . If you introduce the surface current  $\vec{J}$  as an additional unknown, then the right hand side of equation (2) can be simplified to

$$-\int_{S_1} \vec{T} \cdot \hat{n} \times ((1/\mu_r) \nabla \times \vec{E}^I) ds_1 = j\omega\mu_0 \int_{S_1} \vec{T} \cdot \hat{n} \times \vec{H} ds_1 = j\omega\mu_0 \int_{S_1} \vec{T} \cdot \vec{J} ds_1$$

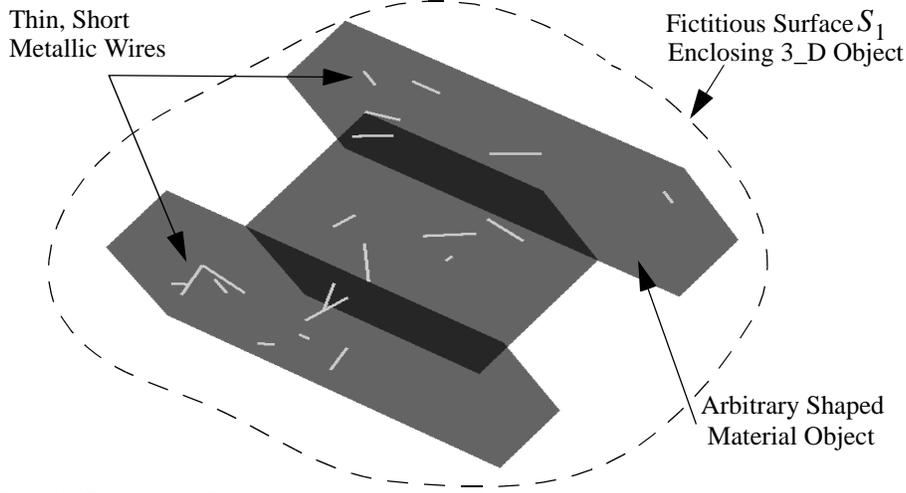


Figure 1 Geometry of arbitrary shaped material object embedded randomly with thin and short metallic wires.

The equation (2) can then be written as

$$\int_V ((1/\mu_r) \nabla \times \vec{T} \cdot \nabla \times \vec{E}^I - k_0^2 \epsilon_r \vec{E}^I \cdot \vec{T}) dv - j\omega\mu_0 \int_{S_1} \vec{T} \cdot \vec{J} ds_1 = 0 \quad (3)$$

In equation (3), in addition to the unknown electric field, the surface current  $\vec{J}$  is introduced as an extra unknown and to generate additional equations, continuity of tangential electric field across the fictitious surface boundary can be used. Hence,

$$\vec{E} \Big|_{S_1} = -\nabla \times \vec{F} - j\omega\mu_0 \vec{A} + 1/(j\omega\epsilon_0) \nabla \nabla \cdot \vec{A} + \vec{E}_{inc} \quad (4)$$

where  $\vec{A}$  and  $\vec{F}$  are the vector potential functions for the region outside the surface  $S_1$  and  $\vec{E}_{inc}$  is the incident field. To facilitate the solution of equations (3) and (4) the region inside the fictitious surface  $S_1$  is discretized into tetrahedron and the electric field over each tetrahedron can be expressed as

$$\vec{E}(x, y, z) = \sum_{n=1}^N \sum_{m=1}^6 b_m^n \vec{W}_m^n(x, y, z) \quad (5)$$

where  $b_m^n$  and  $\vec{W}_m^n(x, y, z)$  are, respectively, the unknown amplitude of electric field and the vector basis function associated with the  $m^{th}$  edge of  $n^{th}$  tetrahedron. The surface currents  $\vec{J}$  and  $\vec{M}$  required for the calculations of vector potentials can also be expressed in terms of vector basis functions  $\vec{W}$  as

$$\vec{J} = \sum_{i=1}^3 I_i \hat{n} \times \vec{W}_i, \quad \vec{M} = \sum_{i=1}^3 I_i \vec{W}_i \times \hat{n} \quad (6)$$

Using the expressions (5), (6), and the Method of Moments (MoM), the equations (3) and (4) are converted into set of simultaneous equations:

$$\begin{bmatrix} [S_{11}] & [S_{12}] \\ [S_{21}] & [S_{22}] \end{bmatrix} \begin{bmatrix} [b] \\ [I] \end{bmatrix} = \begin{bmatrix} [0] \\ [v] \end{bmatrix} \quad (7)$$

If  $N_{ed}$  are the number of tetrahedron edges and  $N_{apt}$  are the edges lying on the surface  $S_1$ , then the coefficient matrix is of the order  $(N_{ed} + N_{apt}) \times (N_{ed} + N_{apt})$ . which can be solved for  $b_m^n$  and  $I_i$ . From the knowledge of the surface currents, the EM scattering properties of an arbitrarily shaped object are estimated.

The simulation technique presented so far estimates the EM scattering from a homogenous material object. To estimate the EM scattering due to an arbitrarily shaped body populated randomly with short thin wires we follow the procedure described below. Let us assume that when the object is embedded with thin wires, a wire occupies one of the positions of edges out of total  $N_{ed}$  edges. To populate randomly the homogeneous object with  $N_{wire}$  number of wires, we generate  $N_{wire}$  random numbers between  $\{1 \sim N_{ed}\}$ . The edges corresponding to these random numbers are assumed to be occupied by thin metallic wires. Consequently,  $b_m^n$  coefficients on these edges will be zero. The modified matrix equation for the object randomly populated with thin metallic wires can be simply obtained by eliminating rows and columns corresponding to those edges where metallic wires are assumed to be present. Hence the matrix equation (6) when applied to the object loaded with thin metallic wires gets modified to

$$[S_{red}] [b] = [v_{red}] \quad (8)$$

where the  $[S_{red}]$  and  $[v_{red}]$  are the matrices obtained from  $[S]$  and  $[v]$  by eliminating the rows and columns corresponding to the edges where metallic wires are assumed to be present.

### III. Numerical Results and Discussion:

For a numerical experiment we consider a material sphere of radius  $k_0a = 1.0$  with permittivity  $\epsilon_r = 4.0 - j0.0$ , permeability  $\mu_r = 1.0 - j0.0$  illuminated by a plane EM wave. To estimate bistatic radar cross section of the material sphere we assume that the sphere is illuminated by a plane wave with incident angle  $\theta_{in} = 180^\circ$ ,  $\phi_{in} = 0^\circ$ . To facilitate the bistatic RCS calculation the sphere is discretized using the COSMOS/Geostar as is shown in Figure 2(a). Using the procedure described above the bistatic RCS of the material sphere is calculated and presented in Figure 2(b). The number of tetrahedron used to discretized the sphere were 123 resulting in  $N_{ed} = 238$  and  $N_{apt} = 132$ . The numerical data presented in Figure 2(b) are confirmed with the other published results. A good agreement between the results obtained using the present procedure and other published results confirms the validity of the present formulation.

Now to estimate the bistatic RCS for the material sphere doped with  $N_{wire}$  number of very thin metallic wires,  $N_{wire}$  random numbers uniformly distributed between 1 and  $N_{ed} = 238$  are generated. If  $P_d$  is the percentage of doping, then the number wires to be used for doping can be calculated from

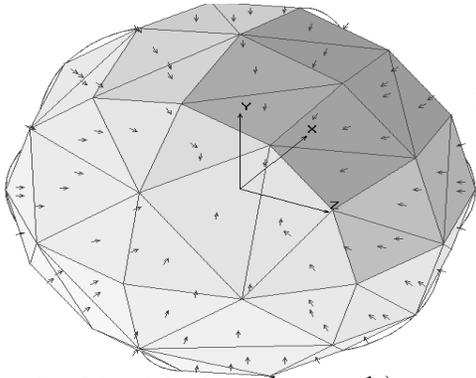


Figure 2(a) Material sphere ( $k_0 a = 1$ ) discretized using tetrahedron elements

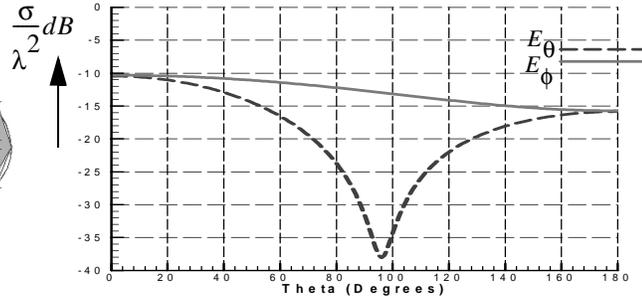


Figure 2(b) Bistatic RCS of material sphere shown in Figure 2(a)

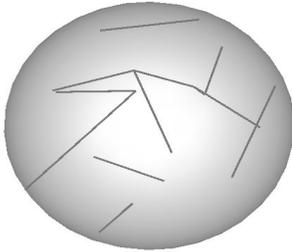


Figure 3(a) Material sphere ( $k_0 a = 1$ ) populated randomly with thin metallic wires

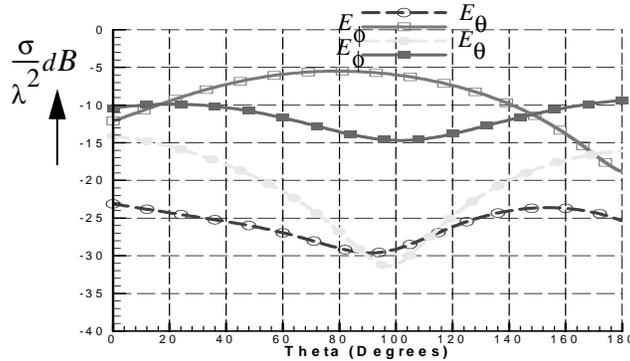


Figure 3(b) Bistatic RCS of material sphere shown in Figure 3(a)

$N_{wire} = \frac{P_d}{100} \times N_{ed}$ . For the present example, with  $P_d = 5$  and  $N_{ed} = 238$  choices the number of

wires used for doping were  $N_{wire} = 12$ . The edges, corresponding to these 12 random numbers (uniformly distributed between 1 and 238) are assumed to be thin metallic wires. The doped material sphere with the randomly oriented wires is shown in Figure 3(a). Following the FEM procedure described above, the bistatic RCS for the new structure is estimated and is shown in Figures 3(b). Various numerical examples were simulated for varying concentrations of wires, their orientations and placing. These results will be discussed at the time of presentation.

#### IV Conclusions

A new and modified finite element methodology (FEM) has been successfully developed to study electromagnetic scattering from an arbitrarily shaped material object populated randomly with thin metallic wires. Bistatic RCS of material objects in spherical, cylindrical, and cubical shapes randomly populated with thin metallic wires have been studied.

#### V References

- [1] John L. Volakis, et al., "Finite Element Method for Electromagnetics," IEEE Press, , New York, 1998.
- [2] J. Jin, "The Finite Element Method in Electromagnetics," John Wiley & Sons, Inc., New York, 1993
- [3] J. K. Abraham, et. al., "Tailoring the dielectric properties of meta materials, " 2003 IEEE MTT-S Digest, pp. 1837-1840.
- [4] M. D. Deshpande, et al., " A new approach to estimate complex permittivity of dielectric materials at microwave frequencies using waveguide measurements," IEEE Trans. Microwave and Theory Techniques, Vol. 5, No. 3 , pp. 359-366, March 1997.