Heat Transfer on a Flat Plate with Uniform and Step Temperature Distributions

Parviz A. Bahrami
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Nomenclature

- $C_p$ = specific heat of free stream
- $K$ = turbulent kinetic energy
- $k_\infty$ = thermal conductivity of free stream
- $k_t$ = turbulent thermal conductivity
- $l$ = overall length of the plate
- $Ma$ = free-stream Mach number
- $Pr$ = Prandtl number, $(Pr = \frac{C_p }{k_\infty} \frac{\mu_\infty}{\mu_\infty})$
- $Pr_t$ = turbulent Prandtl number, $(Pr_t = \frac{C_p }{k_t} \frac{\mu_t}{\mu_t})$
- $q$ = heat flux
- $Re$ = Reynolds number, $(Re = \frac{\rho_\infty U_\infty X}{\mu_\infty})$
- $St$ = Stanton number, $(St = \frac{q}{C_p \rho_\infty U_\infty \Delta T})$
- $T_\infty$ = temperature of free stream
- $T_O$ = stagnation temperature of free stream
- $T_W$ = wall temperature
- $T_r$ = recovery temperature
- $u_t$ = friction velocity, $(\tau_w / \rho_w)^{0.5}$
- $U_\infty$ = free-stream velocity
- $x$ = stream-wise distance from the plate leading edge
- $x_o$ = length of uncooled section
- $y$ = distance perpendicular to the plate
- $y^+$ = wall-related Reynolds number, $(\rho_u u y / \mu_u)$
- $\varepsilon$ = absolute rate of turbulent dissipation
- $\omega$ = specific rate of turbulent dissipation, $(\varepsilon / K)$
- $\mu_\infty$ = dynamic viscosity
- $\mu_t$ = turbulent dynamic viscosity
- $\rho_\infty$ = density at the wall
- $\rho_w$ = free-stream density
- $\tau_w$ = wall shear
- $\Delta T$ = free-stream to wall temperature difference, $(T_\infty - T_W)$
Heat Transfer on a Flat Plate With Uniform and Step Temperature Distributions

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Abstract

Heat transfer associated with turbulent flow on a step-heated or cooled section of a flat plate at zero angle of attack with an insulated starting section was computationally modeled using the GASP Navier-Stokes code. The algebraic eddy viscosity model of Baldwin-Lomax and the turbulent two-equation models, the K-ω model and the Shear Stress Turbulent model (SST), were employed. The variations from uniformity of the imposed experimental temperature profile were incorporated in the computations. The computations yielded satisfactory agreement with the experimental results for all three models. The Baldwin-Lomax model showed the closest agreement in heat transfer, whereas the SST model was higher and the K-ω model was yet higher than the experiments. In addition to the step temperature distribution case, computations were also carried out for a uniformly heated or cooled plate. The SST model showed the closest agreement with the Von Karman analogy, whereas the K-ω model was higher and the Baldwin-Lomax was lower.

Introduction

Computational fluid dynamics and heat transfer (CFDHT) as an effective design tool for aerospace vehicles and systems has lead to widespread utilization of and reliance on computer solutions in place of laboratory and flight tests. When studying complex phenomena, laboratory experimentation, apparatus, and instrumentation may also be substituted by judicious use of computer experimentation by numerically modeling representative cases.

Clearly, an integral and crucial element of CFDHT as a design tool for accurate simulations of flow field and heat transfer is turbulence modeling. CFDHT can be applied with confidence to a wide range of problems, provided its ability to produce accurate solutions to the actual problems under investigation is assessed. This assessment is generally referred to as experimental validation, which constitutes an impetus for the present research.

Validation of a CFDHT code is not a trivial task. It is highly desirable to employ laboratory experimental results for validation and calibration of CFDHT codes. The picture that emerges from consideration of various experimental and computational studies indicates that the differences in definitions of Stanton number and the heat transfer coefficient, as well as experimental and computational factors, require careful consideration to enable precise comparisons. Furthermore, to assess the accuracy of the computational results likely obtainable for a particular problem under investigation, one should also compare the results obtained with selected turbulence models for related geometry with available experimental data.

In an important class of problems, non-uniform surfaces are encountered in design of modern aerothermodynamic vehicles owing to, for example, material discontinuities or variations in surface emissivities. The situation may be analogous with jumps in surface catalytic efficiency, which accompany sudden surges in heat transfer. Computation by point-wise temperature specification for these important aerothermal applications may be necessary, for example, for cooled wall sections or situations where abrupt changes in the thermal conductivity of the wall material is encountered due to structural or manufacturing considerations.

The present work is concerned with determination of heat transfer through turbulent flow on heated or cooled regions of a vehicle’s surface with step-wise wall temperature distribution and comparison with the experimental results reported in the open literature. In addition to the aforementioned important applications, a wide range of other applications can be associated with this study, for example, for application to hot spots, or when local cooling of portions of a vehicle may be needed because they are subjected to high or non-uniform temperature and the material limitations of the thermal protection material or the structure may necessitate temperature control.

Thermal control can be employed by external means, for example, transpiration or film cooling, or with internal conductive or convective heat transfer. Transpiration and
film cooling, whereby fluid at a lower temperature than
the surface is added to the boundary layer, are effective
means of temperature control. A desirable feature of film
cooling is that the cooling actually extends some distance
downstream of the opening. The opening may, therefore,
be placed where the structure allows it and away from such
things as optical windows. In other applications it may be
desirable to avoid employing openings in the vehicle
walls for the purposes of film or transpiration cooling,
while producing some of their salient features. In these
cases portions of the wall may be held at specified
temperatures. Internal cooling can be achieved in instances
where the thermal conductivity of the wall material is
sufficiently high to allow heat transfer to the interior
region and removal by internal means.

To assess the accuracy of the computations, the results are
compared to available experimental data. Moretti and Kays
(1965) reported heat transfer data for a step distribution in
wall temperature (ref. 2). A more recent study by Orlando,
Moffat, and Kays (1974) is concerned with step wall
temperature distribution, but is limited to a strong adverse
pressure gradient (ref. 3). Taylor, Love, Coleman, and
Hosni (1990) presented experimental and computational
data for incompressible turbulent flow of air with an
unheated starting length followed by a constant wall heat
flux length (ref. 4). The experimental data of Moretti and
Kays (1965) was used in the present study to assess the
validity and accuracy of the present computations when
applied to variable wall temperature flow (ref. 2).

Non-Uniform Surface Temperature
Correlation

Wall temperature is often variable in practical
applications, while analyses are often carried out for
constant wall temperature conditions. A considerable
amount of work has been reported on heat transfer in
turbulent boundary layers, but few heat transfer
measurements have been reported that investigate heat
transfer with imposed variable wall temperature. A
comprehensive study of the turbulent flow over a flat plate
conducted by Reynolds, Kays, and Klein (1958) was
published in four reports (refs. 5–8). Heat transfer rates,
velocity profiles, and temperature profiles for turbulent
incompressible flow of air over a flat plate with a constant
surface temperature were measured and reported by
Reynolds, Kays, and Klein (1958) (ref. 5). The turbulent
heat transfer measurements agreed well with their power
representation of the Von Karman analogy for turbulent
incompressible flow of air.

\[ St = 0.0296 \ Re^{-0.2} \left( \frac{Pr \ T_w}{T_\infty} \right)^{-0.4} \]  

Heat transfer rates and temperature profiles for flow over a
flat plate with a stepwise temperature distribution were
reported by Reynolds, Kays, and Klein (1958) (ref. 6). An
integral analysis was conducted that allowed calculation of
heat transfer from a flat plate with a step wall temperature
distribution. The analysis was in agreement with all
available data. The data were used in a differential analysis
that allowed prediction of the temperature profiles. Other
analyses of varying wall temperature are reported by
Reynolds, Kays, and Klein (1958) and Kays, W. M., and

Experimental results of Moretti and Kays (1965) and
Moretti’s Ph. D. dissertation (1965) are of considerable
interest to this study (refs. 2, 10). They extend the results
of Reynolds, Kays, and Klein (1958) (refs. 5–8). The
results agreed with the momentum integral method of
Ambrok (1957) (ref. 11). The bulk of Moretti and Kays
(1965) (ref. 2) experiments pertain to flow conditions with
various rates of free-stream acceleration, but include an
important step wall temperature distribution case with
uniform free-stream conditions (ref. 2). The other reported
cases are incompressible constant-density flows and
include variations of longitudinal wall temperature and
pressure distributions with non-zero rates of free-stream
acceleration.

Geometry and Flow Description

Flat plate geometry is fundamental to fluid dynamics, but
more importantly, heat transfer from a flat plate with a
step temperature distribution may provide the basis for
analysis of complex wall temperature distribution
problems by employment of superposition. For the
present study, a flat plate geometry with the unheated
starting length and non-uniform surface heat flux
conditions was selected. Experimental data in the open
literature appear to be limited to the experiments of
Moretti and Kays (1965) (ref. 2), extending the results of
Reynolds, Kays, and Klein (1958) (refs. 2, 6). They
pertain to step wall temperature distributions of
incompressible turbulent flow over a flat plate without
pressure gradient. In the present computations, the flow
conditions were chosen to correspond to the conditions
attained in the laboratory wind tunnel experiments of
Moretti and Kays (1965) (ref. 2). The flow was uniform
and parallel to the plate in the free stream, and the
boundary layer was tripped and fully turbulent.
A schematic diagram of the physical problem under study is presented in figure 1. The sketch is a cross-sectional view of the configuration. As seen therein, the cooled region, enclosed by dashed lines, started at a distance \( x_o \) from the leading edge of the plate and continued in the flow direction through the rest of the plate length. Figure 1 also contains the dimensional nomenclature, as well as the \( x, y \) coordinates used to identify the position on the computational domain. Although the actual dimensions in the computational domain will be indicated shortly, it is relevant to note that the results have more general applicability when they are expressed in terms of dimensionless quantities. The dimensionless quantities, which govern the heat transfer results, may be written as Stanton number, \( St \), and Reynolds number, \( Re \).

\[
St = \frac{q}{C_p \rho \Delta T} \quad (2)
\]

\[
Re = \frac{\rho U_x x}{\mu} \quad (3)
\]

The present computations are for flow of air with the free-stream velocity, \( U_x = 19.39 \) m/s, free-stream temperature, \( T_x = 309.4 \) K, and free-stream density, \( \rho_x = 1.185 \text{ kg/m}^3 \). The laminar and turbulent Prandtl numbers are \( Pr = 0.72 \) and \( Pr_t = 0.9 \), respectively. The starting length of the plate, leading edge to \( x_o = 0.61 \) m, is insulated, whereas the rest of the length encounters spatially non-uniform heat flux conditions. The overall length dimension \( l \) of the plate is \( 1.83 \) m.

**GASP Code and Turbulent Models**

GASP, a commercially available finite-volume Navier-Stokes code, served as the means of computation. GASP is a fluid dynamics and heat transfer computational code that solves the integral form of time-dependent Reynolds Averaged Navier-Stokes (RANS) equations in three dimensions, employing the user-specified initial and boundary conditions. The code is fully conservative and shock capturing. The primitive variables, density, Cartesian velocity components, pressure, and temperature are stored throughout the code. A number of physical and thermodynamic models, as well as curve fits for transport properties, are selectable. In the present computations, a point-wise specification of the wall temperature was employed in the viscous no-slip boundary, and the Sutherland empirical formula for molecular viscosity was used.

For the present research the algebraic or zero-equation turbulence model of Baldwin-Lomax (1978) (ref. 12), the turbulent two-equation models, \( K-\omega \) model of Wilcox (1991) (ref. 13) and Shear Stress Turbulent model (SST) of Menter (1991) (ref. 14), were employed. All three models are eddy viscosity models. The zero-equation Baldwin-Lomax model is well adapted to attached flows, where a single well-defined layer can be identified. For more complex flows in which it may be difficult to define the velocity and length scales, the more advanced turbulence two-equation \( K-\omega \) model has been developed. The two-equation \( K-\omega \) model is one of several two-equation eddy viscosity models available.

Although a detailed discussion of these models is beyond the scope of this paper, since the equations in the \( K-\omega \) model are less stiff than the K-\( \epsilon \) model near the wall and a damping function is not required in the viscous sublayer, the \( K-\omega \) model was employed in the present research as an alternative to the algebraic turbulent eddy viscosity model of Baldwin-Lomax. The SST two-equation model was also used. It combines the \( K-\omega \) and K-\( \epsilon \) models and extracts some of the mixed derivatives, which allows determination of the specific rate of turbulence dissipation, \( \omega \), in the region between the wall and the boundary layer edge. The model constants, values used in Menter (1991) (ref 14.), were employed for all computations.

The GASP code uses a finite-volume spatial discretization method. The computational grid can be structured so that the state variables are stored at the centers of each control volume. GASP solves for the control volume averages and interpolates the solution to the boundaries. The wall temperature boundary condition values are specified at the internal control volume wall face center.

**Description of Grid**

For the present solutions, a grid containing 8000 points was created that resolved the viscous and thermal phenomena associated with this problem. There were 100 grid points in the stream-wise direction and 80 points in the normal direction to the free-stream direction. A simple stretching scheme was employed, which increased the spacing of the grid points with the distance from the wall in a fashion resembling a geometric series. As a guide, calculations were made to evaluate \( y+ \), which in essence is a wall-related Reynolds number. The first grid point was set to correspond to a \( y+ \) value of order unity or smaller than unity. By adjusting the values of the constants in the stretching functions, the grid could further be compressed near the wall. The resulting grid was more compressed in the \( y \) direction normal to the plate. It provided uniform, but relatively fine, spacing in the general flow direction,
nature provided a continuously varying temperature profile variations of order 2 K. Furthermore, as may be expected, somewhat non-uniform in temperature, with the mean and somewhat non-uniform. The cooled section was also and the free stream. The temperature of this uncooled and the plate. There also existed a small, but non-zero, temperature difference between the starting section of the plate and the free stream. A line is passed through the data points for clarity. Although the experiments were planned and carried out with care, some non-uniformity in the wall temperatures could not be avoided, due to experimental difficulties in completely insulating the starting section of the plate. There also existed a small, but non-zero, temperature difference between the starting section of the plate and the free stream. The temperature of this uncooled section of the plate was 2 K cooler than the free stream and somewhat non-uniform. The cooled section was also somewhat non-uniform in temperature, with the mean variations of order 2 K. Furthermore, as may be expected, nature provided a continuously varying temperature profile in the region between the cooled and uncooled sections, rather than a sharp discontinuity. The overall uncertainties in measurement of temperature, heat flux, and velocity in the cooled section were estimated to be approximately 3%, 2%, and 1%, respectively.

From the foregoing discussion it should be clear that the conditions in the laboratory experiments deviated somewhat from the ideal: an adiabatic and uniform temperature starting section and a sharp step function temperature imposed in the non-adiabatic section. In order to model this experiment accurately, it was useful to incorporate the temperature profile of the entire length of the plate, especially since the temperature differences between the cooled section of the plate and the free stream, ∆T, were relatively small. This reduced the possibility that not accounting for the small temperature anomalies might affect the temperature history of the thermal boundary layer and alter the local heat flux in comparison to the ideal case. The point-wise plate temperature profile was entered into the GASP input file and the computations were initiated. The convergence criteria were set at 10^-6 for successive values of all variables. The velocity and temperature fields converged in less than 4000 iterations and remained unchanged when the number of iterations were doubled. The laminar sub-layer was well defined and the velocity and temperature profiles were linear in the inner wall region.

As mentioned earlier, the dimensionless quantities, which govern the heat transfer results, were chosen as Stanton number, St, and Reynolds number, Re. While the code determines the values of the physical properties for each control volume, the use of free-stream values in calculation of the Reynolds and Stanton numbers is preferred, since end users are more likely to possess the free-stream data. Use of the free-stream values in calculation of the dimensionless parameters also reduces the confusion often associated with the variations in reference temperature concept when employing CFDHT results.

Before discussing the step temperature results, it is useful to examine the computational results that were obtained for the same conditions employed above, but with uniform wall temperature distribution on the entire plate. The results of these computations are shown in figure 3 together with the curve of Reynolds, Kays, and Klein (1958) for the Von Karman analogy for comparison (ref. 5). As shown therein, the Shear Stress Turbulent two-equation model demonstrated the closest agreement with the Von Karman analogy, while the turbulent two-equation K-ω model was higher and the algebraic eddy viscosity model of Baldwin-Lomax was lower. Although a satisfactory level of agreement is evident for both

Results and Discussion

The experimental heat transfer results of Moretti and Kays (1965) (ref. 2) for an approximate step function wall temperature distribution and non-accelerating flow conditions provided a unique opportunity for a substantive comparison with the present computations. Most of the pertinent data collected during experimentation was reported in Moretti (1965) (ref. 10). In their experiments, the starting length of the plate was insulated, whereas electrical currents to strip heaters further downstream were adjusted to achieve a pre-selected and relatively uniform temperature jump. In order to assess the accuracy of the present computations, the experimental conditions were computationally modeled in the present study. The actual experimental free-stream and surface temperature data were employed as boundary conditions in the GASP code. When the free-stream values for the three components of velocity, the air temperature, and density were set according to the experimental values stated earlier, the resulting Mach number was Ma = 0.055 and the unit Reynolds number was Re/x = 1,215,400 m^-1.

In order to discuss the variations of the wall temperature in Moretti (1965), the wall temperature data have been recast in the SI units and plotted as a function of the Reynolds number, Re, in figure 2 (ref. 8). The symbols are the measured data, and the Reynolds number is based on the length of the plate from the leading edge, using the free-stream conditions. A line is passed through the data points for clarity. Although the experiments were planned and carried out with care, some non-uniformity in the wall temperatures could not be avoided, due to experimental difficulties in completely insulating the starting section of the plate. There also existed a small, but non-zero, temperature difference between the starting section of the plate and the free stream. The temperature of this uncooled section of the plate was 2 K cooler than the free stream and somewhat non-uniform. The cooled section was also somewhat non-uniform in temperature, with the mean variations of order 2 K. Furthermore, as may be expected, nature provided a continuously varying temperature profile in the region between the cooled and uncooled sections, rather than a sharp discontinuity. The overall uncertainties in measurement of temperature, heat flux, and velocity in the cooled section were estimated to be approximately 3%, 2%, and 1%, respectively.

From the foregoing discussion it should be clear that the conditions in the laboratory experiments deviated somewhat from the ideal: an adiabatic and uniform temperature starting section and a sharp step function temperature imposed in the non-adiabatic section. In order to model this experiment accurately, it was useful to incorporate the temperature profile of the entire length of the plate, especially since the temperature differences between the cooled section of the plate and the free stream, ∆T, were relatively small. This reduced the possibility that not accounting for the small temperature anomalies might affect the temperature history of the thermal boundary layer and alter the local heat flux in comparison to the ideal case. The point-wise plate temperature profile was entered into the GASP input file and the computations were initiated. The convergence criteria were set at 10^-6 for successive values of all variables. The velocity and temperature fields converged in less than 4000 iterations and remained unchanged when the number of iterations were doubled. The laminar sub-layer was well defined and the velocity and temperature profiles were linear in the inner wall region.

As mentioned earlier, the dimensionless quantities, which govern the heat transfer results, were chosen as Stanton number, St, and Reynolds number, Re. While the code determines the values of the physical properties for each control volume, the use of free-stream values in calculation of the Reynolds and Stanton numbers is preferred, since end users are more likely to possess the free-stream data. Use of the free-stream values in calculation of the dimensionless parameters also reduces the confusion often associated with the variations in reference temperature concept when employing CFDHT results.

Before discussing the step temperature results, it is useful to examine the computational results that were obtained for the same conditions employed above, but with uniform wall temperature distribution on the entire plate. The results of these computations are shown in figure 3 together with the curve of Reynolds, Kays, and Klein (1958) for the Von Karman analogy for comparison (ref. 5). As shown therein, the Shear Stress Turbulent two-equation model demonstrated the closest agreement with the Von Karman analogy, while the turbulent two-equation K-ω model was higher and the algebraic eddy viscosity model of Baldwin-Lomax was lower. Although a satisfactory level of agreement is evident for both
Baldwin-Lomax and the K-ω turbulent models, deviations of approximately 10 percent were present.

The results of the computations for the step temperature profile are shown in figure 4 together with the experimental data of Moretti and Kays (1965) (ref. 2). The experimental data are shown by circle symbols. It should be noted that, since Stanton number contains the ratio of the heat flux to the temperature difference, for small values of heat flux as a result of small temperature differences, small errors including neglecting the radiation effects, extraneous heat losses, and variations in physical properties, can alter its value markedly. This is indeed the case for the adiabatic section of the plate. For this reason only the Stanton number values for the heated or cooled section of the plate have significance, especially when comparisons between experiments and computations are made.

As evident in figure 4, the algebraic eddy viscosity model of Baldwin-Lomax showed close agreement with the experiments, while the two-equation Shear Stress Turbulent model was higher and the K-ω was still higher. As with the uniform wall temperature distribution case, a similar trend of agreement is evident between the three models. The agreement of the SST model results are within approximately 10 percent, while the K-ω model results are within approximately 15 percent of the experimental data. As mentioned earlier, a detailed discussion of turbulent modeling is beyond the scope of the present report. However, it should be pointed out that the turbulent thermal conductivity, \( k_t \), is extracted from the definition of the turbulent Prandtl number,

\[
Pr_t = \frac{C_p \mu_t}{k_t}
\]

and applied in the Boussinesq approximation of the turbulent heat transfer. The three models predict the heat transfer somewhat differently, not only because of the differences in the convective velocity, but also because of the implicit dependence of \( k_t \) on \( \mu_t \).

Inspection of figures 3 and 4 further reveals that the shapes of the curves for the heated or cooled section of the step temperature case resemble those for the uniform wall temperature case. This suggests that the results of the uniform wall temperature case may be applicable to the step wall temperature case, if the values of Reynolds number were shifted by the value at the start of the heated or cooled section. To investigate this possibility, the data of figures 3 and 4 are superimposed in figure 5, but the values of Reynolds number for the uniform wall temperature case were uniformly increased by 740,910, the value corresponding to the Reynolds number at the start of the cooled section. Figure 5 indicates that there is a remarkable level of agreement between each pair of curves, corresponding to each turbulent model far downstream from the step in wall temperature or the plate leading edge. This result indicates the diminishing influence of the early momentum boundary layer on the far downstream region. The heat transfer close to the leading edge and for some distance downstream from the leading edge, however, is higher due to the thinner momentum boundary layer of the uniform wall temperature case. One may, for first approximation purposes, reasonably (within 20 percent) estimate the heat transfer in the cooled section of the step wall temperature problem by employing the results from the uniform wall temperature case.

It should be noted that for the present conditions, the radiation effects had introduced only minor errors due to the relatively low temperatures employed in the experiments. The uncertainty in measurement of temperature, heat flux, and velocity in the cooled section were estimated by Moretti and Kays (ref. 2) to be approximately 3%, 2%, and 1%, respectively. This appears to have caused only minor errors. The stagnation temperature of the free stream, \( T_o \), and the recovery temperature, \( T_r \), were very close in value to the free-stream temperature, \( T_\infty \). The differences in definitions of Stanton number and the heat transfer coefficient, often encountered when attempting to compare reported experimental and computational results due to varied employment of \( T_r \) and \( T_o \), did not come into play in the present investigations.

**Concluding Remarks**

Heat transfer on a flat plate with uniform and step temperature distribution was successfully modeled. The pointwise wall temperature specification feature of the GASP code was employed to obtain the computational results. Employment of the algebraic eddy viscosity model of Baldwin-Lomax and the turbulent two-equation models, the K-ω model and the Shear Stress Turbulent model, for the local heat transfer on the plate with step temperature distribution resulted in solutions in agreement with the experiments of Moretti and Kays (1965) within 15 percent (ref. 2). The computational results obtained for uniform plate temperature were within 10 percent of the Von Karman analogy. For the present investigation, the full three-dimensional Navier-Stokes form was utilized, where primitive variables were density, Cartesian velocity components, pressure, and temperature. These agreements lend support to the applicability of the code and its
implementation of the turbulence models as a tool for engineering design application, as well as for computer experimentation in studying related problems with non-uniform wall temperature. It was shown that, for engineering design purposes, one could estimate the heat transfer to the cooled section of the step wall temperature problem by employing the results of the uniform wall temperature case if the magnitude of the temperature difference applied is relatively small.

References

Figure 1. Flat plate with step wall temperature distribution.

Figure 2. Distribution of the free stream to wall temperature difference.
Figure 3. Comparison of the heat-transfer results for uniform wall temperature with the Von Karman analogy.

Figure 4. Comparison of the heat-transfer results for step wall temperature with the experiments.
Figure 5. Comparison of the heat-transfer results for step and uniform wall temperature, when the plates are shifted.
Heat transfer associated with turbulent flow on a step-heated or cooled section of a flat plate at zero angle of attack with an insulated starting section was computationally modeled using the GASP Navier-Stokes code. The algebraic eddy viscosity model of Baldwin-Lomax and the turbulent two-equation models, the K-ω model and the Shear Stress Turbulent model (SST), were employed. The variations from uniformity of the imposed experimental temperature profile were incorporated in the computations. The computations yielded satisfactory agreement with the experimental results for all three models. The Baldwin-Lomax model showed the closest agreement in heat transfer, whereas the SST model was higher and the K-ω model was yet higher than the experiments. In addition to the step temperature distribution case, computations were also carried out for a uniformly heated or cooled plate. The SST model showed the closest agreement with the Von Karman analogy, whereas the K-ω model was higher and the Baldwin-Lomax was lower.

Turbulent flow, Heat transfer, Experimental fluid dynamics, Turbulence modeling