Universal Dynamics of a Strongly-Interacting Fermi Gas

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04/16/2003
• Current work in atomic DFGs
• New frontier: Universal Behavior in Strongly-Interacting Fermi Gases
• Free Expansion Experiment
• Extraction of a Universal Parameter
• Summary
Over the past several years, there have been a number of atomic degenerate Fermi gases created around the world. In every case, these degenerate gases were produced via evaporative cooling in a neutral atom trap.

At JILA and in our own work at Duke, the evaporation is driven by collisions between two hyperfine states of a single atomic species. The other experiments use “sympathetic cooling” where the fermionic species shares the trap with a bosonic partner in thermal equilibrium.

The NASA fundamental physics in microgravity program is well represented in the field---the Rice, Duke and MIT groups are all speaking at this conference.
An exciting motivator in the field of atomic Fermi-physics is the possibility, first suggested only a few years ago, that the superfluid transition temperature might be extraordinarily high in the region where the interatomic interactions are enhanced by a Feshbach resonance. The system characterized by this transition lies in the “cross-over region” between weakly-bound cooper pairs and a BEC of tightly-bound molecular dimers.

The transition temperature for a harmonically-trapped degenerate gas of fermions is predicted to be as high as half of the Fermi temperature (several orders of magnitude larger than the transition temperature in metallic superconductors).

In $^6$Li, our atom of choice, to access the resonance needed for this effect requires magnetic fields in the range of 850-1100 G.
Strong interatomic interactions are produced in the system by a Feshbach resonance. In a Feshbach resonance, the energy of the colliding atoms is energetically tuned into resonance with a bound state on a closed exit channel. In $^6$Li, the two relevant channels are the single and triplet potentials. The triplet state tunes in a magnetic field, so the relative spacing between the potentials can be varied by changing an externally-applied magnetic field.

The result is a magnetically-tunable interaction, characterized by the zero-energy $s$-wave scattering length, which can take on all values.
This plot shows the s-wave scattering length as a function of magnetic field for the two lowest hyperfine groundstates of $^6$Li. Two Feshbach resonances are shown, a narrow resonance at approximately 550G and a very broad resonance centered near 850 G.

In addition, it is important to note another nice feature of this atomic system—-the scattering length, and hence the interactions, are zero at zero magnetic field. Thus we can change from an interacting system to a non-interacting one (or vice-versa) simply by applying or removing a magnetic field.

\[ \sigma = 4\pi a_s^2 \]
Unitarity Limit

- Interaction length scale cannot truly become infinite. Limited to length scale of wavepacket.
  \[ a_{\text{eff}} \approx \pm 1/k \text{ as } a \to \pm \infty \]
- In a highly-degenerate Fermi system \( k_{\text{max}} = k_F \)
  - Scattering length saturates at smaller magnitude than in equivalent Bose system
- Majority of predictions in this field do not consider unitarity. Need to revise predictions now that many experiments operate in this limit.

There is, however, a problem with the simplistic view presented so far---it ignores the quantum-mechanical requirement of unitarity.

Because of the requirement of unitarity, the interaction strength cannot truly become infinite. The effective scattering length is, in fact, limited to approximately the inverse of the wavenumber. This has interesting implications in a Fermi system. For a degenerate Fermi system, there will be wavenumbers up to the Fermi wavenumber. In contrast, in a degenerate Bose system, the wavenumbers are all much smaller and correspond to the ground state of the confining potential. This means that the scattering length saturates to a much smaller value in Fermi systems than in Bose systems.

Perhaps because unitarity is not particularly important in Bose systems, many of the early predictions about fermions did not consider this important fact. Now that all the major experiments are operating in regimes where the scattering length is resonantly enhanced, it is crucial to review earlier predictions in light of unitarity.
### Intermediate-Density Regime

<table>
<thead>
<tr>
<th>Density Regime</th>
<th>Condition</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>Low Density:</td>
<td>$k_F</td>
<td>a</td>
</tr>
<tr>
<td>High Density:</td>
<td>$k_F R \geq 1$</td>
<td>In the extreme limits $R \to 0$, $</td>
</tr>
<tr>
<td>Intermediate Density:</td>
<td>$R \leq 1/k_F \leq</td>
<td>a</td>
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Things become even more interesting when we start to consider many-body effects.

There are two natural limits when discussing these types of systems: the low density limit where the interaction length scale is much smaller than the interatomic spacing, so that many body effects are unimportant, and the high density limit where the interaction length scale is much larger than the range of the interaction potentials, in which case mean-field treatments are excellent approximations.

In the intermediate density regime, neither of these limits apply, and initially one might suspect the system is intractable. However, a very interesting thing happens in this limit. The two length scales of the interaction (the scattering length and the range of the potential) become effectively either infinity or zero—leaving the inverse Fermi wavenumber as the only length scale in the problem.

It has been suggested that systems in this regime are universal. The behavior of the system is independent of the details of the interaction and is applicable to all fermion systems.
If universal behavior does exist in the intermediate density regime, then all physical parameters become proportional to the Fermi parameters (which themselves are proportional to powers of the Fermi wavenumber).

Measurements of these proportionality constants should then be applicable to all Fermi systems in the intermediate density regime, regardless of the nature of their interactions.

The hunt for resonance superfluidity can then be viewed in this light as a quest to measure the universal proportionality constant between the transition temperature and the Fermi temperature.
This is a schematic of the sequencing of our experiment.

We create a MOT of $^6$Li atoms in a UHV system. We then pass a single, focused CO$_2$ laser beam into the system through ZnSe windows, placing the focus in the center of the MOT.

Several million $^6$Li atoms at 150 microKelvin are transferred into the dipole force trap formed at the focus, and the MOT beams are extinguished. We change the orientation of one magnetic field coil and apply 900G to the sample, initiating rapid evaporative cooling via the Feshbach resonance.

Once we have a highly degenerate sample, the CO$_2$ laser is extinguished, allowing the gas to expand. After a desired expansion time, we pass a resonant absorption beam through the cloud and capture the shadow image on a CCD array.
We can release the cloud for varying times and then image using absorption imaging to get the profile of the atomic cloud. The cloud starts in a long, cigar shape and expands in a very anisotropic manner. The narrow direction of the cloud expands quickly, while the initial long direction hardly moves.

This is quite different from the behavior of a non-interacting degenerate Fermi gas, where the isotropy of the momentum would lead to the cloud taking on a spherical shape at long times.

This behavior was initially suggested as a hallmark of superfluidity, although we have shown that it can also arise as a result of unitarity-limited collisions. Thus, while we may have produced a superfluid, we cannot claim so at this point.

The image on the left show the cloud as it expands in time.

The plot on the right shows the expansion of the initially long dimension as a function of time.
This plot shows the expansion of the initially narrow dimension as a function of time. It is quite clear that this direction expands much more rapidly than the long dimension.
This is a plot of the measured aspect ratio as a function of time (red dots with error bars). The red line which closely approximates the data is for purely hydrodynamic expansion with no free parameters.

The blue data is for a non-interacting cloud. Note that it approaches an aspect ratio of 1 at long time---as we would expect. The curve passing through the blue data is for ballistic expansion with no free parameters.

The orange and green curves show what the expansion would look like if the system were not hydrodynamic, but did have repulsive or attractive mean field interactions, respectively.
The previous slide showed that hydrodynamic expansion seemed to explain the observed data, however there was a non-negligible deviation at long times. This plot shows the growth of both the initially narrow (red) and initially long dimensions of the cloud.

The solid curves are once again hydrodynamic expansion with no free parameters. It is clear that the radial expansion is well explained by this model. We do not yet have an explanation why the axial dimension of the cloud walks off the curve at long time.
Universal Parameter $\beta$

We define the universal parameter $\beta$, the ratio of the mean field energy to the local Fermi energy

$$U_{MF}(x) = \beta \varepsilon_F(x)$$

Measurement of $\beta$ is equivalent to a measurement of the mean-field energy per particle.

Exact value is an open question. Predictions spanning over 30 years:

- Steele, nucl-th/0010066 (2000)

From the expansion data we can make a first measurement of a universal parameter of a degenerate Fermi gas in the intermediate density regime.

In the universal regime, the contribution of the mean field to the chemical potential is proportional to the local Fermi energy. We define the constant of proportionality as $\beta$.

Measuring $\beta$ is equivalent to measuring the mean-field energy per particle in units of the local kinetic energy. This parameter has been the subject of theoretical predictions for well over 30 years.
We can extract a measurement for $\beta$ from our data by relating the kinetic energy of the released cloud to its size after expansion. We account for the finite temperature of the cloud with a Sommerfeld expansion of the density and for the mean-field interaction using the parameter $\beta$. 

\[
\langle KE \rangle = \frac{3}{8} k_B T_F \sqrt{1 + \beta}
\]

\[
\langle KE \rangle = \frac{3}{8} k_B T_F \sqrt{1 + \beta} \left[ 1 + \frac{2\pi}{3} \left( \frac{T}{T_F} \right)^2 \right]
\]

@ $k_F a \simeq -7.4$ We Measure:

- $\langle E_x \rangle = 1.44 \mu K$
- $T_F = 7.9 \mu K$
- $T/T_F \simeq 0.1 - 0.15$
- $\beta = -0.26 \pm 0.07$
Comparison to Theory

Duke Measurement: $\beta = -0.26 \pm 0.07 \, @k_F a \sim -7.4$

| Predictions $@ |k_F|a| \to \infty$ | Predictions $@ k_F a = -7.4$ |
|----------------|-----------------|
| Randeria:      | $<-0.410$       | Steele:          | -0.460 |
| Baker:         | -0.674          | Heiselberg:      | -0.540 |
|                | -0.432          |                  |       |
| Steele:        | -0.674          |                  |       |
| Heiselberg:    | -0.674          |                  |       |
|                | -0.330          |                  |       |
| Carlson:       | -0.560          |                  |       |

Recent Measurement: $\beta \simeq -0.3$

Bourdel, et. al.,
cond-mat/0303079 (2003)

We measure beta=-0.26.

Most theoretical predictions are for the limit where the product of the Fermi wavenumber and the scattering length is infinite. Only a few prediction have explicit $k_F a$-dependence which we can evaluate.

We find qualitative agreement with the theoretical predictions. Quantitatively, we differ by roughly a factor of two. Where does this discrepancy come from? Recent experiments at ENS in Paris give an answer in close agreement with our measurement. Additionally, some of the best theoretical predictions explicitly ignore terms of order 25%. Further refinements to theory will no doubt help to reconcile the two values. Also, the initial experiments are not as sensitive to beta as we would like. Further experiments should allow us to refine our estimate and reduce the likelihood of systematic errors.
Summary

- Unitarity limit has now become relevant for DFGs
- Possibility of universal behavior in strongly-interacting Fermi systems leads to useful connections to other branches of physics (nuclear, ??)
- First experiments in intermediate-density regime
  - Duke
  - JILA, Paris, MIT
- Demonstration of mechanical stability (submitted to PRL)
- Theoretical treatment of unitarity-limited elastic collision rate (paper in preparation)
- Looking for superfluid “smoking gun”
- Further studies of universal behavior

In summary, all the DFG experiments are now working in the strongly-interacting unitarity-limited regime. This limits the applicability of many previous theoretical predictions and may serve as a test for recent theories of strongly-interacting Fermi systems.

An exciting new feature of DFG physics is the possibility of universal behavior in these systems.

The first experiments in the intermediate-density regime have been performed in the past year.

Our work has shown that strongly attractive degenerate Fermi gases are mechanically stable, in contrast to Bose systems. We have also begun to explore the theoretical implications of unitarity on these systems.

We are currently working towards a clear-cut test for superfluidity in our experiment and are beginning further studies of universal behavior.
The Team

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