Analytic Modeling of the Hydrodynamic, Thermal, and Structural Behavior of Foil Thrust Bearings

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Abstract

A simulation and modeling effort is conducted on gas foil thrust bearings. A foil bearing is a self acting hydrodynamic device capable of separating stationary and rotating components of rotating machinery by a film of air or other gaseous “lubricant.” Although simple in appearance these bearings have proven to be complicated devices in analysis. They are sensitive to fluid structure interaction, use a compressible gas as a lubricant, may not be in the fully continuum range of fluid mechanics, and operate in the range where viscous heat generation is significant. These factors provide a challenge to the simulation and modeling task. The Reynolds equation with the addition of Knudsen number effects due to thin film thicknesses is used to simulate the hydrodynamics. The energy equation is manipulated to simulate the temperature field of the lubricant film and combined with the ideal gas relationship, provides density field input to the Reynolds equation. Heat transfer between the lubricant and the surroundings is also modeled. The structural deformations of the bearing are modeled with a single partial differential equation. The equation models the top foil as a thin, bending dominated membrane whose deflections are governed by the biharmonic equation. A linear superposition of hydrodynamic load and compliant foundation reaction is included. The stiffness of the compliant foundation is modeled as a distributed stiffness that supports the top foil. The system of governing equations is solved numerically by a computer program written in the Mathematica computing environment. Representative calculations and comparisons with experimental results are included for a generation I gas foil thrust bearing.

Nomenclature

\( CF \) coefficient function used in the Reynolds equation solution
\( CF \theta \) coefficient function used in the Reynolds equation solution
\( CF_r \) coefficient function used in the Reynolds equation solution
\( C_P \) specific heat capacity for the lubricant
\( E \) material modulus of the top foil
\( g \) static (no load) film thickness function
\( h \) film thickness function, static plus structural deflections
\( i \) finite differencing index variable for \( x \) or \( r \)
\( j \) finite differencing index variable for \( y \) or \( \theta \)
\( K_c \) stiffness of the compliant foundation
\( K_{b,f} \) bending stiffness of the top foil
\( Kn \) Knudsen number at the bearing leading edge
\( m \) mass flow rate
\( p \) hydrostatic pressure
\( Q_s \) heat transfer between the lubricant and surroundings
\( RR \) inner radius ratio for the cylindrical thrust bearing
\( r \) radial direction coordinate in cylindrical thrust bearings
Introduction

A foil bearing is a self-acting hydrodynamic device capable of separating rotating components of rotating machinery by a film of air or other gaseous “lubricant.” The benefits of such bearings in high speed turbomachinery applications can be quite significant. Among these benefits are reduced machine weight due to the elimination of the oil systems, removal of the traditional diameter–rotational speed limit imposed by rolling element bearings, and a synergistic use of working fluid as a lubricant enabling a contaminant free working fluid. However in order to realize these benefits, foil bearing must be able to support the required machine load and operate reliably and without burden to the machine. Excess burden to a turbomachine usually manifests itself in the detrimental use of working fluid as bleed flow to cool or pressurize specific components.

A typical generation I thrust bearing is shown in figure 1. It consists of a rigid, stationary backing plate that supports a number of discrete pads. These pads are supported by a compliant foundation or bump foil and are topped with a smooth petal or top foil. The foil journal bearing has received much research and development attention over the past two decades with the primary objective of increasing load capacity and understanding stiffness and damping characteristics of the bearings. Until recently the foil thrust bearing has received very little research and development attention. This circumstance is primarily due to the fact that until recently, the thrust loads in machines using foil bearings could be managed and reduced to acceptable levels through clever machine designs. However potential future applications include those in which the foil bearing supported turbomachinery is part of a larger system or a closed loop working fluid. For example, aviation turbofan engines and space nuclear Brayton-cycle alternators operate in such modes. In these applications the thrust loads are no longer merely a function of machine design, but are imposed on the machine by specific operating conditions. In order to extend the utility of foil bearings to these applications, the physics and details of the foil thrust bearing must be exposed through experiments and analysis.

Governing Equations

The generalized challenge of foil thrust bearing simulations entails the simultaneous solution of nine unknown variables. These unknowns include the three velocity components, pressure, temperature, density, viscosity, specific heat capacity, and structural deflection. Of course, nine equations or relationships and their appropriate boundary conditions are required in order to solve this simulation. These relations include the three scalar conservation of momentum equations, conservation of mass, conservation of energy, and an equation of state, two property equations, and a structural model. The complete simulation of the structural behavior of the foil thrust bearing would introduce a host of additional unknowns and requisite equations. Therefore a simplified model is introduced in the...
hydrodynamic simulation. In order to reduce the size of this simulation even further a scaling analysis is performed that includes the traditional thin layer hydrodynamic assumptions. These assumptions allow for the elimination of the three velocity components and the related momentum equations. The simulation can then be reduced to three partial differential equation and three algebraic equations. The resulting simulation is then solved for the pressure, temperature, film thickness, density, viscosity, and heat capacity fields.

**Reynolds Equation**

The Reynolds equation is used for the hydrodynamic simulation of the foil thrust bearing. It is developed from the basic conservation laws of mass and momentum of a continuum fluid element [1, 2]. The range of applicability for the equation can be extended into the sub-continuum range for Knudsen number greater than 0.01 if slip flow is accounted for when the velocity boundary condition is applied. Following the schematic and nomenclature of figure 2 the non-dimensional Reynolds equation can be written in the form of equation (1). The assumptions that facilitate the construction of this equation include: non-inertial and time-steady flow, concentric bearing/runner operation, and invariant fluid properties across the film.
The Energy Equation

Equation (1) contains the primitive flow variable, density. Typically in hydrodynamics, an assumption is made that couples the density field to the pressure field. The most common density field assumption is the isothermal assumption, which when coupled with the ideal gas law allows one to replace density with pressure in the Reynolds equation. Another density field assumption is that of a polytropic relation between the pressure and density. In this assumption variable temperature effects can be analyzed. Although these assumptions eliminate the need to solve a system of differential equations, the Reynolds equation becomes non-linear. In highly loaded gas bearings, the viscous dissipation function may be sufficiently large to render any analysis based on such assumptions invalid. In order to include the effects of viscous heat generation into the simulation of foil thrust bearings, the conservation of energy equation must be analyzed to account for thermal effects on the density and viscosity fields. Applying the conservation of energy equation, in enthalpy form, to a notional control volume shown in figure 3, provides the energy equation shown in equation (2). The second boundary condition on the energy equation comes from the assumption of non-inertial flow and concentric runner/bearing operation. For these assumptions radial massflow can only be caused by radial pressure gradients. Therefore along the locus of points connected by the condition of zero radial pressure gradient, the radial massflow and pressure gradients are zero and the energy equation becomes an ordinary differential equation in angular coordinate. The energy equation can be further simplified if the radial convection is assumed to be much smaller than the angular convection. The resultant energy equation is presented in equation (3).

\[
\text{Boundary Conditions} \\
\bar{p} \text{ (on boundary)} = 1
\]

\[
\frac{1}{r} \frac{\partial}{\partial \theta} \left( \rho \left( \frac{\rho h}{\mu} \left( 1 + \frac{6 K h}{\rho h} \frac{\partial p}{\partial \theta} \right) \right) \right) + \frac{\partial}{\partial r} \left( r \rho \left( \frac{\rho h}{\mu} \left( 1 + \frac{6 K h}{\rho h} \frac{\partial p}{\partial r} \right) \right) \right) = \Lambda r \frac{\partial}{\partial \theta} (\rho h)
\]
\[
\frac{\gamma C_p}{\gamma - 1} \left\{ \frac{\dot{m}_r}{\psi} \frac{\partial T}{\partial r} + \frac{\dot{m}_0}{\psi \theta} \frac{\partial T}{\partial \theta} \right\} = -\frac{6Knh^2}{\Lambda \psi} \left[ \left( \frac{\partial \dot{p}}{\partial r} \right)^2 + \left( \frac{1}{\psi \theta} \frac{\partial \dot{p}}{\partial \theta} \right)^2 \right] + \frac{\Lambda \mu r^2}{3h \left( 1 + \frac{2K_n \mu}{\psi} \right)^2} + \bar{Q}_s
\]

where,
\[
\bar{Q}_s (\bar{r}, \theta) = \frac{\Lambda h_1}{3\mu_a (\omega r_o)^2} Q_s
\]

Boundary Conditions
\[
\bar{T} (\theta = 0, \bar{r}) = 1
\]

\[
\frac{d\bar{T}}{d\theta} \text{ (along } \frac{d\bar{p}}{d\bar{r}} = 0) = \gamma - 1 \frac{\bar{r}}{C_p} \left\{ \frac{\dot{m}_r}{m_0} \frac{\partial T}{\partial r} \right\} = \frac{6Knh^2}{\psi \Lambda \psi} \left[ \left( \frac{1}{\psi \theta} \frac{\partial \dot{p}}{\partial \theta} \right)^2 + \frac{\Lambda \mu r^2}{3h \left( 1 + \frac{2K_n \mu}{\psi} \right)^2} + \bar{Q}_s \right]
\]

\[
\frac{\dot{m}_r}{m_0} \ll 1, \text{ or } \frac{-\bar{r}^2 \frac{\partial \dot{p}}{\partial \theta}}{\frac{\partial \dot{p}}{\partial \bar{r}} + \bar{r}^2} \ll 1
\]

\[
\frac{\gamma C_p}{\gamma - 1} \left\{ \frac{\dot{m}_0}{\rho \theta} \frac{\partial T}{\partial \theta} \right\} = -\frac{6Knh^2}{\psi \Lambda \psi} \left[ \left( \frac{\partial \dot{p}}{\partial r} \right)^2 + \left( \frac{1}{\psi \theta} \frac{\partial \dot{p}}{\partial \theta} \right)^2 \right] + \frac{\Lambda \mu r^2}{3h \left( 1 + \frac{2K_n \mu}{\psi} \right)^2} + \bar{Q}_s
\]

**Structural Model**

A single equation is used to model the deflections of the compliant foundation/top foil combination. The models have their genesis in elementary beam theory and Stephen Timoshenko’s theories for thin plates and shells [3, 4]. The major assumptions for the top foil model are that the foil is thin enough such that the entire membrane behaves like the neutral axis and that top foil stiffness is bending dominated such that the tensile stresses can be neglected. Additionally, the foil bending stiffness is assumed to be constant throughout the angular and radial extent of the bearing pad. This assumption allows the bending stiffness to be moved to the denominator of the right hand side of the equation. These assumptions limit the range of applicability to conditions of modest foil sag between discrete compliant foundation contacts. However it does provide a useful and practical model for the investigation of fluid-structure interactions. The stiffness of the compliant layer is modeled empirically by a distributed stiffness function, \( K_c (r, \theta, \delta) \). Initially a mathematical function is derived that simulated the contact lines between the bump foil and the top foil. Bump foil flattening is then modeled by distributing this contact line across a finite width, creating a step function. The final step of this modeling is to define the magnitude of this step function, which essentially sets the structural stiffness of the bearing. Figure 4 shows the graphical representation of the distributed compliant foundation stiffness function. By modeling the compliant foundation in such
Figure 4.—Compliant foundation distributed stiffness function.

In a manner two important physical effects are captured. These effects are top foil sag and bump foil flattening. The variable temperature effects on the top foil material properties and the detailed motion of the bump foil are not captured by this modeling method. The structural model is summarized in equation (4) with boundary conditions shown in equation (5). Two boundary conditions are applied along all the edges of the foil to satisfy the fourth order equation. Along the leading edge the structural condition is clamped where deflection and slope are zero. Along the remaining three edges the structural conditions are “free” where moment and shear are zero.

\[
4 \frac{\partial^2 \delta}{\partial \theta^2} + \frac{\partial^4 \delta}{\partial \theta^4} + r \frac{\partial \delta}{\partial r} - 2r \frac{\partial^3 \delta}{\partial \theta^2 \partial r} + r^4 \frac{\partial^4 \delta}{\partial r^4} + 2r^3 \frac{\partial^3 \delta}{\partial r^3} - r^2 \frac{\partial^4 \delta}{\partial r^2 \partial \theta^2} + 2r^2 \frac{\partial^4 \delta}{\partial r^2 \partial \theta^2} = r^4 \left( \frac{r_0}{h_1} \right)^4 \left( \frac{1}{r_1} - 1 \right) - \overline{K_C}(\overline{r}, 0, \overline{\delta}) \overline{\delta}
\]

\[
\overline{K_C}(\overline{r}, 0, \overline{\delta}) = \frac{K_C(r, 0, \delta) \delta}{P_a h_1}
\]

\[
\overline{K_b, f} = \frac{K_{b, f}}{P_a h_1^3}
\]

\[
K_{b, f} = \frac{E h_1^3}{12(1 - \nu^2)}
\]


\[
\begin{align*}
\text{Clamped} \\
\bar{\delta}(\theta = 0) &= 0, \quad \left. \frac{\partial \bar{\delta}}{\partial \theta} \right|_{\theta = 0} = 0 \\
\text{Free} \\
\left. \frac{\partial^2 \bar{\delta}}{\partial \theta^2} \right|_{\text{on boundary}} &= 0, \quad \left. \frac{\partial^3 \bar{\delta}}{\partial \theta^3} \right|_{\text{on boundary}} = 0 \\
\left. \frac{\partial^2 \bar{\delta}}{\partial r^2} \right|_{\text{on boundary}} &= 0, \quad \left. \frac{\partial^3 \bar{\delta}}{\partial r^3} \right|_{\text{on boundary}} = 0
\end{align*}
\]

(5)

\textbf{Numerical Technique}

The previously developed equations are solved in the Mathematica programming environment using finite differencing and built-in Mathematica functions [5, 6, 7]. The derivatives of pressure, temperature, and deflection are discretized using central differencing. A sparse version of the numerical grid is show in figure 5. The number of nodes in the r and \(\theta\) directions is independently specified, but the grid spacing is uniform. The current calculations are carried out using a 50 by 50 grid in the radial and angular directions. Numerical experiments have proven this to be a well balanced point between computational time and resultant accuracy. No a prior solutions are utilized in the initialization of the iteration procedure for this simulation and modeling program. The basic programming logic is outlined in the flowchart of figure 6. The standard set of inputs required to begin a calculation includes bearing number, fluid properties, undeflected film thickness, compliant foundation stiffness, foil bending stiffness, and density model. The solution procedure then commences by solving the Reynolds equation for an incompressible, rigid wall, bearing. Once the pressure field is known, the equations yielding structural deflections and temperature fields can be solved. The density field can then be calculated through the use of an equation of state. The updated density and hydrodynamic film thickness fields are then used to calculate new coefficient functions for the Reynolds equation and the second iteration progresses as the first with the exception of a relaxation factor added to the calculated deflections and temperatures in which the values from the two

![Figure 5.—Typical numerical grid in thrust pad geometry.](image-url)
Previous iterations are combined in a weighted average. This method damps the oscillations of the convergence criterion and arrives at a solution more quickly than an unrelaxed method. The convergence criteria are calculated upon the completion of each iteration to ensure a self-consistent solution is reached at some user defined threshold of convergence criterion. Typically bearing hydrodynamic load is used as a convergence criterion. Setting the desired bearing conditions in the initial iteration is herein referred to as direct convergence. At times direct convergence is not possible due to the desired conditions being far removed from the rigid wall, incompressible case. In such instances calculating bearing behavior along a speed line is the most useful approach to obtaining high speed and load solutions. In this approach the bearing performance is initially calculated at a low speed such that the incompressible, rigid wall assumptions are fairly accurate. When a low speed solution is reached, the speed is incrementally increased until the desired condition is obtained. Figure 7 shows a typical convergence of the program as a plot of iteration number versus nondimensional load.

**Comparison of Simulations and Experiments**

A standard generation I 8-pad thrust bearing is modified by removing every other pad to create a 4-pad thrust bearing in which each pad had an angular extent of 43°. The exposed area between the pads is further modified to include air feed hole near the leading edge mid-span of each remaining pad. This modification was made to ensure that no pad to pad hot gas carry over would occur during the experiments. A post-test (post-failure) photograph is shown in figure 8. Thermocouples are installed under the bump foils at approximately the 37.5 percent span and 32° location. The bearing is tested at
nominal conditions of 50,000 rpm, 67 Newton (15 pound) axial load, and leading edge air inlet temperatures of 16 to 21 °C (60 to 70 °F). These conditions result in bearing temperature measurements in the range of 49 to 59 °C (120 to 137 °F). No torque data was possible due to the rotational constraint of the cooling air line. The bearing performance is simulated numerically using the Reynolds equation. The power law density and viscosity model having a polytropic exponent 1.41 relating the pressure to density and a square root relation between the temperature and viscosity were used. The compliant foundation and top foil structural models were also used. Initial (zero load) film thickness is shown in figure 9. It consists of a typical ramp-flat design in which the ramp height is 50 μm (0.002 inches) and the flat is separated from the runner by 16 μm (0.000650 inches). Figure 10 shows the results of the numerical simulation in terms of hydrodynamic pressure, temperature, and foil deflection. The minimum film thickness at the trailing edge, which shows an increase in the film thickness due to the
Figure 9.—Static Film Thickness.

Figure 10.—Graphical Simulation Results.
membrane effects, is plotted in figure 11. The most significant result of these calculations is shown in figure 12. This figure plots the 37.5 percent span line for a 60 and 70 °F inlet temperature as well as the thermocouple data from the test. The agreement between the measured temperature and the hydrodynamic performance is very good. This example demonstrates the utility of applying the power law density model to the Reynolds equation. At low speeds and load where the viscous heating may still be considered insignificant, the power law model can correlate temperatures to pressures. This is significant because it is possible to obtain thermocouple data from foil bearing experiments, but it is nearly impossible to measure pressure directly without corrupting the bearing surfaces, lubricant flow field, and top foil stiffness.

Figure 11.—Static and Deflected trailing edge film thickness.

Figure 12.—Calculated and Measured bearing temperatures.
Conclusion

A simulation and modeling effort is conducted on foil gas thrust bearings. Although simple in appearance these bearings have proven to be very complicated devices. The conservation equations of mass, momentum, and energy are applied to the problem. A computer program written in the Mathematica computing environment is created to solve the set of equations and relations developed to analyze the foil thrust bearing. Quantitative comparisons are produced that demonstrate the ability of this method to predict the temperature field of foil thrust bearings and the coupling of the temperature field to the hydrostatic pressure field. The calculations show that the analyses which couple the Reynolds, energy, and structural equation will accurately model future bearing performance. An analytic model is developed that can provide insight into the hydrodynamic, thermal, and structural interactions of thrust foil bearings for future bearing designs.

References

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