CAPILLARY PRESSURE OF A LIQUID BETWEEN UNIFORM SPHERES ARRANGED IN A SQUARE-PACKED LAYER

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Summary The capillary pressure in the pores defined by equidimensional close-packed spheres is analyzed numerically. In the absence of gravity the menisci shapes are constructed using Surface Evolver code. This permits calculation the free surface mean curvature and hence the capillary pressure. The dependences of capillary pressure on the liquid volume constructed here for a set of contact angles allow one to determine the evolution of basic capillary characteristics under quasi-static infiltration and drainage. The maximum pressure difference between liquid and gas required for a meniscus passing through a pore is calculated and compared with that for hexagonal packing and with approximate solution given by Mason and Morrow [1]. The lower and upper critical liquid volumes that determine the stability limits for the equilibrium capillary liquid in contact with square packed array of spheres are tabulated for a set of contact angles.

EXTENDED SUMMARY

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Fig. 1. Dependence of dimensionless capillary pressure, $\Delta P$, on dimensionless liquid volume, $V$, (liquid on bottom) for pores formed by square close-packed spheres. Numbers on curves denote the value of the contact angle $\theta$ (in degrees). For $\theta \neq 90^\circ$, the dependence consists of two branches. The vertical dotted lines represent transitions between two branches of capillary surfaces. The segments shown by dash lines are valid only for a single-layer packing.

Our calculations show that the limiting cases of hexagonal and square packing diverged considerably. The maximum capillary pressures for $90^\circ \leq \theta \leq 180^\circ$ (the minimum capillary pressures for $0 \leq \theta \leq 90^\circ$) differ by $1.75 \pm 2.5$ times. There are also qualitative distinctions. In the case of hexagonal packing and $90^\circ < \theta \leq 180^\circ$, the existence of two branches of equilibrium configurations has been illustrated in [2] only for $\theta = 100^\circ$ and $110^\circ$, and there is no evidence that the same holds for $120^\circ \leq \theta \leq 180^\circ$. For the case of $\theta = 110^\circ$ that was analyzed in [2], the value of the dimensionless volume $V$ at the extreme right point of the long branch (LB) is less than that at extreme right point of the short branch (SB), i.e. $V^*_s < V^*_l$. 