

Uncertainty Analysis of Heat Transfer to Supercritical Hydrogen in Cooling Channels

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Sound understanding of the cooling efficiency of supercritical hydrogen is crucial to the development of high pressure thrust chambers for regeneratively cooled LOX/LH₂ rocket engines. This paper examines historical heat transfer correlations for supercritical hydrogen and the effects of uncertainties in hydrogen property data. It is shown that uncertainty due to property data alone can be as high as 10%. Previous heated tube experiments with supercritical hydrogen are summarized, and data from a number of heated tube experiments are analyzed to evaluate conditions for which the available correlations are valid.

Nomenclature

C_p	= constant pressure specific heat
D	= inner diameter of tube
h	= convection coefficient
k	= thermal conductivity
Nu	= Nusselt number
Pr	= Prandtl number
Pr	= reduced pressure
Re	= Reynolds number
T	= temperature
T_r	= reduced temperature
U	= uncertainty
V	= velocity
x	= distance downstream from entrance
x	= any property
μ	= dynamic viscosity
ν	= kinematic viscosity
ρ	= density
σ	= standard deviation

Subscripts

b	= property evaluated at bulk temperature reference
e	= critical condition
$calc$	= calculated value
exp	= experimental value
f	= property evaluated at film temperature reference
ref	= reference temperature type
s	= condition at inner-wall surface
0.4	= property evaluated at 0.4 temperature
\int	= property evaluated by integral method

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I. Introduction

Analytical performance predictions and design margins for regeneratively cooled liquid rocket engines utilize correlations to predict the heat transfer from the combusting gas to the thrust chamber wall and from the wall into the cooling fluid. These semi-empirical heat transfer correlations are highly dependant on the fluid transport properties and the conditions at which the fluid is flowing. Although a number of coolant-side correlations have been proposed for hydrogen acting as the coolant, the ability of the correlations to accurately predict the heat transfer over a wide range of operating conditions has typically been limited.

In addition to the transport properties, heat transfer is highly dependant on cooling channel geometry. Efforts are currently being made to produce accurate correction factors that can be applied to existing heat transfer correlations to account for the effects of curvature, roughness, asymmetric heating, entrance effects, and high aspect ratio cooling channels for improved thrust chamber design. For these correction factors and new correlations to have any merit, it is important to understand the limitations of these correlations for the simplest case: a straight, uniformly heated, circular tube. This paper examines existing correlations for straight tube geometries using supercritical hydrogen as the coolant. The uncertainty on the heat transfer correlations due to inherent uncertainties in the equation-of-state and transport properties of supercritical hydrogen are evaluated. Previous experimental data is also analyzed with existing published correlations over a wide range in the supercritical region to determine the regions of validity for each correlation.

II. Heat Transfer Correlations and Studies

In typical rocket applications, hydrogen enters the coolant channels as a liquid above the critical pressure (1.3 MPa) and at temperatures as low as 25 K, which is below the critical temperature of 33 K. High-pressure thrust chambers using hydrogen as a regenerative coolant operate at very high ($>10^6$) Reynolds number flow conditions in the coolant passages.¹ As the hydrogen continues through the channels, it is quickly heated to above the critical temperature, where it becomes a supercritical gas, as shown in Figure 1. The heat transfer mechanisms occurring in these regions are supercritical liquid and supercritical gas forced convection.²

Numerous experimental studies have been performed for hydrogen flowing supercritically in uniformly heated circular tubes. The flow conditions and geometries for many of these studies are summarized in Table 1. From these studies, most of which took place in the 1960s, came a number of correlations to describe the heat transfer characteristics of supercritical hydrogen in cooling tubes. These correlations are all principally based on the conventional Nusselt-type equation of Dittus-Boelter, where the Nusselt number is a function of the Reynolds and Prandtl numbers.³

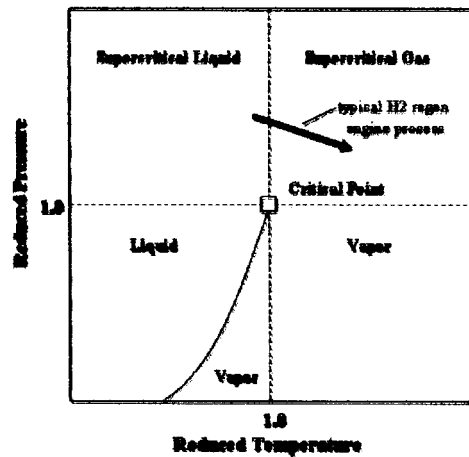


Figure 1: Reduced Pressure-Temperature Diagram

$$Nu = 0.023 Re^{0.8} Pr^{0.4} \quad (1)$$

In 1960, McCarthy and Wolf proposed a modified version of this equation,⁴

$$Nu_b = 0.025 Re_b^{0.8} Pr_b^{0.4} (T_i/T_b)^{-0.55} \quad (2)$$

where the subscript b denotes that the properties are evaluated at the bulk temperature of the fluid and T_i/T_b is the inner-wall-to-bulk temperature ratio. Hendricks, Simonsau and Friedman in 1965 replaced the bulk condition and proposed the correlation,⁵

$$Nu_f = 0.021 Re_f^{0.8} Pr_f^{0.4} \quad (3)$$

where the properties are evaluated at the film temperature,

$$T_f = \frac{T_s + T_b}{2} \quad (4)$$

Hess and Kunz, also in 1965, proposed a modified version of the film temperature-type correlation that included wall-to-bulk viscosity ratio,⁶

$$Nu_f = 0.0208 Re_f^{0.8} Pr_f^{0.4} (1 + 0.01457 \nu_s / \nu_b) \quad (5)$$

Miller, Seader and Trebes (1965) proposed using a new reference temperature, and developed the correlation,⁷

$$Nu_{0.4} = 0.0208 (Re_{0.4})^{0.8} (Pr_{0.4})^{0.4} (1 + 0.00983 \nu_s / \nu_b) \quad (6)$$

where

$$T_{0.4} = T_s + 0.4(T_s - T_b) \quad (7)$$

This approximately constant reference temperature is equivalent to Deissler's comprehensive theory which describes forced convective heat transfer for turbulent flow at supercritical gas conditions sufficiently above the critical temperature. The use of bulk temperature physical properties with a T_s/T_b correction factor, as in the McCarthy & Wolf correlation, has also been shown to be equivalent to the Deissler theory.⁷

Taylor, working in 1968-1970 in support of the NERVA and Phoebus-2 nuclear rocket engines, added a correction factor to the McCarthy & Wolf correlation to improve the heat transfer along the length of the tube:^{8,12}

$$Nu_b = 0.023 Re_b^{0.8} Pr_b^{0.4} (T_s/T_b)^{-(0.37-1.39D/x)} \quad (8)$$

In 1973, Schaet and Quentmeyer proposed a new type of reference condition¹³ such that,

$$Nu_i = 0.025 Re_i^{0.8} Pr_i^{0.4} \quad (9)$$

This correlation uses an integral reference type. Property x is evaluated such that,

$$x = \frac{1}{T_s - T_b} \int_{T_b}^{T_s} x(T) dT \quad (10)$$

These correlations have traditionally been used to calculate the heat transfer to supercritical hydrogen cooling tubes, and are all based on the Dittus-Boelter-type equation for Nu . Ref. 3 provides a good summary of other laminar and turbulent flow forced convection heat transfer correlations. These correlations may also be adapted for supercritical hydrogen cooling tubes.

Table 1: Summary of Straight Circular Tube Heat Transfer Experiments with Hydrogen

Investigator	Year	Ref.	ID(m)	γ/D	T_1/T_2	T_1 (°K)	P (psi)	Heat Flux (BTU/ft ²)	Mass Flow Rate (lb/s)	Bank Reynolds Number	T_1 (°K)	T_r	Pr
Wright & Walters	1959	1*		1.77-2.95	95-134	693-711	0.4-0.57		0.001-0.128	845,000 - 2,000,000		1.60-2.26	3.72-3.82
McCarty & Wolf	1960	4	0.194-0.430	5.8-50.2	135-560	32-1354	0.036-14.8		0.001-0.128		830-2240	2.28-9.46	3.72-3.82
Thompson & Geary	1962	14	0.192	10.3-0.9	55-102	600-1344	0.14-8.0		0.009-0.062	260,000-1,740,000	77-1994	0.93-1.72	3.65-7.22
Scottie	1962	1*		2.91-10.2	69-85	213-315	0.33-1.76			1,490,000-3,120,000		1.17-1.44	1.14-1.69
Reedtschyn	1963	1		1.6-2.26	584-718	664-2476	6.29-34.4			180,000-3,790,000		9.86-12.12	3.57-13.30
Taylor	1964	15	0.116	11.6-77	557-573	41-79	0.79-3.3		0.001-0.002		738-5630	9.40-9.68	0.22-0.42
Taylor	1965	16	0.115	11.6-77	252-325	37-68	0.84-4.57		0.001-0.002	5,700-48,400	350-3300	4.26-5.49	0.20-0.37
Weiland	1965	8*	0.118	2.7-252	222-548	250-1000	0.24-3.0				530-2300	3.75-9.25	1.34-5.37
Hess & Kaniz	1965	6	0.2-0.3	3-14	57-120	235-745	0.61-10.16					0.96-2.03	1.26-4.00
Miller, Sander & Trehan	1965	17	0.211-0.214	4.7-47.4	51-185	458-2486	1.37-24		0.16-0.737	1,930,000-13,400,000	107-1730	0.86-3.12	2.46-13.35
Hendricks, Simonsen & Friedman	1965	5	0.211-0.438	3.4-78.2	60-200	1000-2500	0.6-10.0		0.05-0.40		150-1200	1.01-3.38	5.37-13.43
Hendricks, Graham, Han & Friedman	1966	18	0.188-0.507	4.9-114	45-500	220-820	0.2-3.3		0.028-0.195		45-1311	0.76-8.44	1.18-4.40
Aensjot	1967	19	0.148	6.7-33.9	62-97	696-1371	6.4-27.6		0.070-0.442		430-1681	1.04-1.63	3.74-7.36
Gibbden & Watt	1968	8*	0.335	2.0-127	65-170	18-36			0.004-0.011		250-750	1.10-2.87	0.10-0.19

III. Property Uncertainty Analysis

When conducting any experiment, the measurements of all variables have uncertainties associated with them, as do the values of material properties that are obtained through reference databases. These uncertainties propagate throughout the data reduction equation and produce an overall uncertainty for the calculated result. For the case of our heat transfer analysis, the data reduction equations considered are Eqs. (2), (3), (5), (6), and (8), with

$$Re_{ref} = \frac{\rho_{ref} V D}{\mu_{ref}} \quad (11)$$

and

$$Pr_{ref} = \frac{Cp_{ref} \mu_{ref}}{k_{ref}} \quad (12)$$

where the property values are determined at the reference conditions given by Eqs. (4), (7) and (10).

Property values for hydrogen were obtained from the NIST *Thermodynamic and Transport Properties of Pure Fluids* database²⁰ and the NIST *Chemistry WebBook*.²¹ These sources are based on Ref. 22 for density and specific heat, and Ref. 23 for viscosity and thermal conductivity. They contain data for temperature up to 400 K. For conditions above 400 K, Ref. 24 was used, which is an updated version of Ref. 25 to correct for errors in certain property data.

The uncertainties listed in Table 2 are compiled from those supplied by the referenced sources and are based on accuracies of experimental measurements and inaccuracies introduced in fitting the data. The uncertainties quoted are not meant to represent the critical region where the uncertainties could reach 10 to 20% or higher. Indeed, an analytical function such as the Modified Benedict-Webb-Rubin Equation used by Ref. 22 cannot represent the proper behavior near the critical point, and is not considered to be valid in that region. For the purpose of this analysis, the uncertainty in the Nu correlation is not calculated near the critical region.

To determine the total uncertainty in Nu resulting from the uncertainties of each individual property variable, a general uncertainty analysis was performed. This was accomplished by using a Monte Carlo simulation program that was created in C++. To perform the Monte Carlo simulation, random errors were selected for each variable, based on a uniform parent population with σ equal to one-half of the assumed uncertainty for each variable at the particular temperature and pressure (as per Table 2). A simulated measurement was produced by summing the drawn error with the assumed "true" value of the variable. This was performed for each variable, and the resulting Nu calculated at the specific temperature and pressure. This simulation was repeated 5,000 times at each temperature and pressure with the resulting uncertainty of Nu determined as twice the calculated sample standard deviation.²⁶ The simulation was repeated for each combination of temperature and pressure within the range of interest. The uncertainties in experimentally measured variables (T, P , etc.) were not considered here.

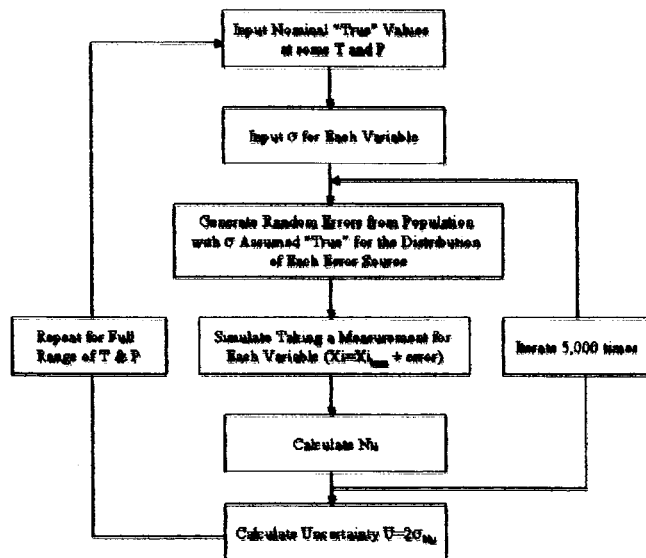


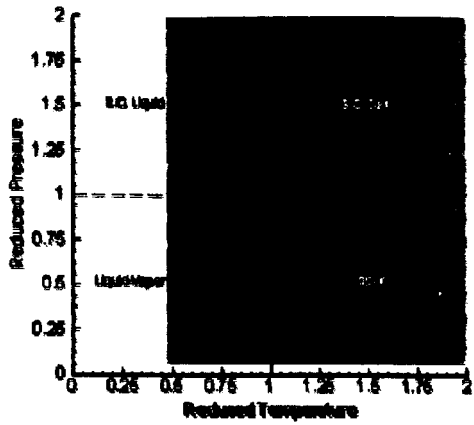
Figure 2: Flow Chart of Analysis Program and Monte Carlo Simulation

Table 2: Summary of Property Data, Uncertainty Values and Sources Used in Analysis

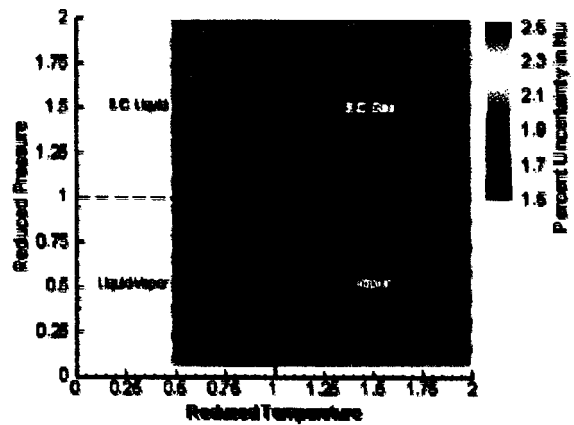
Property	Temperature (K)	Pressure (MPa)	Uncertainty (%)	Ref.
Viscosity	<100	<35	0.5	20, 21, 23
	100-400	<35	4-15*	20, 21, 23
	>400	<100	5	24
	$T_c \pm 1\%$	($\rho_c \pm 20\%$)	Greater, unknown	20, 21, 23
Density	Liquid below T_c	<121	0.1	20, 21, 22
	Gas below T_c	<Pe	0.25	20, 21, 22
	$T_c - 400$	0.005-121	0.2	20, 21, 22
	400-700	0.1-100	0.5	24
	700-3000	0.1-100	1	24
	$T_c \pm 5\%$	($\rho_c \pm 30\%$)	6	20, 21, 22
Specific Heat (Const. Pressure)	Liquid below T_c	<121	3.0	20, 21, 22
	Gas below T_c	-	2.0	20, 21, 22
	Fluid above T_c , <400	0.05-121	3.0	20, 21, 22
	>400	0.01	0.02*	24
	>400	35	3*	
	>400	100	8*	
$T_c \pm 5\%$	($\rho_c \pm 30\%$)	Greater, unknown	20, 21, 22	
Thermal Conductivity	<100	<15	3	20, 21, 23
	<100	15 - 70.9	3-10*	20, 21, 23
	100 < T < 400	-	10	20, 21, 23
	> 400	-	7	24
	$T_c \pm 1\%$	($\rho_c \pm 20\%$)	Greater, unknown	20, 21, 23

*Where ranges of uncertainties are provided, a linear approximation is used to represent the uncertainty in the analysis.

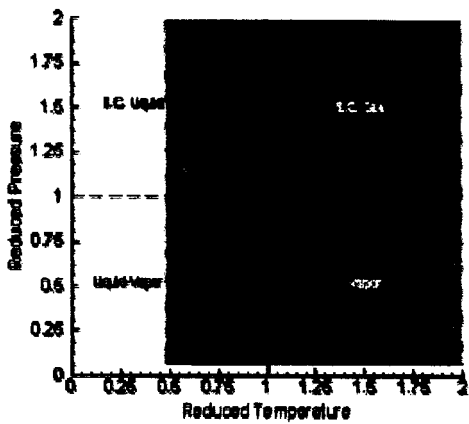
The results of the uncertainty analysis are shown in Figures 3 and 4. The blackened region on each plot is the near critical region where the uncertainties are not defined. In this region the resultant uncertainty is believed to be highest, but no reasonable estimate exists to quantify it precisely. As can be seen from the figures, the McCarthy & Wolf and Taylor correlations, both based on bulk reference type, produce comparable uncertainties. The uncertainty is lowest at approximately 1.8% under the liquid-vapor line, 2% for $Tr < 3$, and 5 to 7% for $3 < Tr < 12$. The reduction in uncertainty at $Tr > 12$ to approximately 4% is a result of switching the source of property data in this region. The Hendricks correlation shows an uncertainty value of about 5.4% for $Tr < 2$, increasing to a maximum of 7% at higher temperatures. The Hess & Kunz and Miller correlation results reveal considerably higher uncertainties for these correlations at low pressures and temperatures and along the liquid-vapor line. The uncertainty is highest at approximately 10%, and is upwards of 7 to 8% to $Pr = 2$. For higher temperatures, the uncertainty is similar to that of the Hendricks correlation at 5 to 7%.



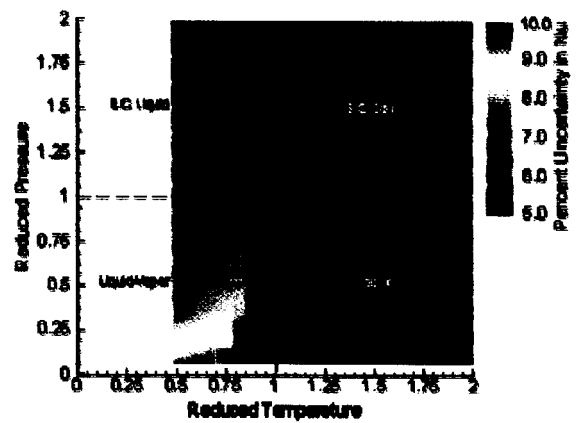
(a) Hendricks



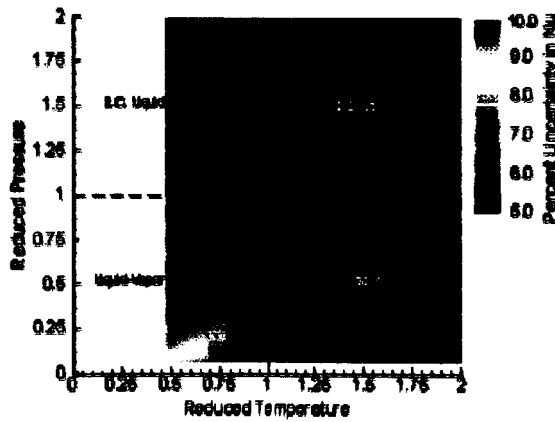
(b) McCarthy & Wolf



(c) Taylor



(d) Hess & Kunz



(e) Miller

Figure 3. Uncertainty Associated with Nu Near the Critical Region Due to Uncertainties in Property Data.

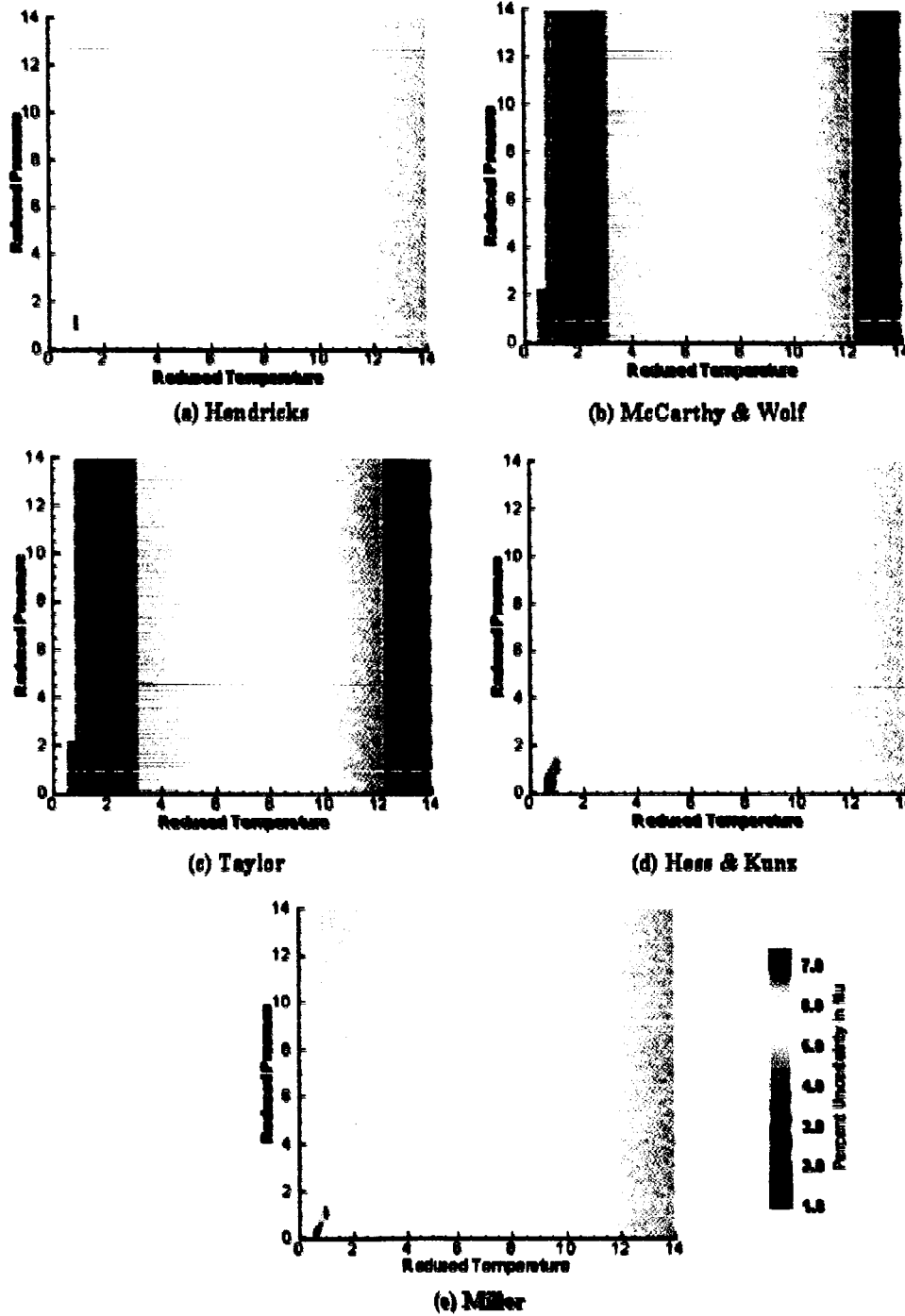


Figure 4. Uncertainties Associated with Nu in Supercritical Region Due to Uncertainties in Property Data.

IV. Experimental Data Study

To assess the possible effects of the property uncertainty, it is useful to compare experimentally determined heat transfer coefficients to those calculated using the heat transfer correlations. With that in mind, the authors gathered published experimental heat transfer data for hydrogen in the supercritical regimes, and reduced that data using more recent hydrogen property databases.

The data used in this report originate from experiments conducted by Hendricks, Simoneau, and Friedman (NASA Lewis, 1965),⁵ Hendricks, Graham, Hsu and Friedman (NASA Lewis, 1966),¹⁸ and Aerojet-General Corporation (1967).¹⁹ The ranges of conditions tested in these experimental programs can be found in Table 1. These sources were selected because their datasets were readily available, and because together they represent a wide range of fluid property conditions. Combined, they represent a total of 2824 data points.

Figure 6 shows the reduced data in terms of the deviation of h_{exp}/h_{calc} , where h_{calc} is determined using the various heat transfer correlations by the relation,

$$h_{calc} = \frac{Nu_{ref} k_{ref}}{D} \quad (13)$$

h_{exp} was calculated by the respective investigators at each point. $h_{exp}/h_{calc} = 1$ represents an exact agreement between predicted and experimental results.

As can be seen in the figure, the McCarthy & Wolf and Taylor correlations (both bulk reference types) over-predict the heat transfer in the region close to the critical point. The Hendricks, Hess & Kunz, Miller and Schact & Quentmeyer all under-predict in this region.

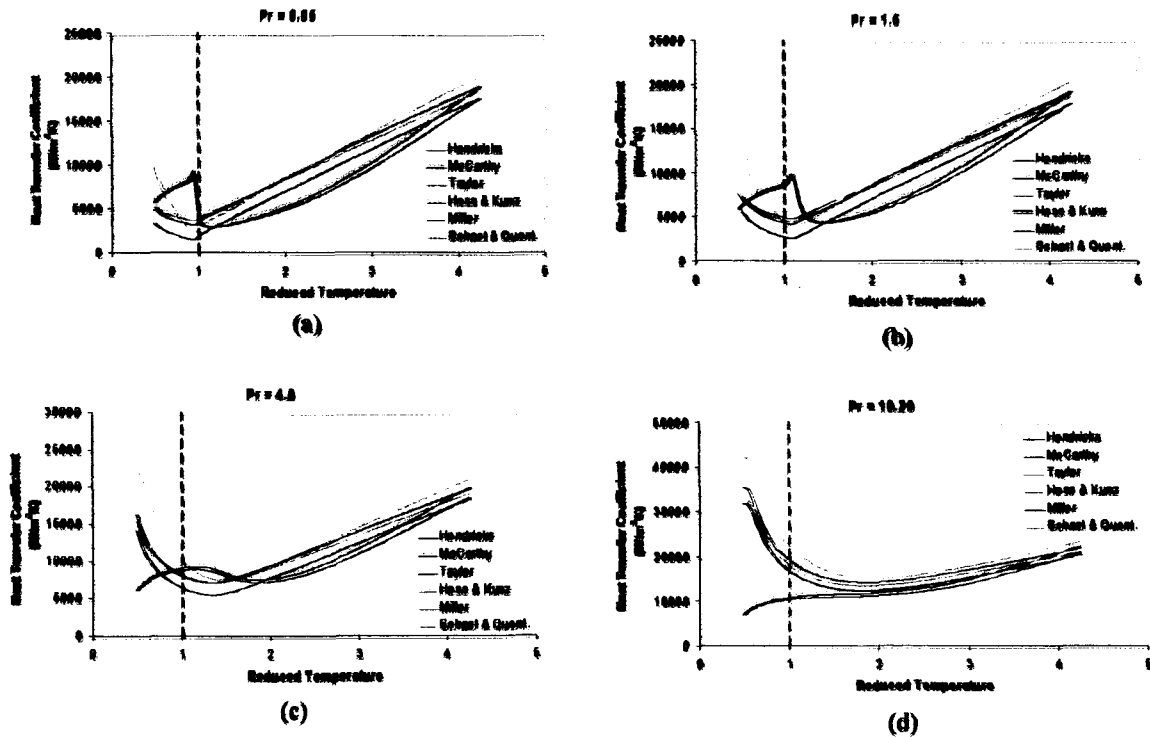


Figure 5. Comparison of Predicted Heat Transfer Coefficient of Correlations for Various Reduced Pressures. All plots are for $Re_b = 2e6$, $T_c = 300\text{ K}$, $D=0.01\text{ m}$, $x/D=40$.

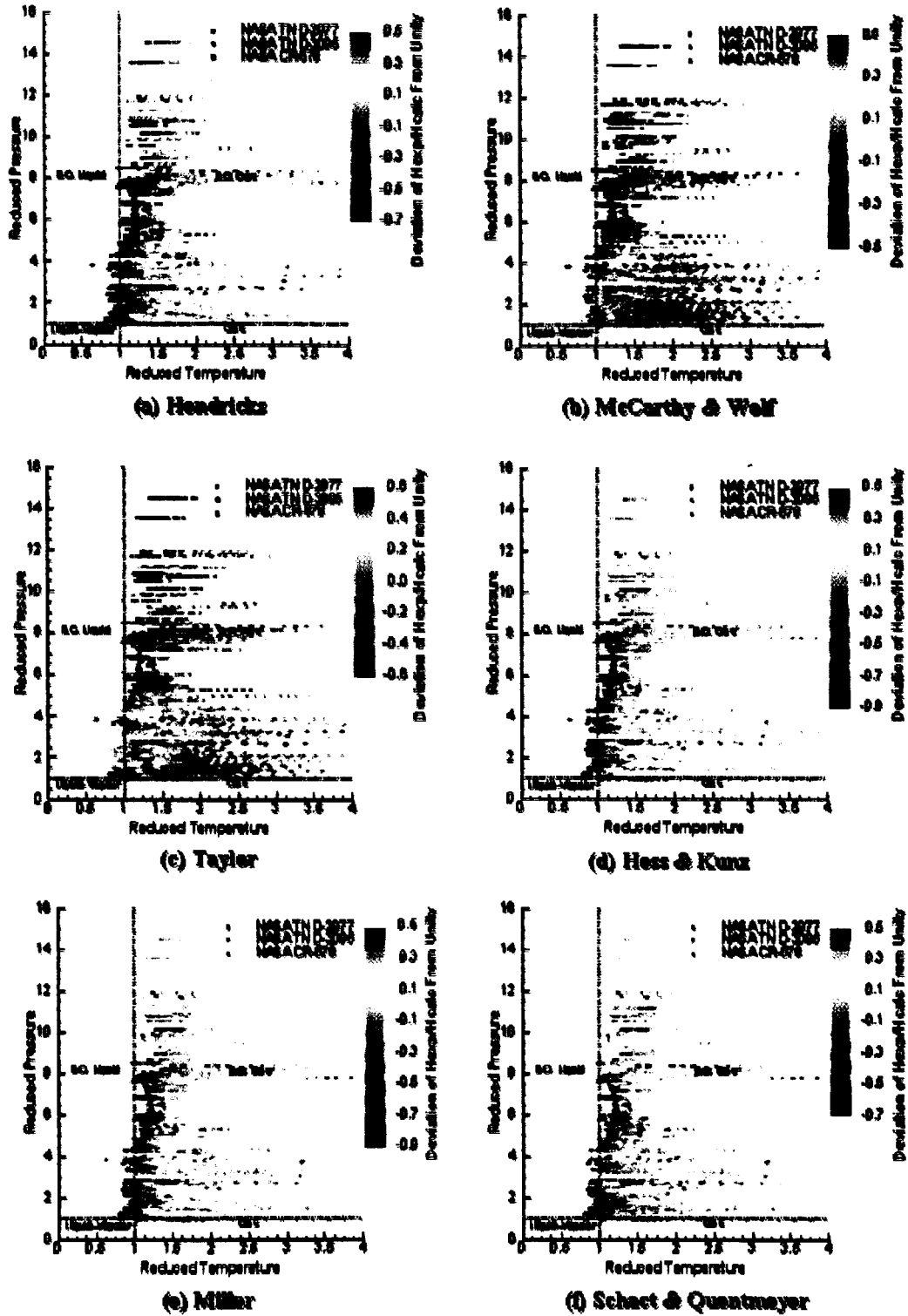


Figure 6. Deviation of h_{exp}/h_{calc} from Unity.

This trend can be seen in a theoretical comparison of the correlations shown in Figure 5 (b) for reduced pressure equal to 1.6. The spike in the prediction of McCarthy & Wolf and Taylor is therefore shown to not represent the true conditions in this region.

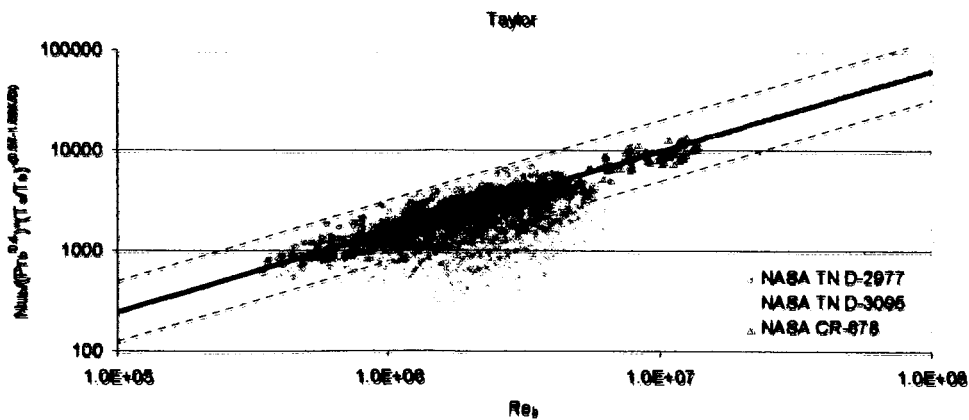
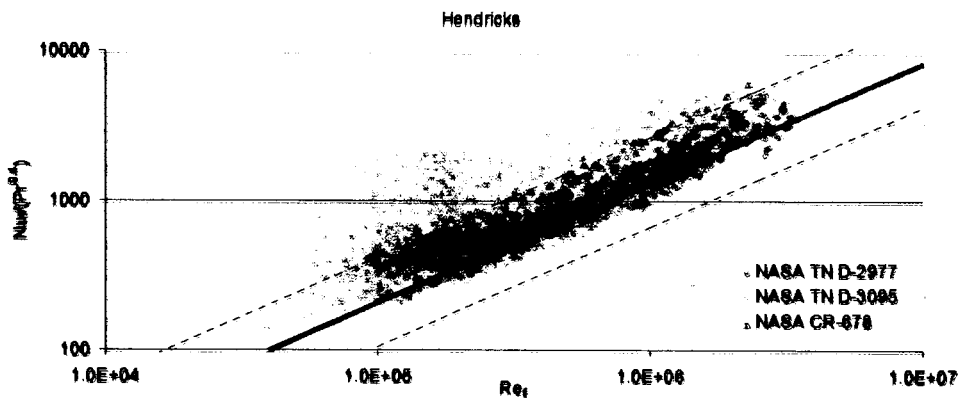
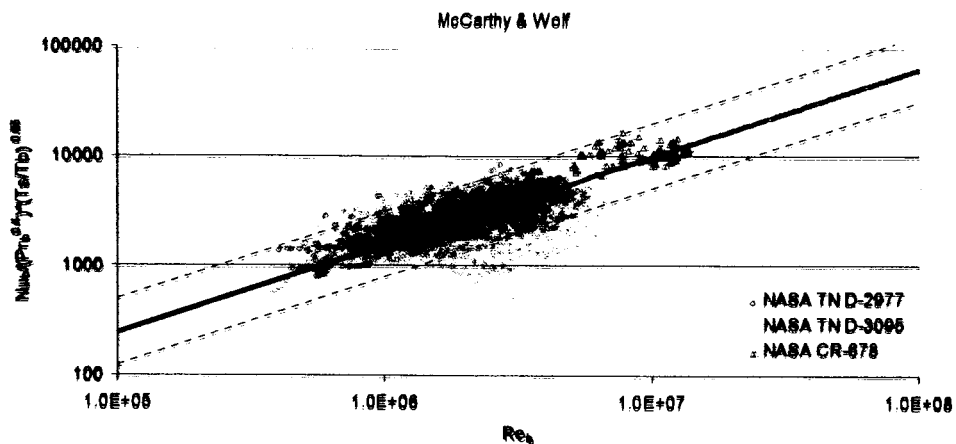
Also from Figure 6, it can be seen that the McCarthy & Wolf and Taylor correlations under-predict in the region of the line of $Tr=1$ for $Pr > 4$. As the pressure is increased, this range of under-prediction extends to larger values of Tr up to about $Tr=2$. This can be seen in Figures 5 (c) and (d) where the trends for McCarthy & Wolf and Taylor can be seen to diverge increasing from the other correlations at low temperatures and higher pressures. The Schact & Quentmeyer appears to give the most accurate prediction close to the $Tr=1$ line for $Pr < 10$.

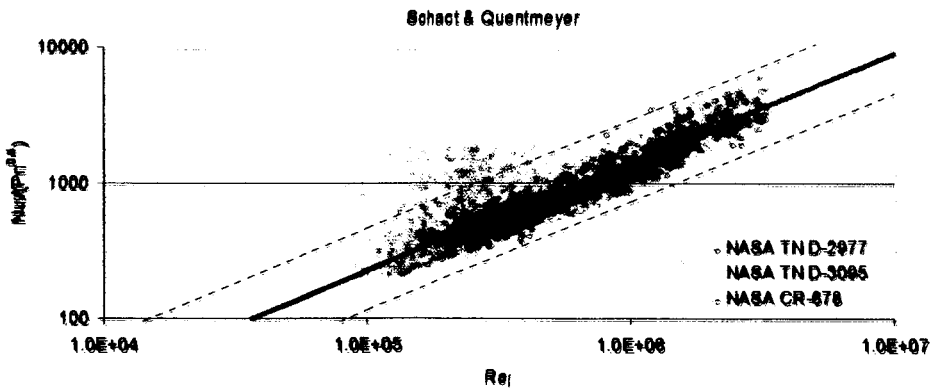
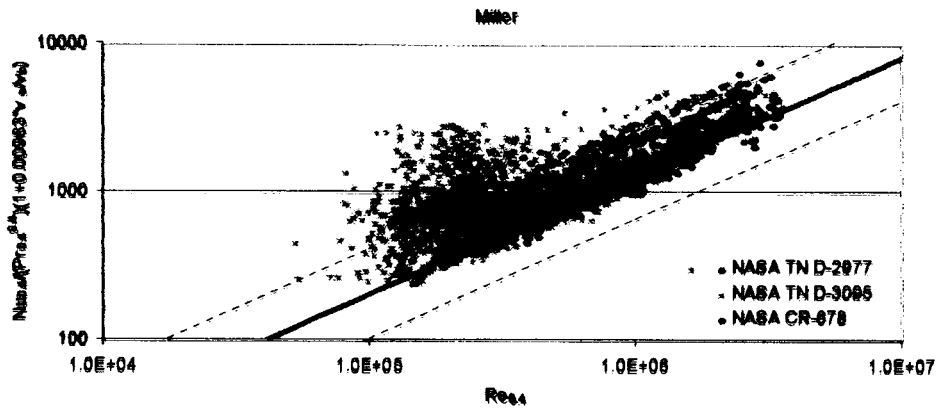
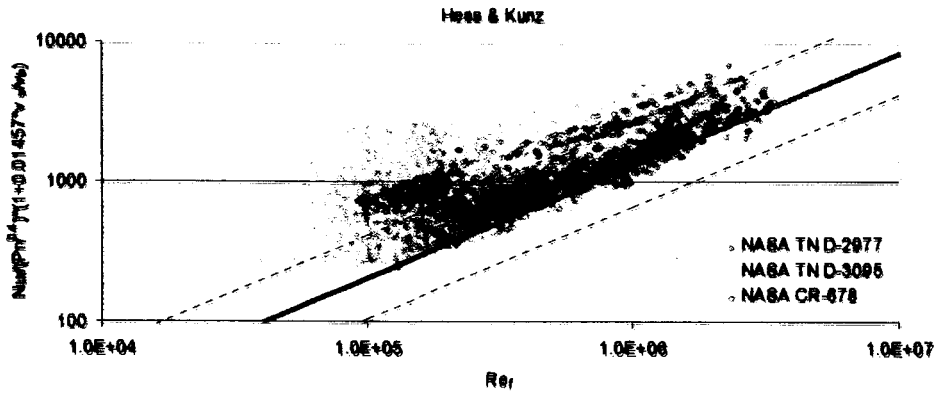
Plots of the experimental data points are shown graphically compared to each correlation in the Appendix. The dashed lines on these plots represent the range of $\pm 100\%$ from the prediction of the correlation. From these plots, along with Figure 6, it is evident that the deviation of the experimental results from those predicted by the correlations are in many cases considerably larger than can be explained by the effects of property uncertainty alone.

V. Conclusions

1. The uncertainty in Nusselt correlations for supercritical hydrogen as a result of uncertainty in property data ranges from about 1.8 to 10%.
2. The uncertainty on Nu is affected by the reference temperature type used. In the low pressure and low temperature range, the bulk reference type correlations have the lowest uncertainty.
3. The deviation of experimental results to those predicted by correlations cannot be explained solely by the uncertainty in property data.
4. The deviations are greatest near the critical region and close to the transition line ($Tr=1$) from liquid to supercritical gas for all pressures. The deviation along the transition line is important for cooling channel applications, and an improved correlation should be developed to better predict heat transfer in this region.
5. The best overall fit to the experimental data came from the Taylor (Eq. 8) and Schact & Quentmeyer (Eq. 9) correlations. The integral property type of Schact & Quentmeyer is promising, and substituting it into other general Nusselt-type equations could lead to improved correlations.

Appendix





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