Stirling Analysis Comparison of Commercial VS. High-Order Methods

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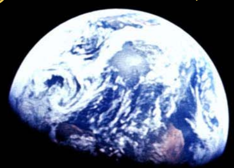
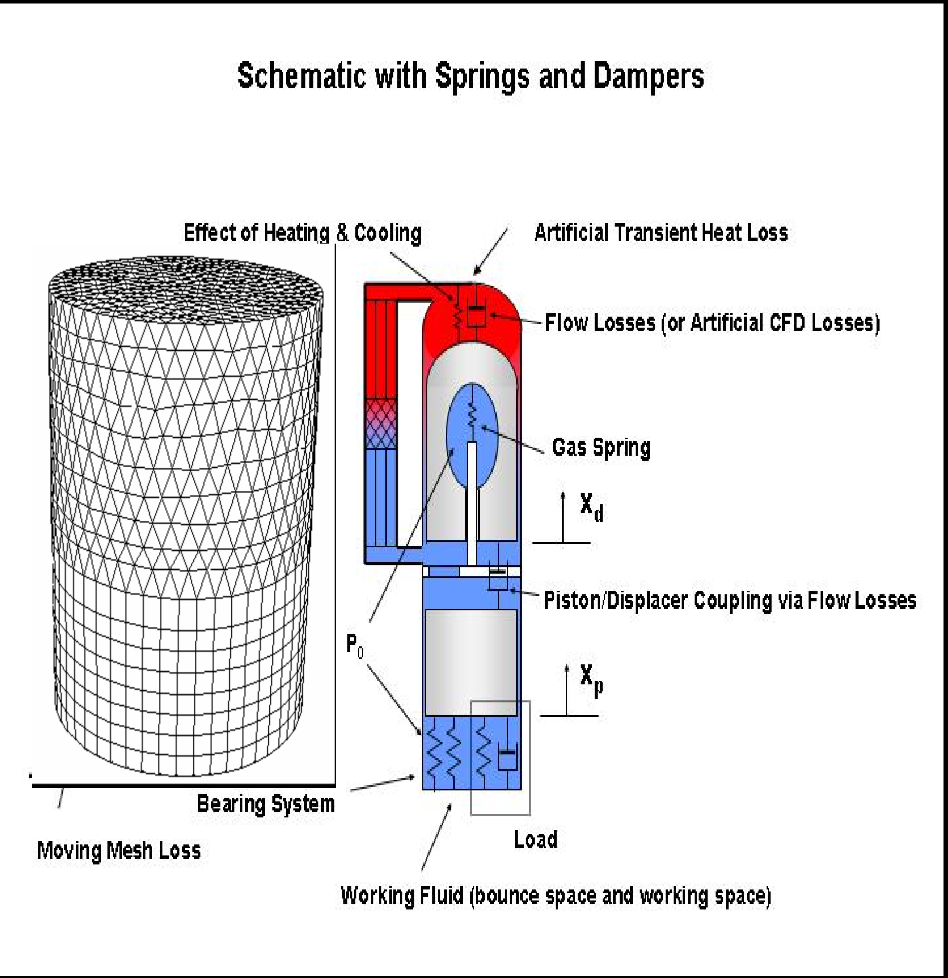


Glenn Research Center at Lewis Field

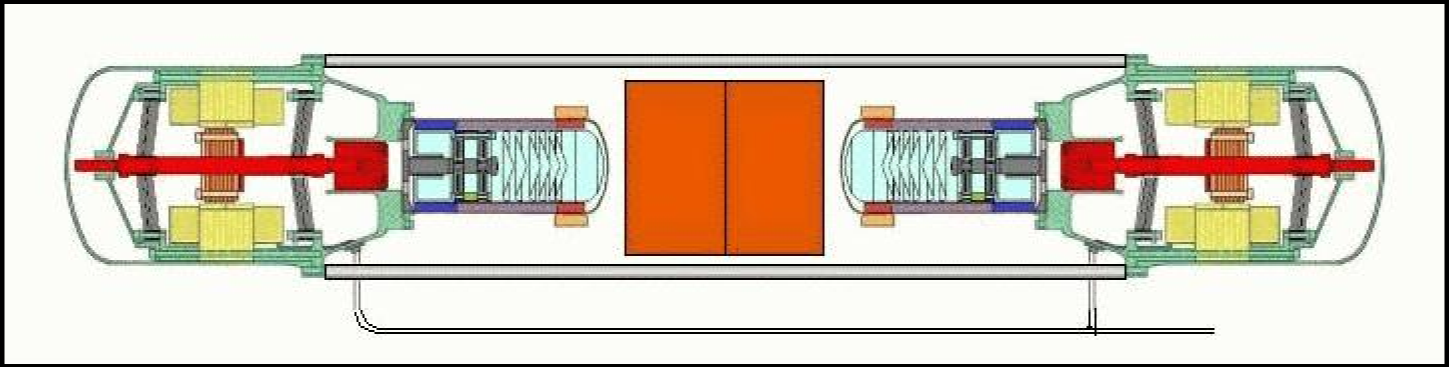
Stirling Simulation Numerical Error

- Remeshing interpolation
- Layering interpolation
- Diffusive time advance
- Low grid quality/skewness
- Sliding interface interpolation
- Artificial entropy
- Turbulence transition



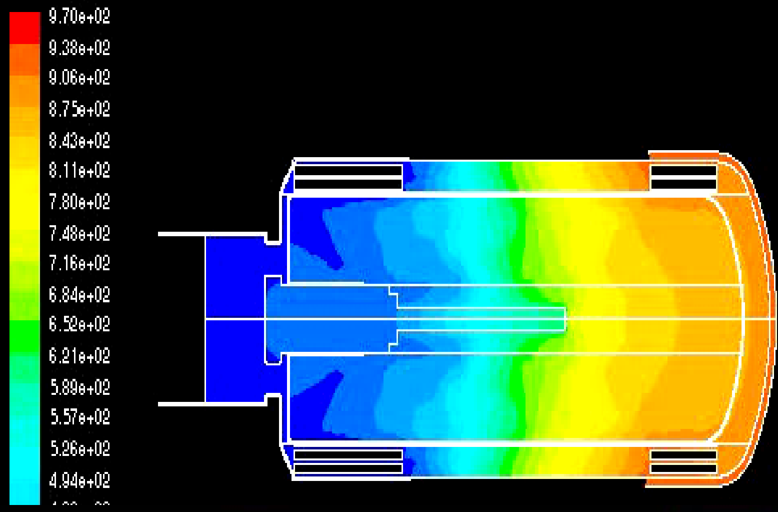
Dual Opposed Convertors

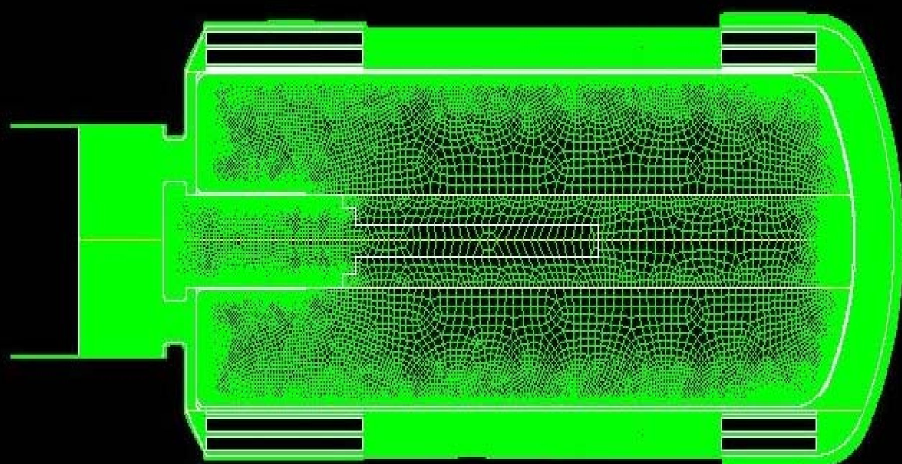
- High Efficiency – Low Mass Space Power



- Free Piston Geometry is Essentially Smooth

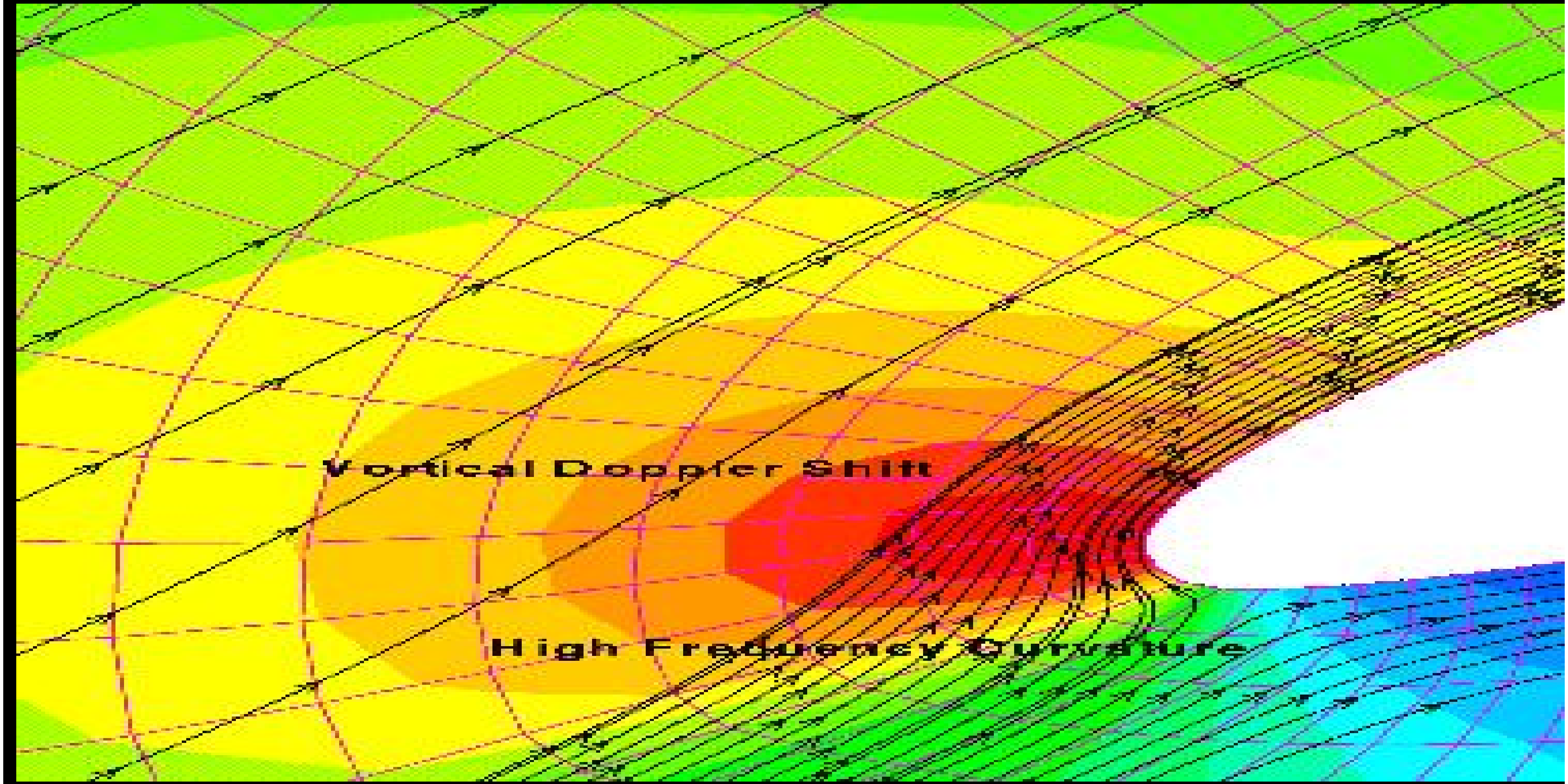
Whole Engine Simulation





- Bounded by Walls – No need for nonreflecting B.C.
- Kolmogorov scales fairly large
- Steep thermal gradients
- No shocks/subsonic/transitioning
- High-order friendly

Curvilinear Features



Stability Analysis

- Courant (CFL) number,
  $r = c \, \Delta t / \Delta x$
- Von Neumann number,
  $v = \mu \, \Delta t / \Delta x^2$
- Linear Viscous Burger's Equation

$$\boxed{\frac{\partial u}{\partial t} + c \frac{\partial u}{\partial x} = \mu \frac{\partial^2 u}{\partial x^2}}$$

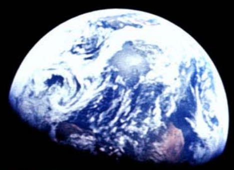
Compact Scheme 6th Order in Space

$$\alpha\left(\frac{\partial u}{\partial x}\right)_{i-1}+\left(\frac{\partial u}{\partial x}\right)_{i}+\alpha\left(\frac{\partial u}{\partial x}\right)_{i+1}=a\,\frac{u_{i+1}-u_{i-1}}{2\Delta x}+b\,\frac{u_{i+2}-u_{i-2}}{4\Delta x}$$

$$\alpha=1/3,\ a=14/9,\ b=1/9$$

$$\alpha\left(\frac{\partial^2 u}{\partial x^2}\right)_{i-1}+\left(\frac{\partial^2 u}{\partial x^2}\right)_{i}+\alpha\left(\frac{\partial^2 u}{\partial x^2}\right)_{i+1}=a\frac{u_{i+1}-2u_i+u_{i-1}}{\Delta x^2}+b\frac{u_{i+2}-2u_i+u_{i-2}}{4\Delta x^2}$$

$$\alpha=2/11,\ a=12/11,\ b=3/11$$



Runge-Kutta 4th Order

$$R(u) = -cu_x + \mu u_{xx}$$

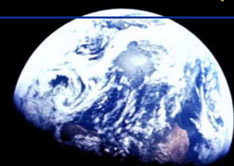
$$u^{(1)} = u^n + \frac{\Delta t}{2} R^n$$

$$u^{(2)} = u^n + \frac{\Delta t}{2} R^1$$

$$u^{(3)} = u^n + \Delta t R^2$$

$$u^{(n+1)} = u^n + \frac{\Delta t}{6}\left(R^n + 2R^{(1)} + 2R^{(2)} + R^{(3)}\right)$$

$$R^{(1)} = R(u^{(1)}), R^{(2)} = R(u^{(2)}), R^{(3)} = R(u^{(3)})$$

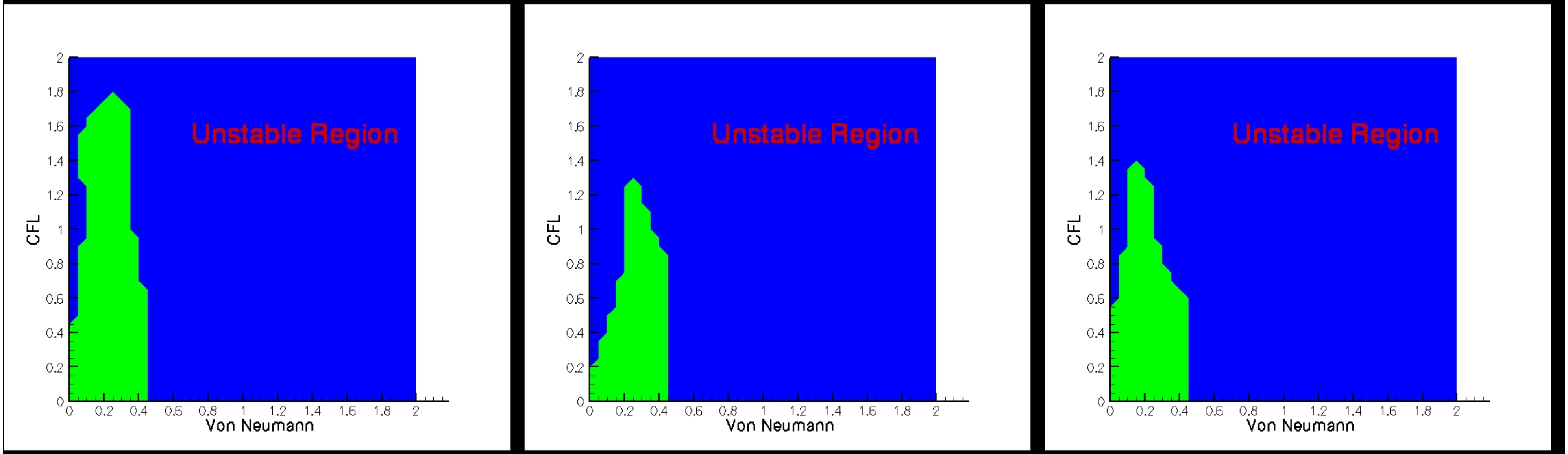


Current Practice

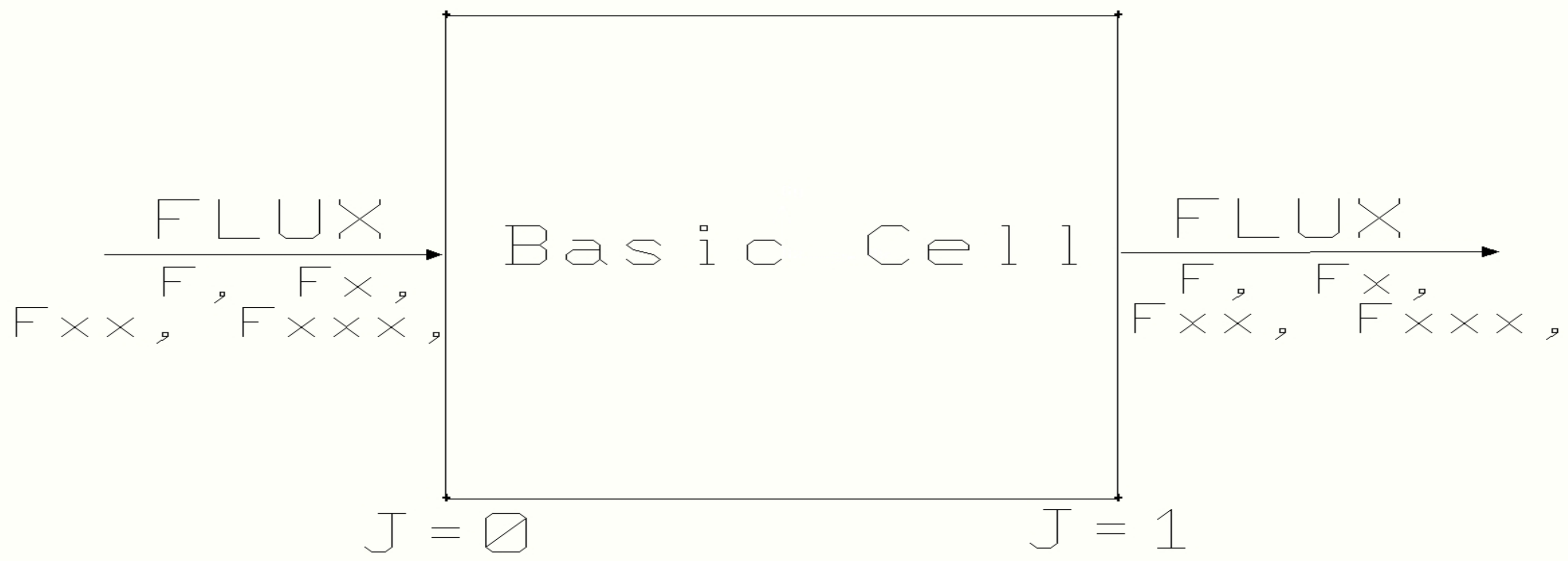
- Implicit 1st or 2nd order in time commercially
- 1st or 2nd order in space implicit
- Explicit/implicit 4th order in time academically
- Implicit 6th order compact scheme in space

Compact Scheme Stability Range

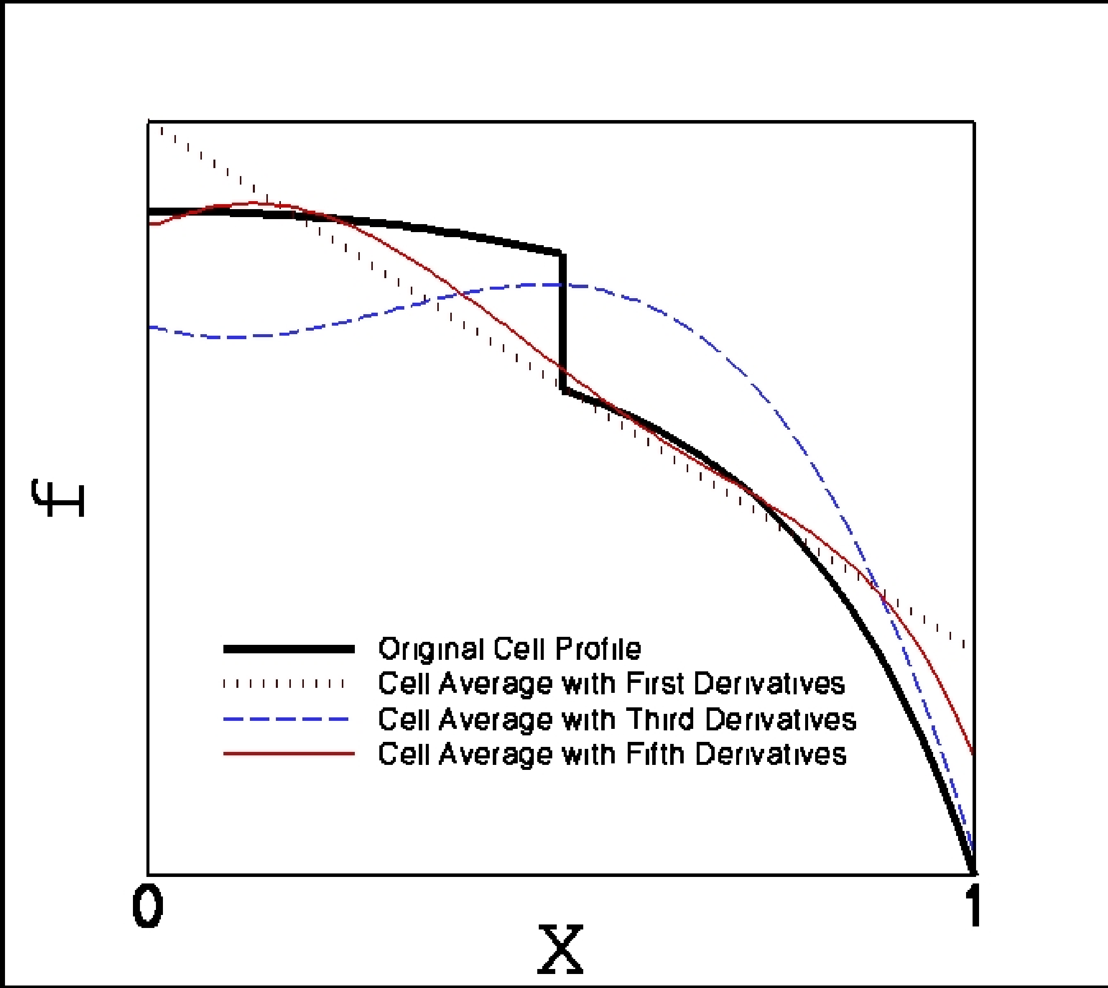
- Domain size affects stability since implicit



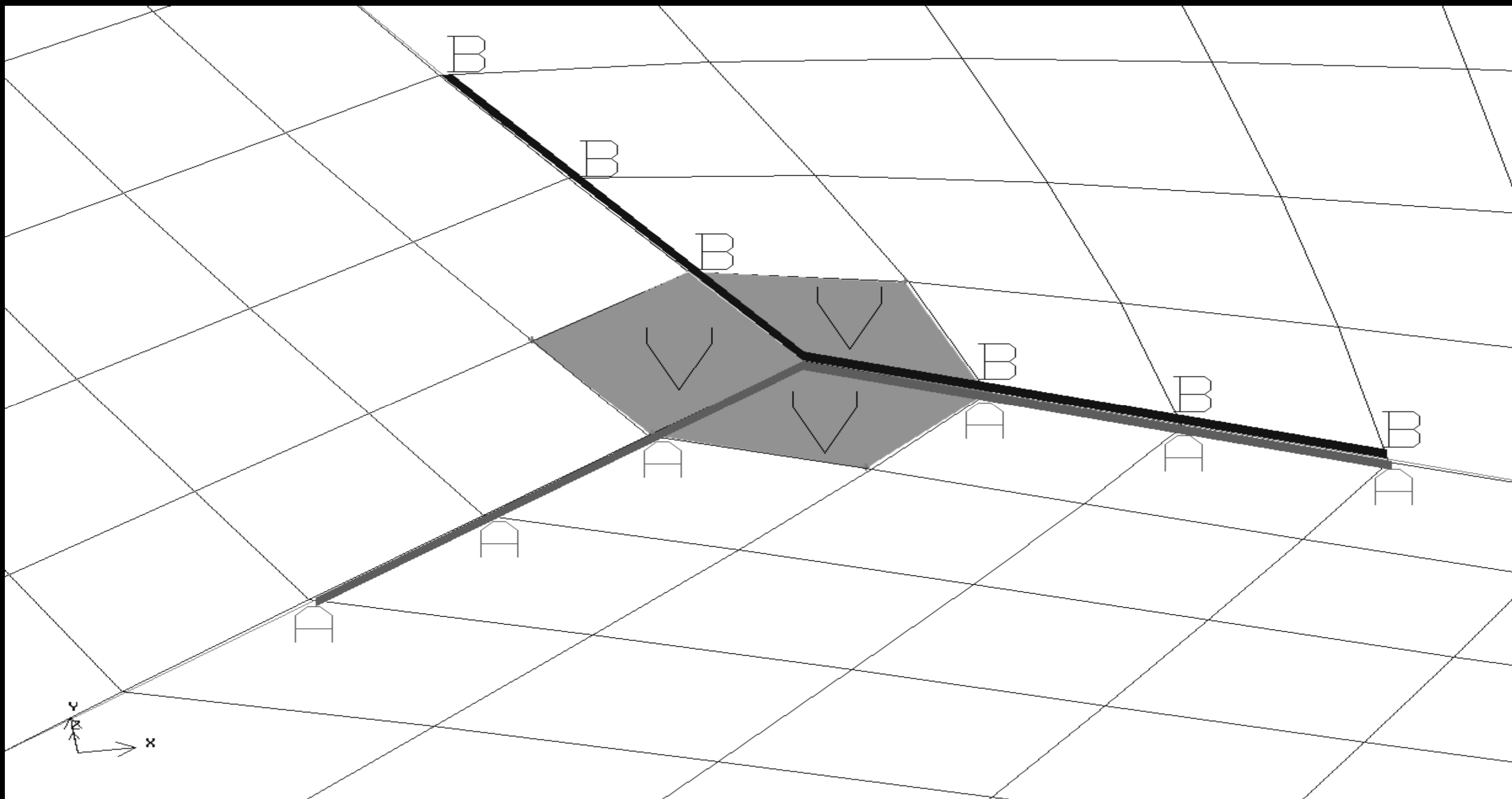
Basic UHF Technique



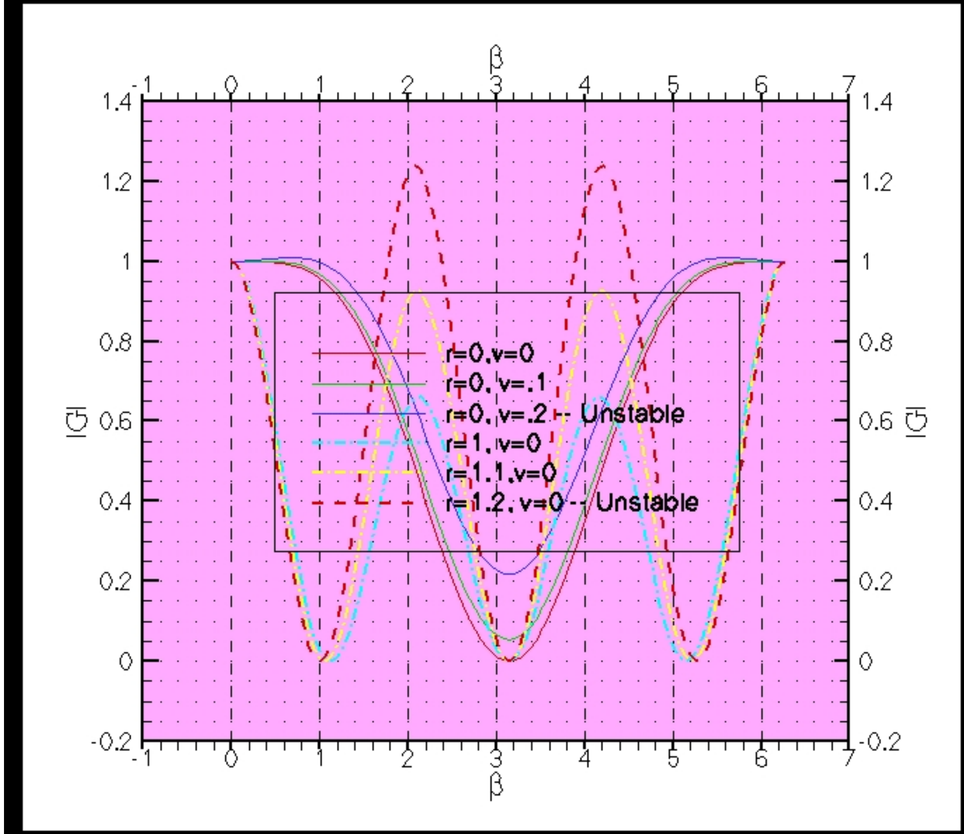
Derivatives of Cell Averages

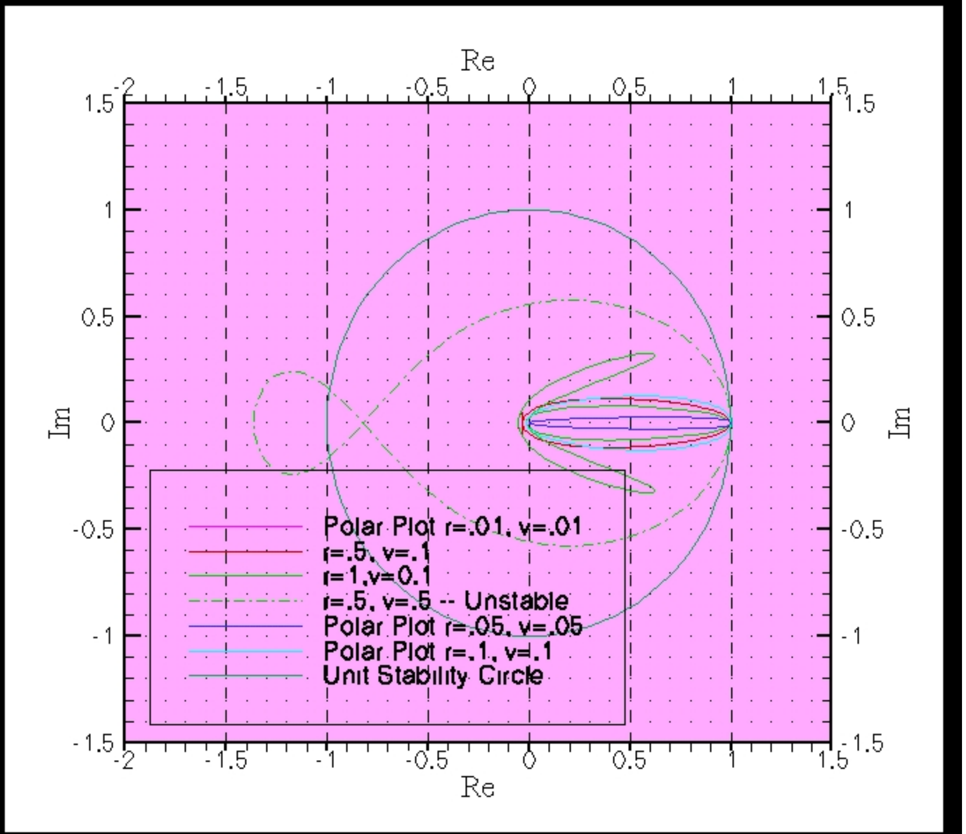


Grid Singularity Resolution

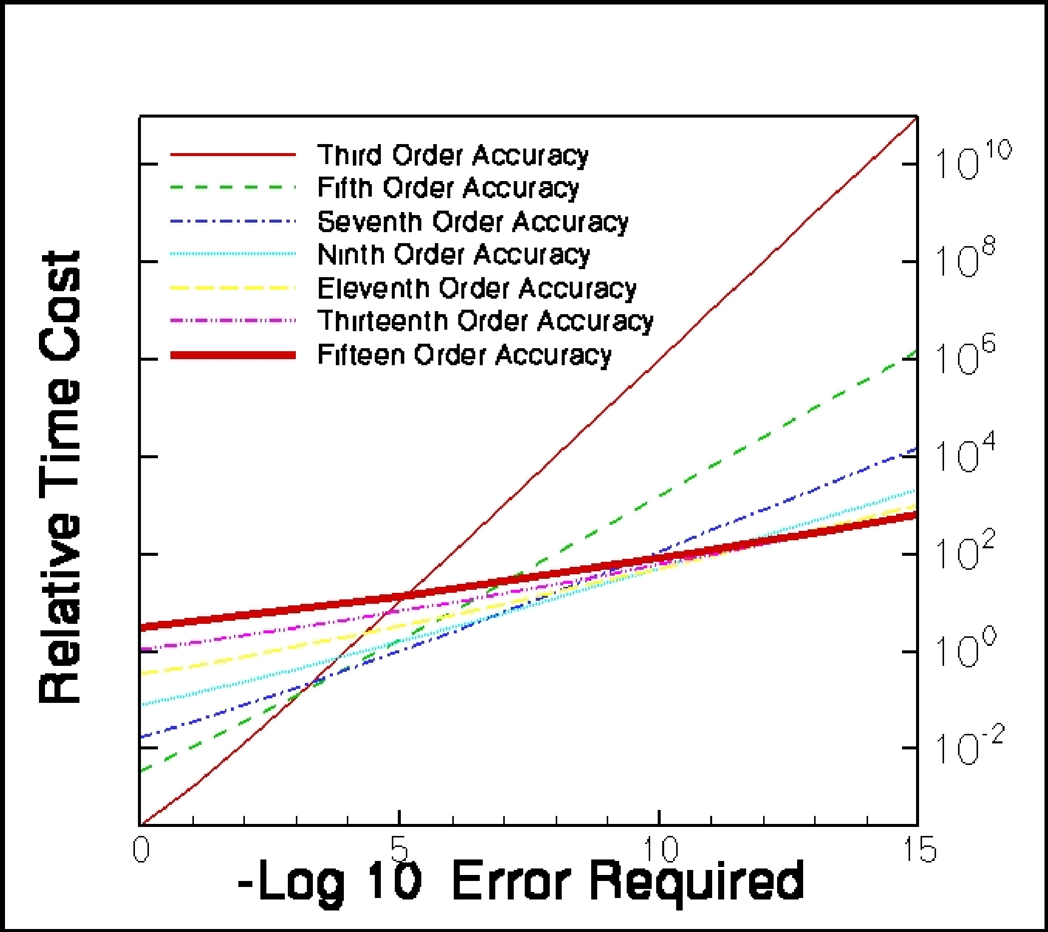


C4o0 Linear Viscous Burger's Equation

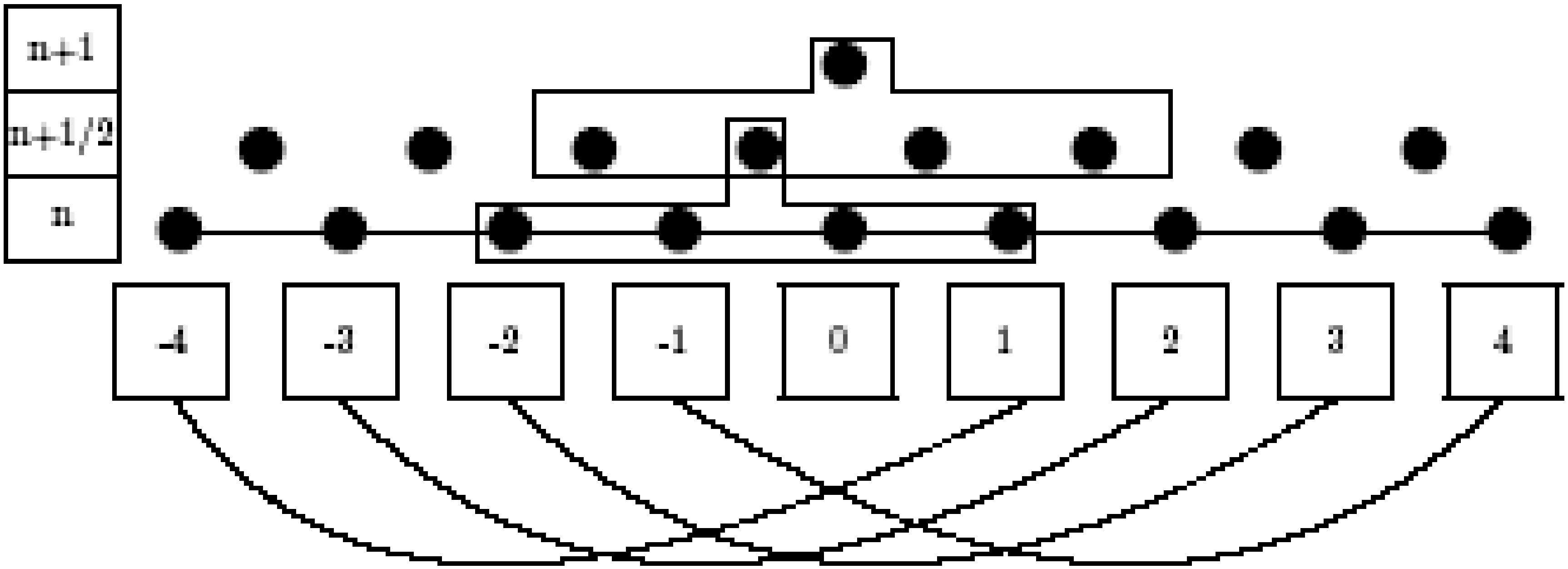




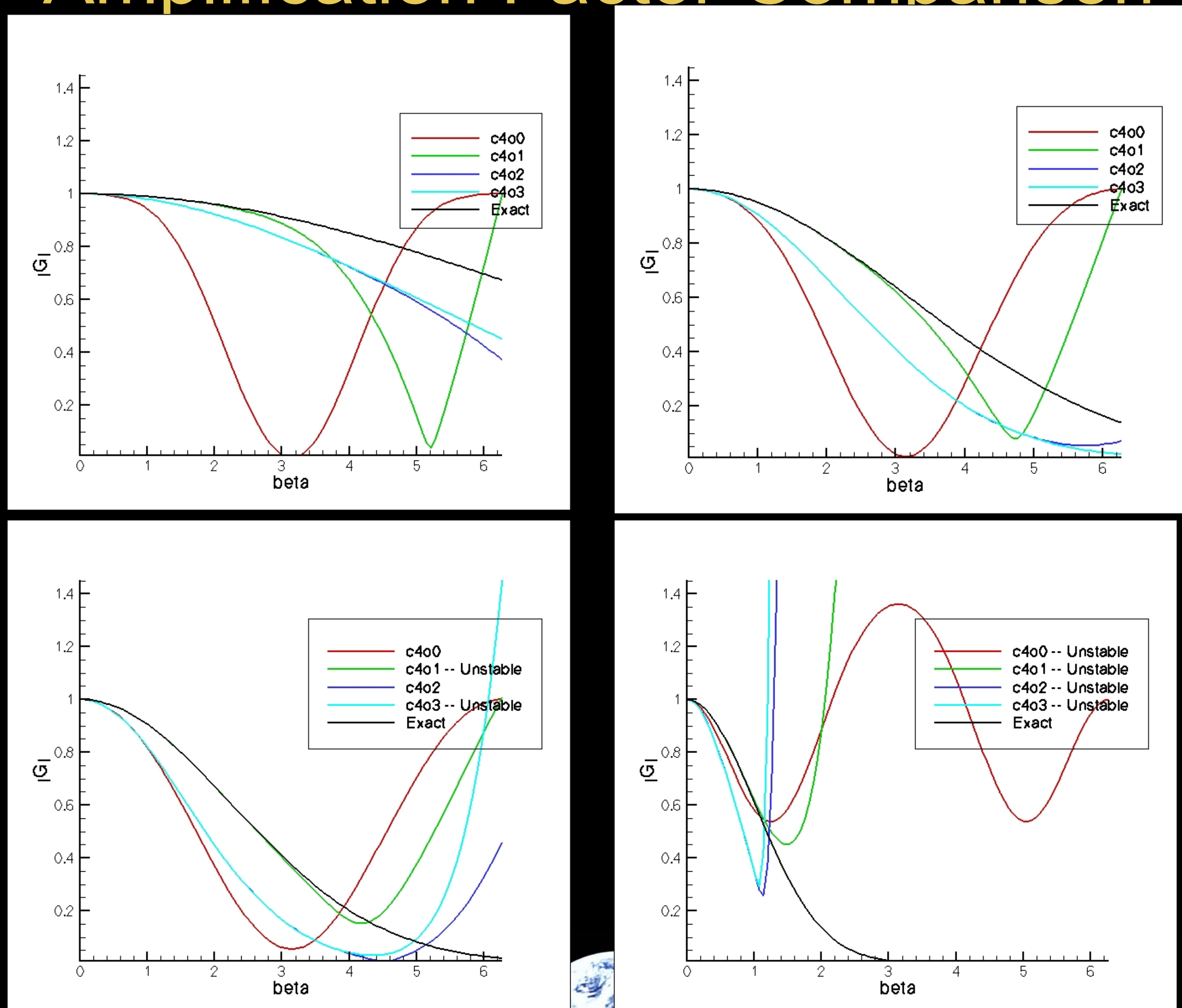
Efficiency Improves with Accuracy



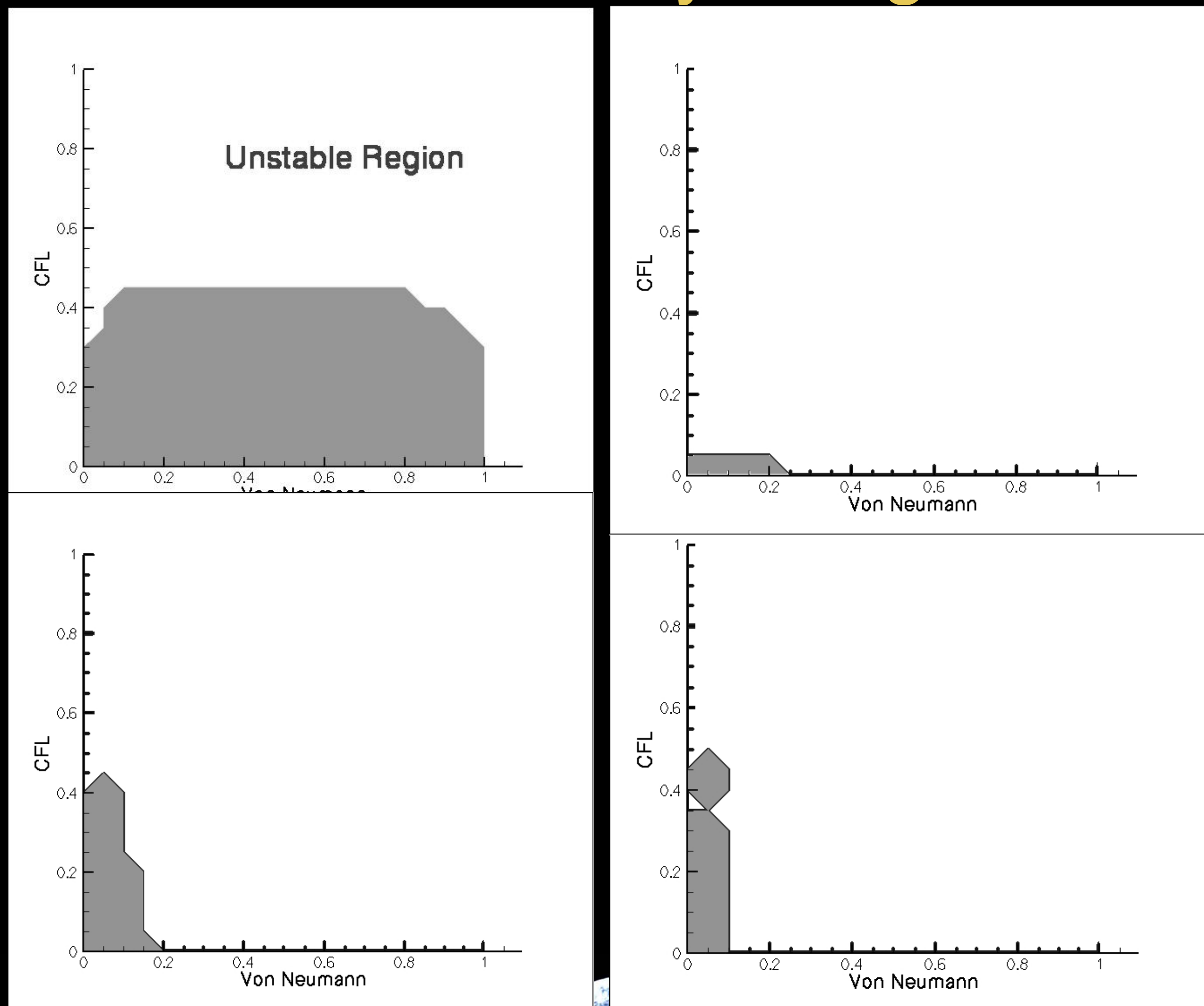
Computational Domain Schematic



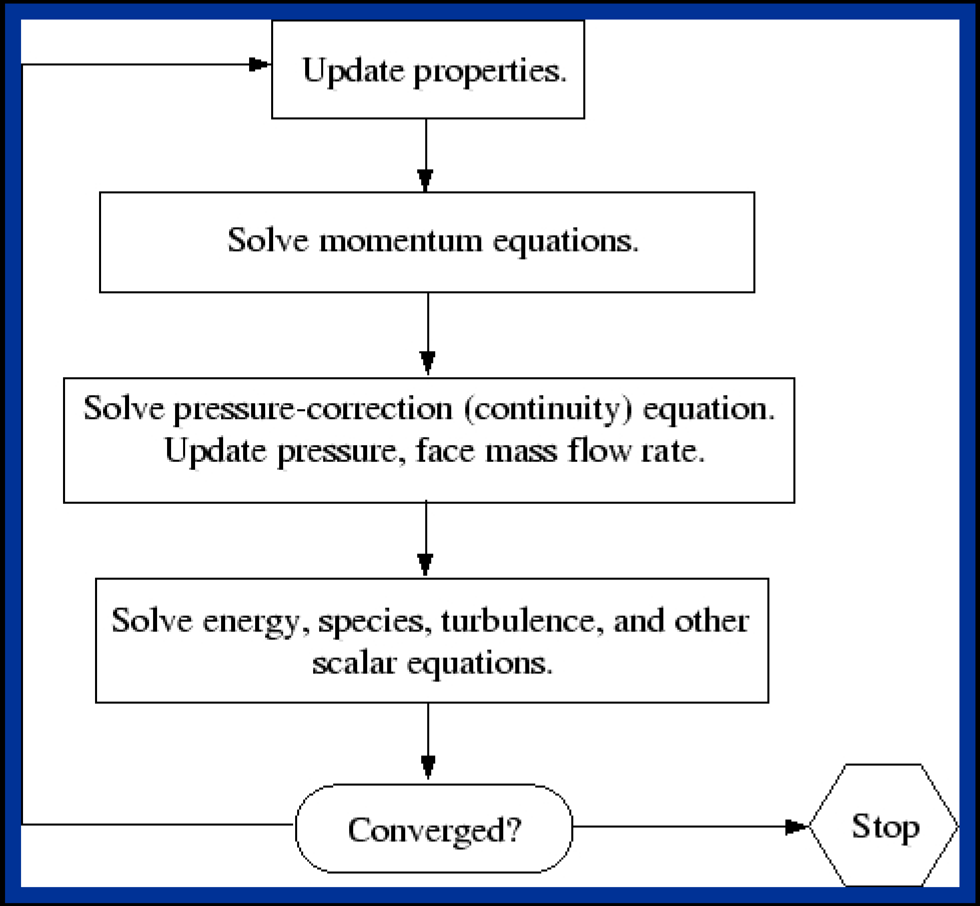
Amplification Factor Comparison



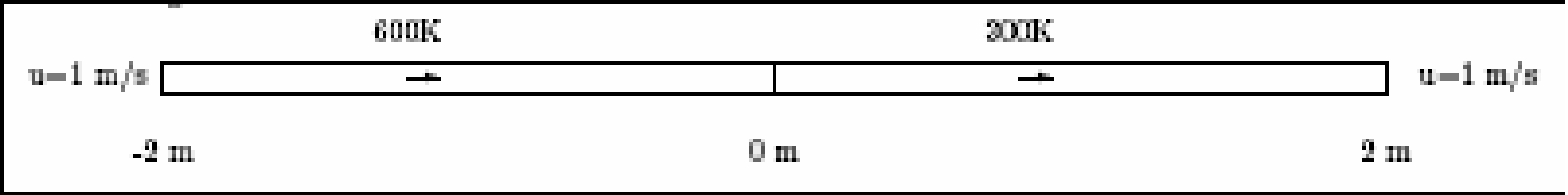
UHF Stability Range



Overview of Segregated Solution



Heat Transfer Test



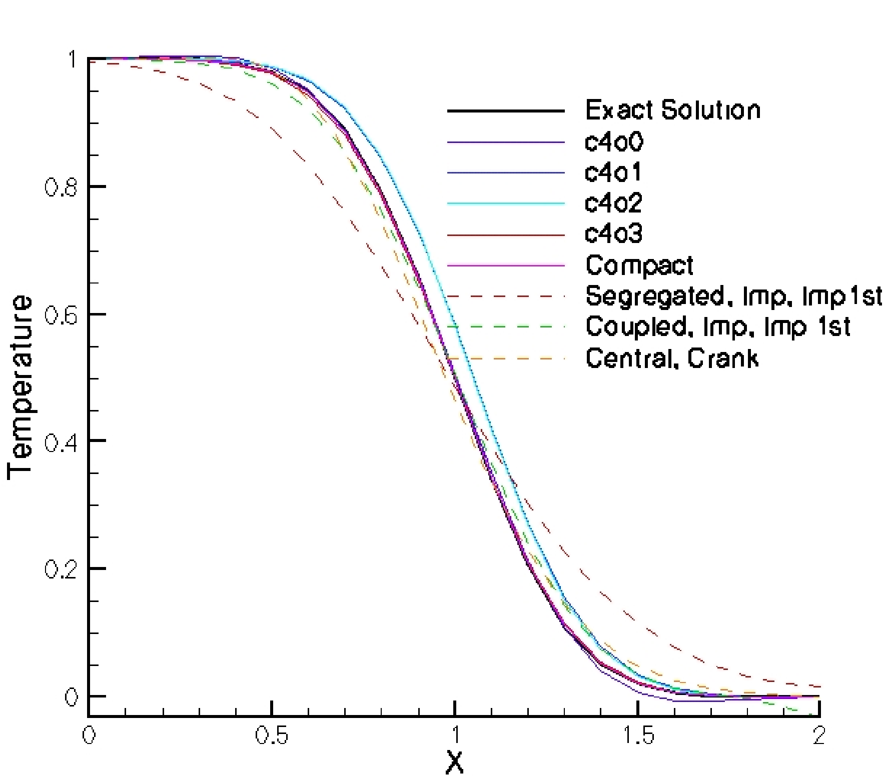
$$\frac{\partial E_t}{\partial t} + \frac{\partial}{\partial x}\left((\rho C_v T + p)u - \frac{4}{3}\mu u u_x + q_x \right) = 0$$

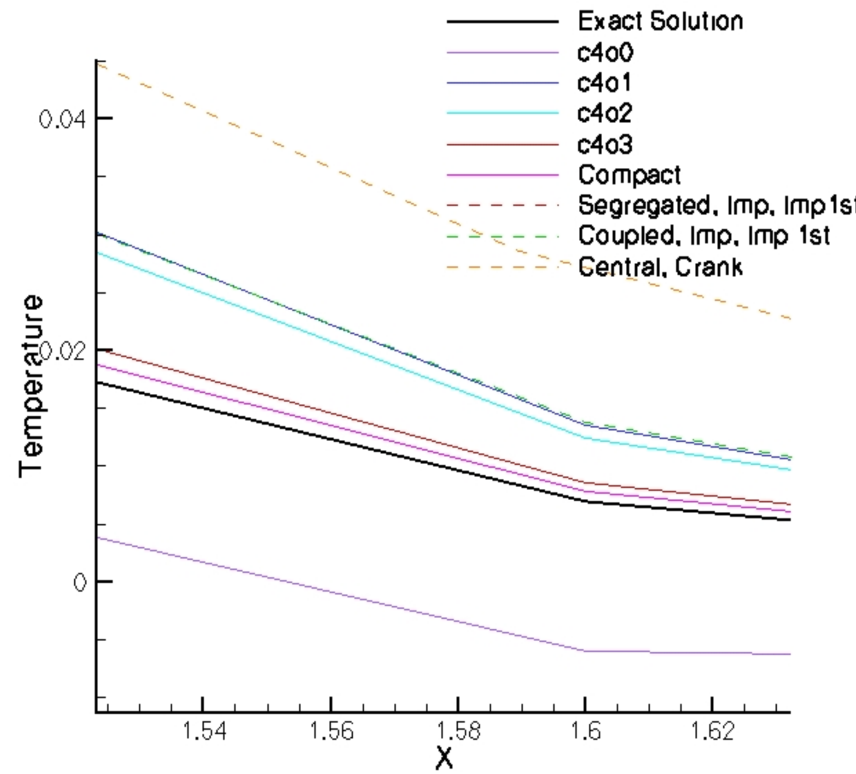
Reduces to:

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = \alpha \frac{\partial^2 T}{\partial x^2},$$

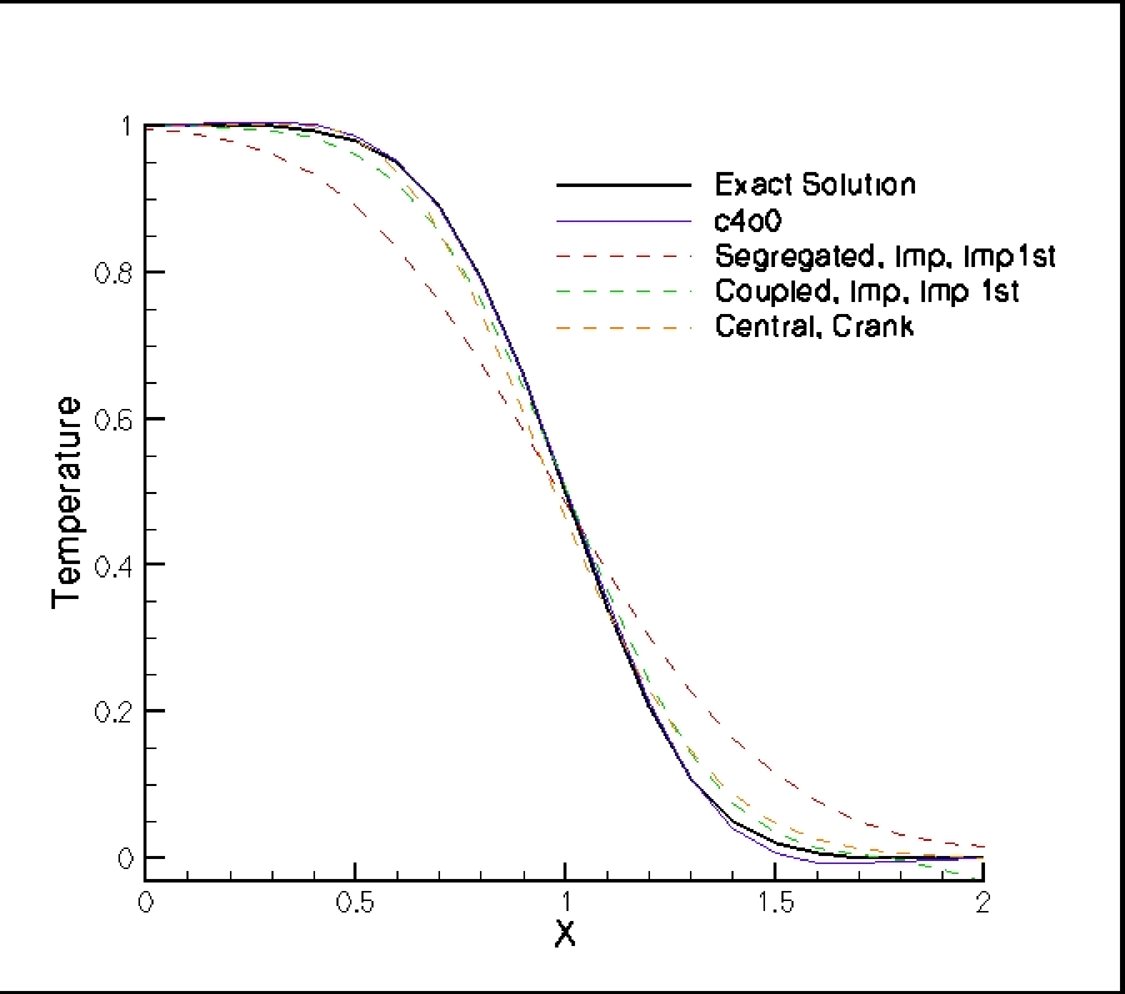
$$\alpha = \frac{k}{\rho C_p}$$

Comparison of Commercial & Advanced

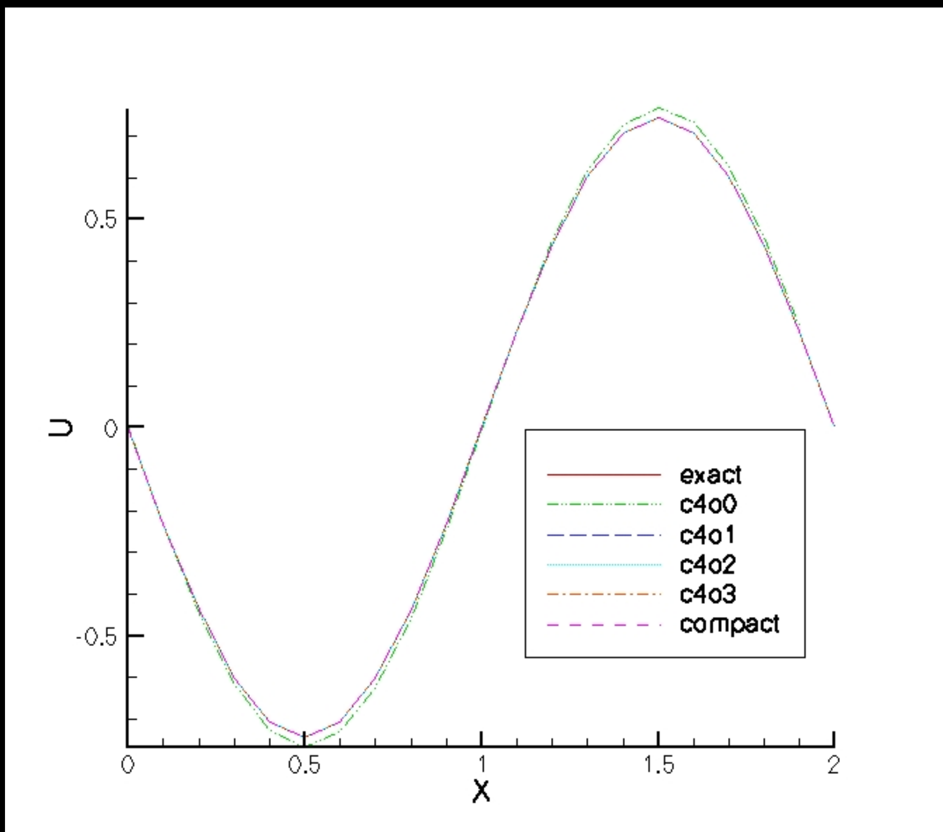




Commercial Comparison

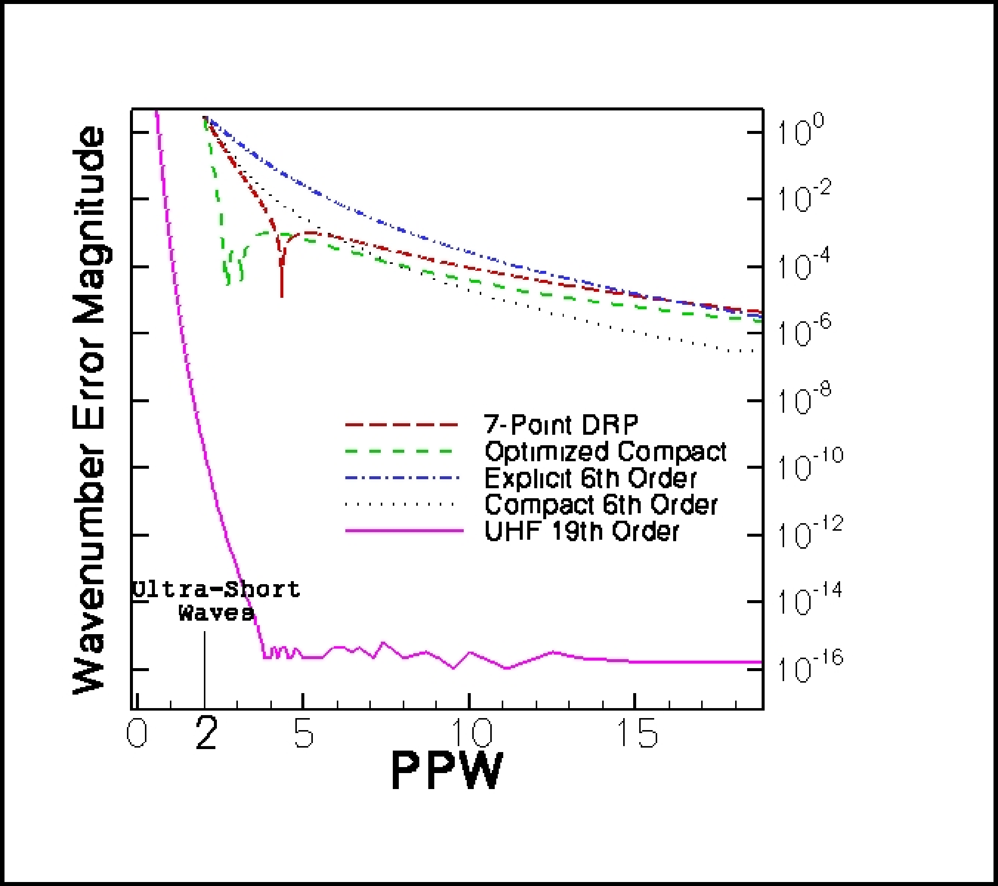


Turbulence Transition Efficiency

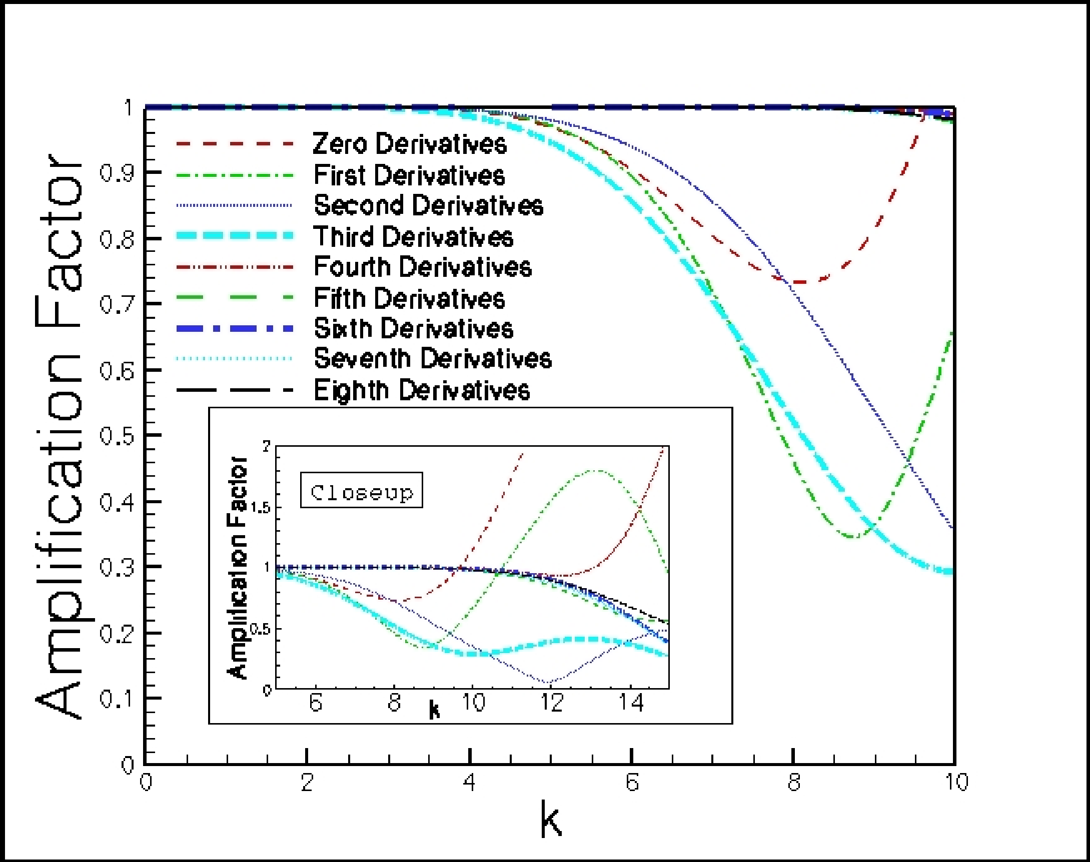


Method	Spacing	Error
c4o0	.1	$2.54299\ 10^{-2}$
c4o0	.2	$4.27563\ 10^{-2}$
c4o0	.4	$4.68577\ 10^{-2}$
c4o1	.1	$3.11163\ 10^{-6}$
c4o1	.2	$2.96551\ 10^{-5}$
c4o1	.4	$8.4702\ 10^{-4}$
c4o2	.1	$1.0178\ 10^{-10}$
c4o2	.2	$3.1935\ 10^{-9}$
c4o2	.4	$6.41079\ 10^{-8}$
c4o3	.1	$3.44169\ 10^{-15}$
c4o3	.2	$2.27818\ 10^{-13}$
c4o3	.4	$3.12925\ 10^{-11}$
compact	.1	$1.0993\ 10^{-6}$
compact	.2	$7.10929\ 10^{-5}$
compact	.4	$5.31245\ 10^{-3}$

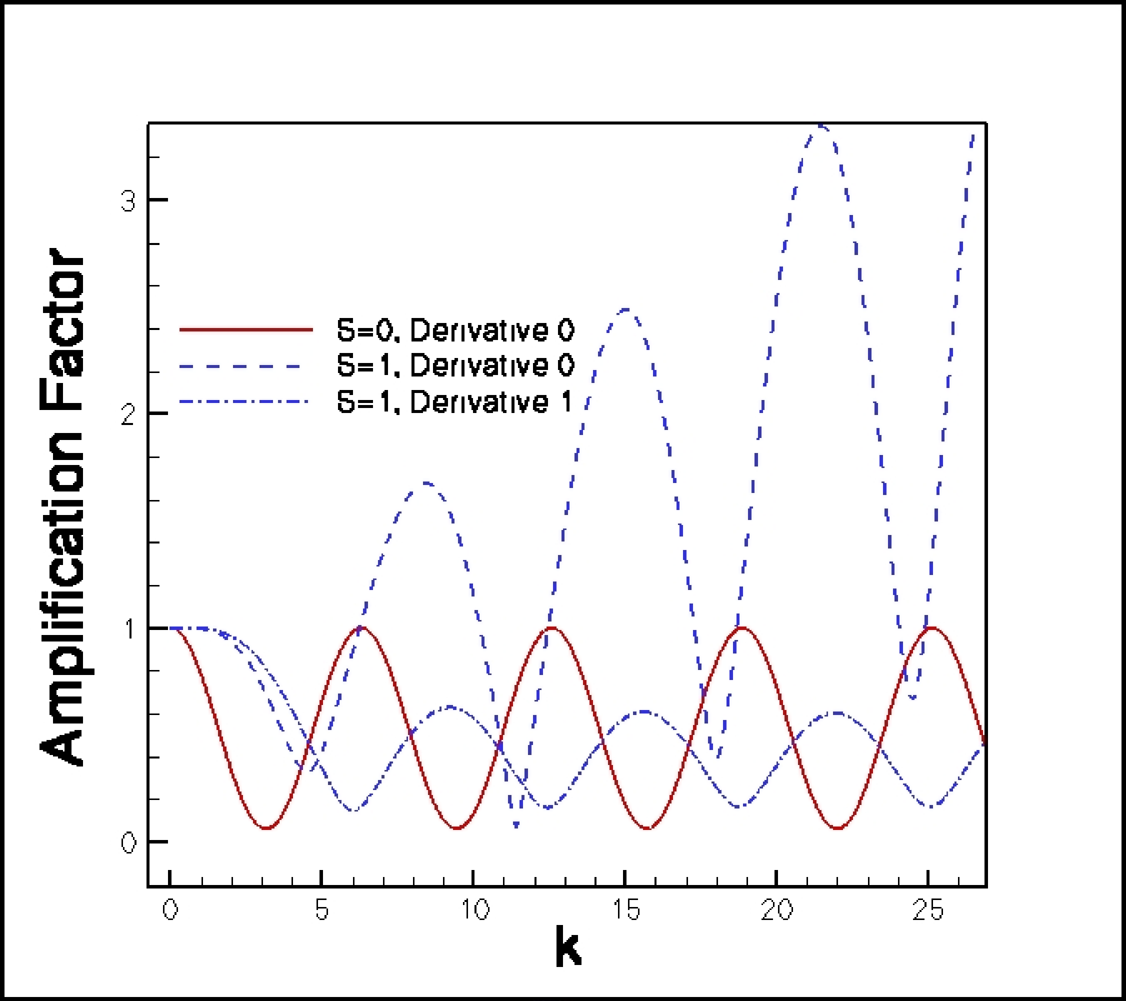
Points per Kolmogorov Wavelength



Wave Equation Amplification

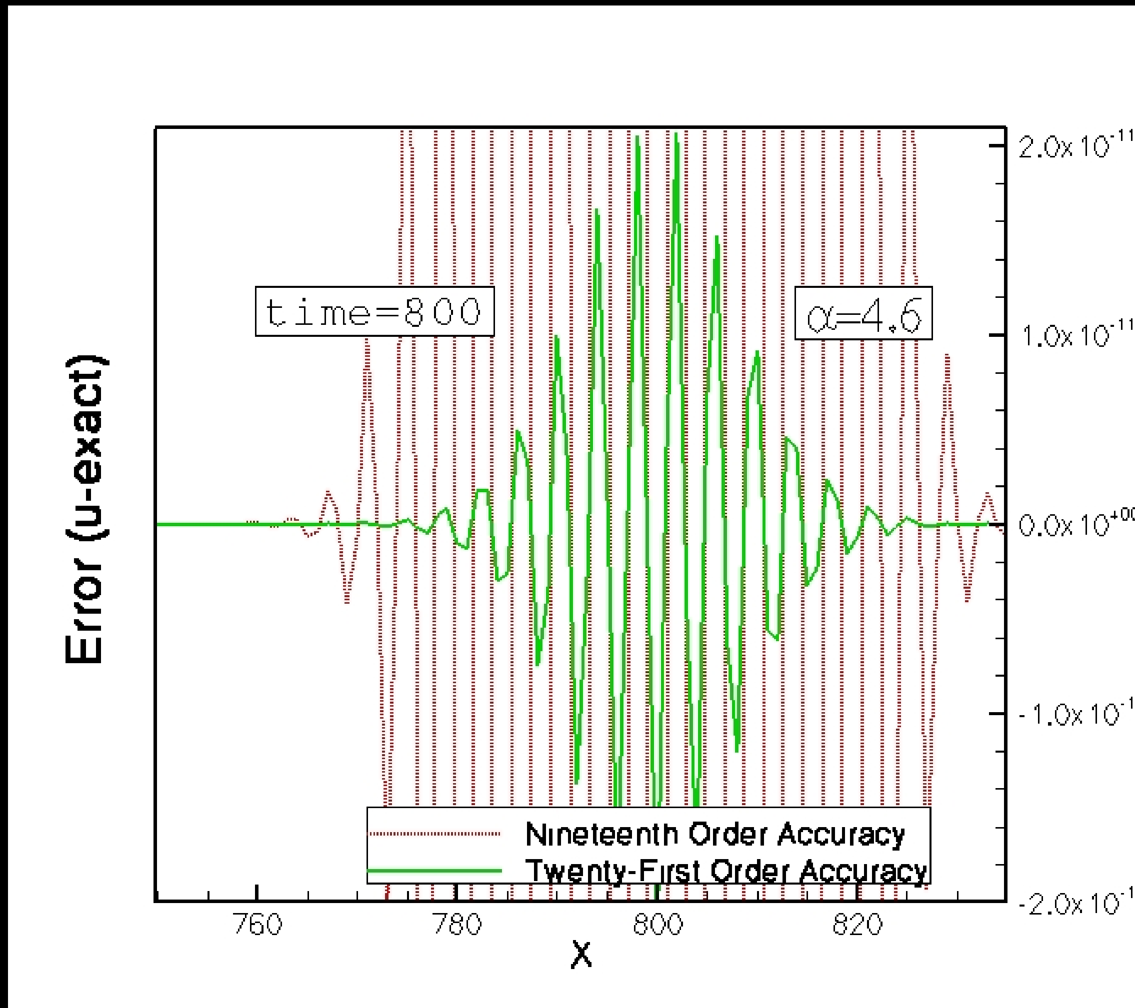


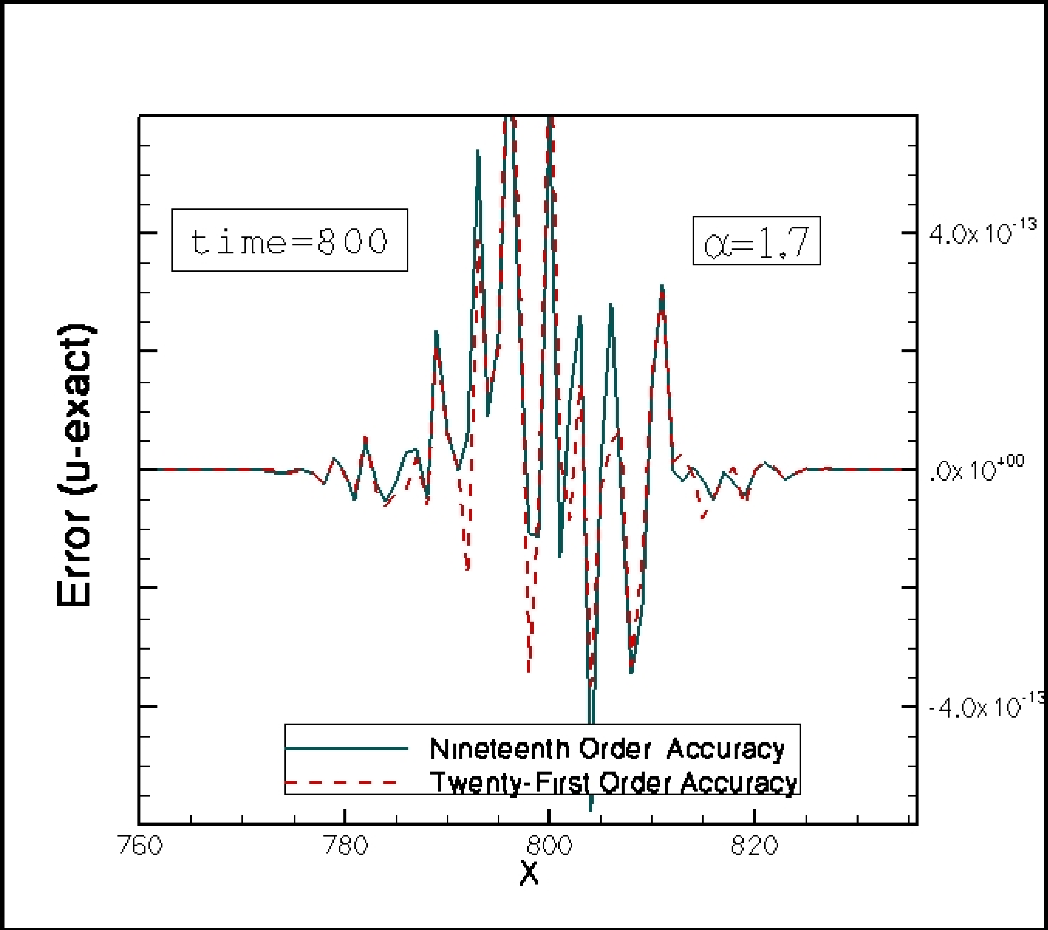
Aliased Frequency Amplification

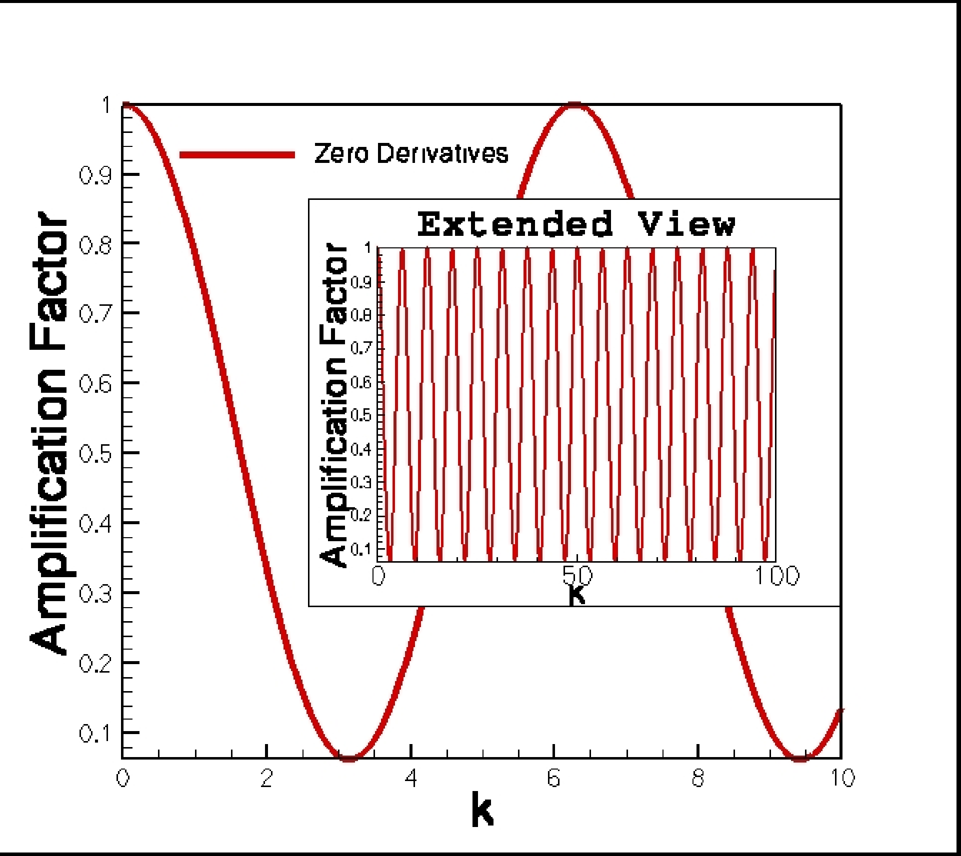


Conclusions

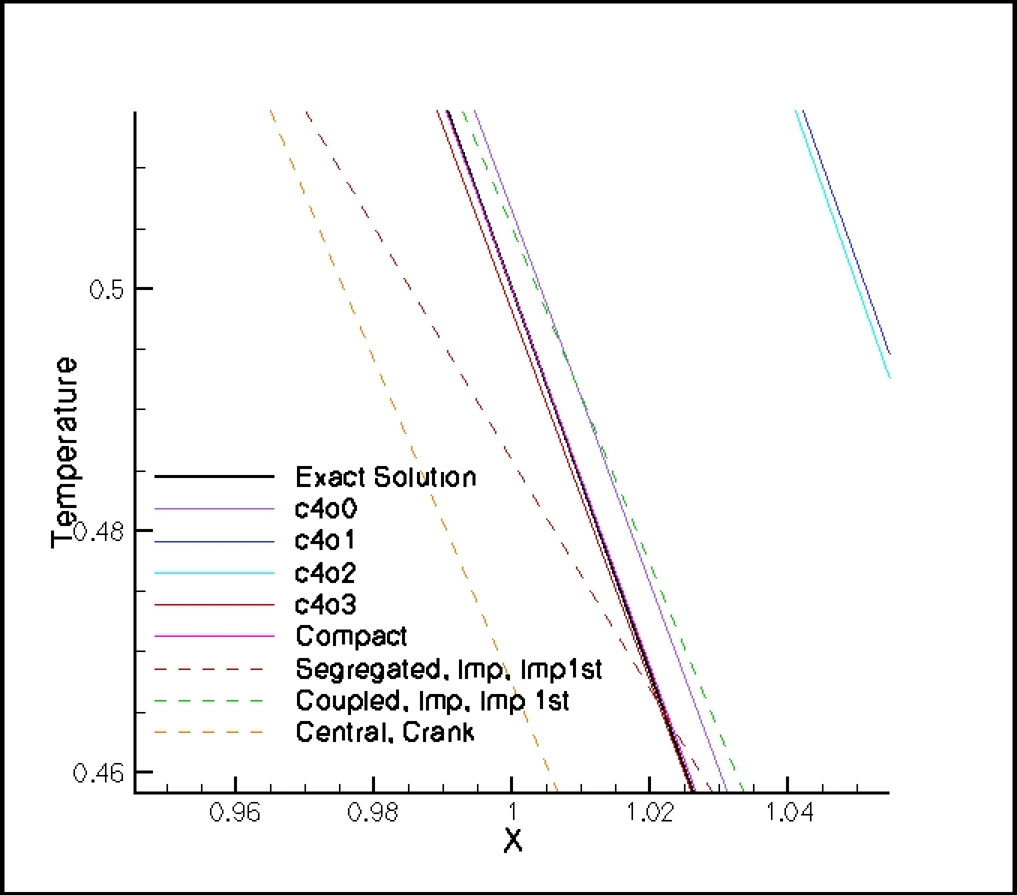
- Low Reynold’s number, wall bounded flow allows economical use of large eddy simulation for turbulent transition modeling
- UHF and Compact comparable at conjugate heat transfer
- UHF much better for turbulence modeling
- Modern methods much more efficient than those currently available commercially


time=800
α=4.6
Error (u-exact)
2.0x10^-11
1.0x10^-11
0.0x10^+00
-1.0x10^-11
-2.0x10^-11
760
780
800
820
X
Nineteenth Order Accuracy
Twenty-First Order Accuracy

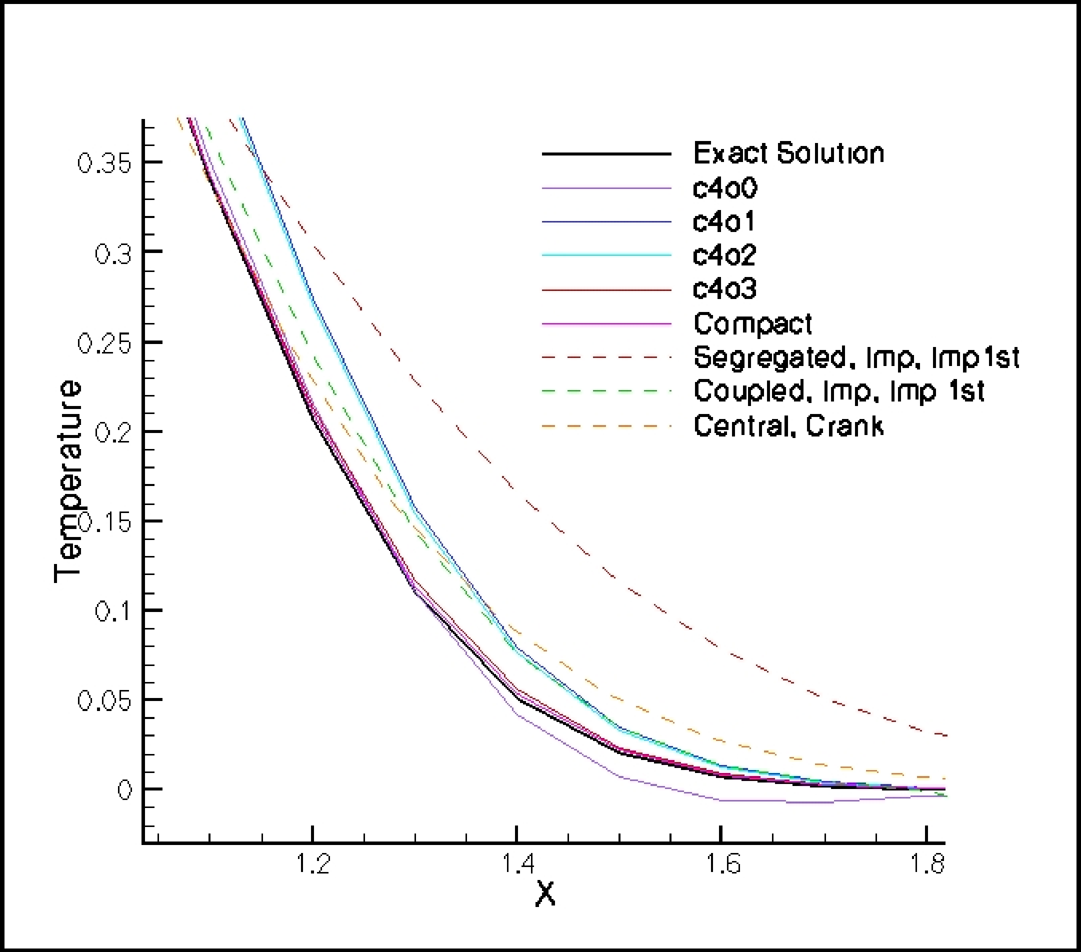

time=800
α=1.7
4.0x10-13
.0x10+00
-4.0x10-13
Error (u-exact)
760
780
800
820
X
Nineteenth Order Accuracy
Twenty-First Order Accuracy

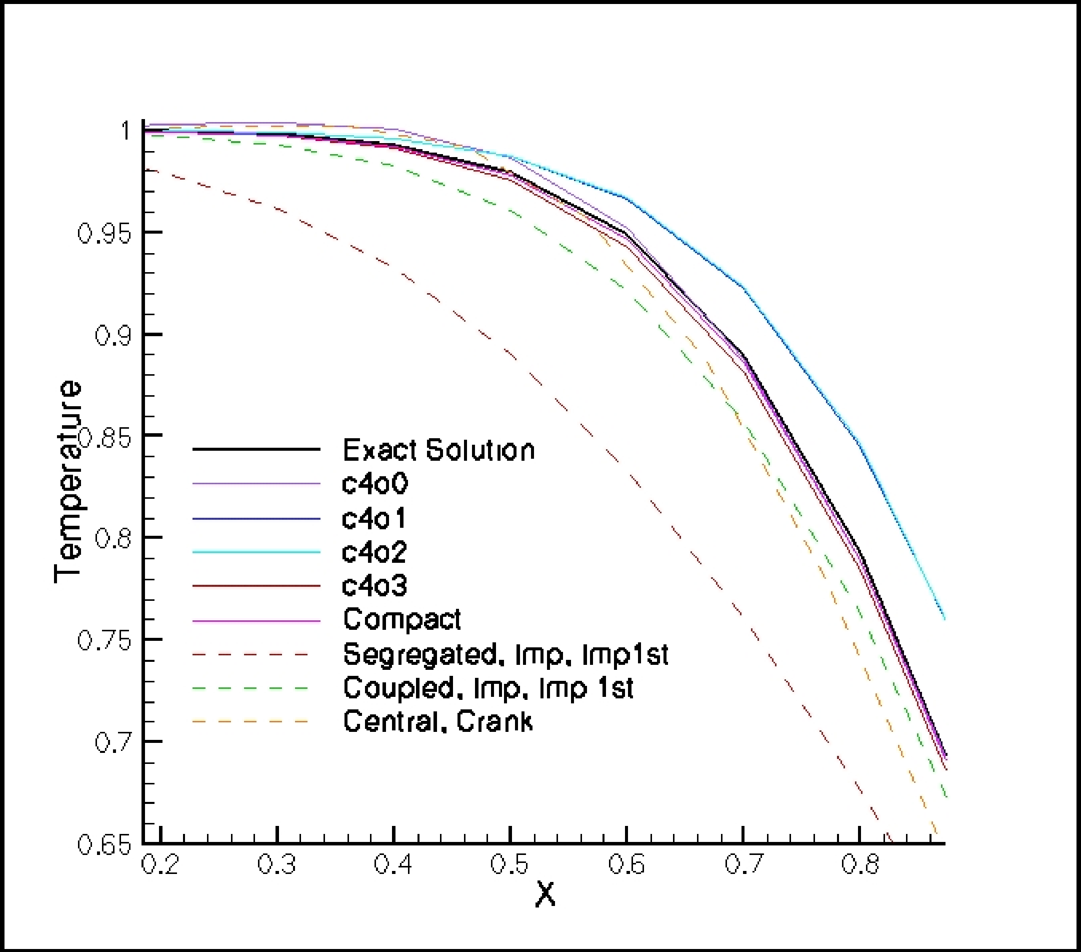


Zero Derivatives
Extended View
Amplification Factor
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
50
100
k
0
2
4
6
8
10

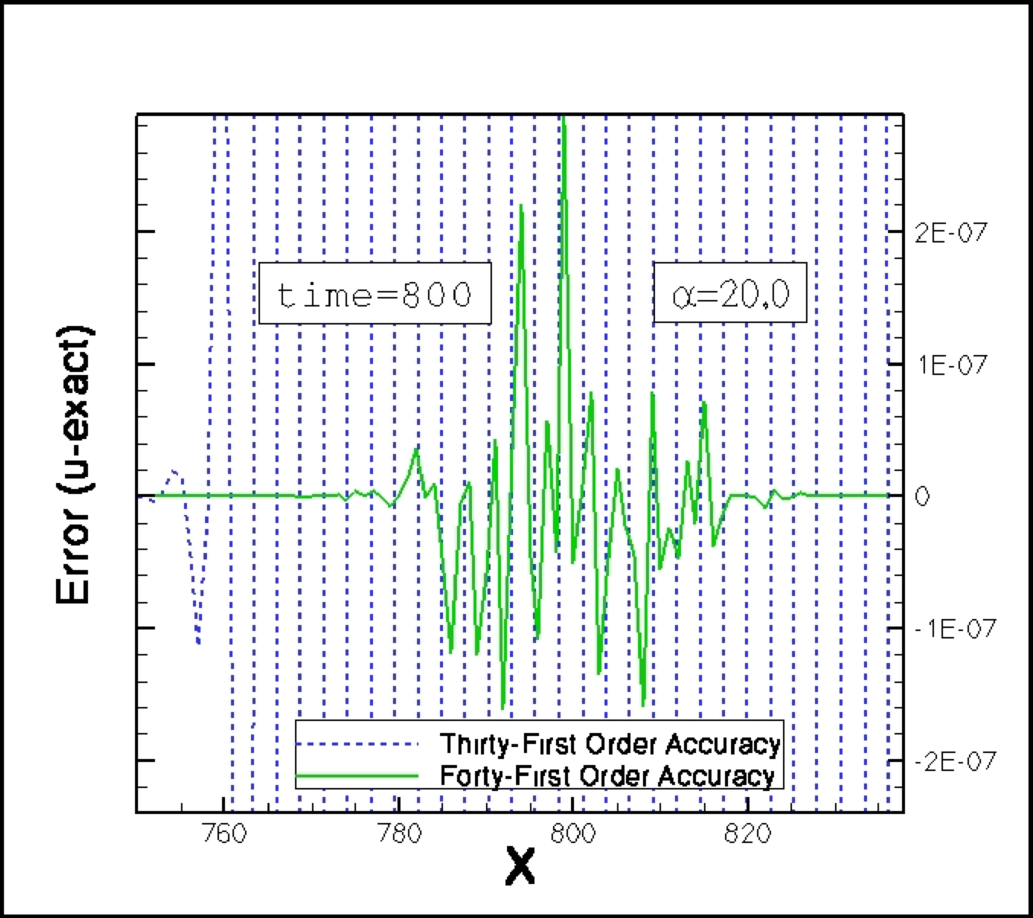


Temperature
X
0.5
0.48
0.46
0.96
0.98
1
1.02
1.04
Exact Solution
c4o0
c4o1
c4o2
c4o3
Compact
Segregated, Imp, Imp1st
Coupled, Imp, Imp 1st
Central, Crank


0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Temperature
1.2
1.4
1.6
1.8
X
Exact Solution
c4o0
c4o1
c4o2
c4o3
Compact
Segregated, Imp, Imp1st
Coupled, Imp, Imp 1st
Central, Crank



Temperature
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.2
0.3
0.4
0.5
0.6
0.7
0.8
X
Exact Solution
c4o0
c4o1
c4o2
c4o3
Compact
Segregated, Imp, Imp1st
Coupled, Imp, Imp 1st
Central, Crank


time=800
α=20.0
Error (u-exact)
X
2E-07
1E-07
0
-1E-07
-2E-07
760
780
800
820
Thirty-First Order Accuracy
Forty-First Order Accuracy