A New Theory of Trajectory Design and NASA's Vision

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Agenda

- Vision for Space Exploration
- Exploration Overview
- Goals and Missions
- Traditional and Innovative Design Methods
- A Historical Perspective of Supported Missions
- Implementation of Chaos
- Mission Applications
- Sun-Earth Libration Missions
- Earth-moon Co-linear Libration Missions
- Lunar Mission Design
Vision for Space Exploration, A “Current” View

Exploration Systems Mission Directorate (ESMD)

“... develop a constellation of new capabilities, supporting technologies, and foundational research that enables sustained and affordable human and robotic exploration.”

Themes:
- Constellation Systems
- Crew Exploration Vehicle (CEV) Development and Launch Vehicles
- Exploration Systems Research and Technology
- Prometheus Nuclear Systems
- Technology and Human Systems

Also part of the ESMD is the Robotic Lunar Exploration Program (RLEP)
- Lunar Reconnaissance Orbiter (LRO)
- 2nd Mission - Lunar Lander
Vision for Space Exploration, A “Current” View

Science Mission Directorate (SMD)
Combines former enterprises of the Space Sciences and Earth Sciences
✓ Solar System Exploration (SSE) (includes the former Moon and Mars exploration)
✓ Earth-Sun System (Sun-Earth-Connections and Earth Sciences)
✓ Universe (includes Origins and Structure & Evolution of Universe)

Examples
- Space interferometry missions,
- James Webb Space Telescope (JWST)
- Terrestrial Planet Finder (TPF)
- Micro Arcsecond X-ray Imaging Mission (MAXIM) Concept

Space Operations Mission Directorate (SOMD)
✓ Shuttle and ISS activities
✓ Space communications systems and the supporting infrastructure.
✓ Ongoing libration orbit missions such as SOHO and WIND missions
A New Theory

Definition: This new theory is defined as the use of chaos to design trajectories and orbits that can be used to meet complex mission goals

Benefits:
- Minimizes fuel cost (related to Delta-V cost)
- Optimizes trajectory profiles
- Provides non-standard and new orbit designs
- Mitigates operational risks

Other ‘synonymous’ terms
- Dynamical Systems
- Invariant Manifolds
- Capture Orbits
- Ballistic Orbits
A Sample of Analysis

So let us look at a few sample ESMD and SMD missions:

• Sun-Earth libration orbits
• Earth-moon libration orbits
• Lunar mission design
• Use of chaos to aid in their design
History, Definitions, & Modeling

Mathematical History

Euler
- Defined three body problem in work on lunar motion.
- Proved existence of co-linear points

Lagrange
- Development of equilibrium points

Poincare
- Stability of motion and use of potential functions
- First to recognize the need for a qualitative approach to three body problem which is unsolvable in closed form

Jacobi
- One exact integral of three body system

Definitions & Modeling
- Easiest to model the system as the Circular Restricted Three Body Problem (CRTBP) where \( m_1 >> m_2 >> m_3 \)
  - \( m_1 \) - primary, \( m_2 \) - secondary, \( m_3 \) - body of interest
  - motion of Earth about Sun is circular
  - motion of \( m_3 \) is in plane of \( m_1 \) & \( m_2 \)
- CRTBP can be solved exactly
- Unfortunately, unmodeled forces (solar radiation pressure, other gravitational bodies - Jupiter, etc.) and physical reality (non-circular motion or EM system about sun) cause perturbations
Libration Points

What Are They??

- Equilibrium or libration points represent singularities in the equations of motion where velocity and acceleration components are zero and the forces are balanced.
- Viewed in the rotating frame: centrifugal (Coriolis-Type) force balances with gravitational forces of the two primaries.
- Libration points are in plane with no Z component. Orbits are mapped to a rotating frame where there are no time dependent forces.
- Our system of interest involves the Sun ($m_1$), the Earth-Moon system ($m_2$) and the spacecraft $m_3$.
- $L_1$ and $L_2$ distance of 1.5 million km.
- $L_4$ and $L_5$ distance of 150. million km.

Where Are They?

- Collinear Points: $L_1$, $L_2$, $L_3$ (unstable)
- Triangular Points: $L_4$, $L_5$ (stable)
Views of a Circular Restricted Three Body Lissajous Orbit

Top (Along z)

Side (Along y)

Through (along x)
# Historical and Future Missions in Libration Orbits

<table>
<thead>
<tr>
<th>Mission</th>
<th>Location / Type</th>
<th>Amplitudes (Ax, Ay, Az)</th>
<th>Launch Year</th>
<th>Total ΔV Allocation (m/s)</th>
<th>Transfer Type</th>
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<tbody>
<tr>
<td>ISEE-3</td>
<td>L1Halo/L2/Comet 1st mission</td>
<td>175000, 660670, 120000</td>
<td>1978</td>
<td>430</td>
<td>Direct</td>
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<tr>
<td>WIND⁺</td>
<td>L1 – Lissajous</td>
<td>10000, 350000, 250000</td>
<td>1994</td>
<td>685</td>
<td>Multiple Lunar Gravity Assist Direct</td>
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<td>SOHO</td>
<td>L1 – Lissajous</td>
<td>206448, 666672, 120000</td>
<td>1995</td>
<td>275</td>
<td>Direct</td>
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<td>ACE</td>
<td>L1 – Lissajous, 1st small amplitude</td>
<td>81775, 264071, 157406</td>
<td>1997</td>
<td>590</td>
<td>Direct (Constrained) Single Lunar Gravity Assist Direct</td>
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<tr>
<td>MAP</td>
<td>L2-Lissajous 1st L2 Mission</td>
<td>n/a, 264000, 264000</td>
<td>2001</td>
<td>127</td>
<td>Direct</td>
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<td>Genesis</td>
<td>L1-Lissajous</td>
<td>250000, 800000, 250000</td>
<td>2001</td>
<td>540</td>
<td>Direct</td>
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<td>Triana</td>
<td>L1-Lissajous Launch Constrained</td>
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<td>620</td>
<td>Direct</td>
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<tr>
<td>JWST</td>
<td>L2-Quasi-Periodic Lissajous</td>
<td>290000, 800000, 131000</td>
<td>#</td>
<td>90</td>
<td>Direct</td>
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<tr>
<td>SPECs</td>
<td>L2-Lissajous Tethered Formation</td>
<td>290000, 800000, 131000</td>
<td>#</td>
<td>Tbd</td>
<td>Direct</td>
</tr>
<tr>
<td>MAXIM Constellation-X</td>
<td>L2 – Lissajous Formation</td>
<td>Large Lissajous</td>
<td>#</td>
<td>#</td>
<td>Direct</td>
</tr>
<tr>
<td></td>
<td>L2 – Lissajous Loose Formation</td>
<td>Large Lissajous</td>
<td>150-250</td>
<td>Single Lunar Gravity Assist</td>
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<td>Darwin</td>
<td>L1-Lissajous</td>
<td>300000, 800000, 350000</td>
<td>2014</td>
<td>#</td>
<td></td>
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<tr>
<td>Stellar Imager</td>
<td>L2 – Lissajous ~30 S/C Formation</td>
<td>Large Lissajous</td>
<td>2015</td>
<td>#</td>
<td>Direct</td>
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<tr>
<td>TPF</td>
<td>L2 – Lissajous Formation?</td>
<td>Lissajous</td>
<td>#</td>
<td>#</td>
<td>#</td>
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</table>
Mission: Investigate Solar-Terrestrial relationships, Solar Wind, Magnetosphere, and Cosmic Rays

Launch: September, 1978, Comet Encounter Sept., 1985

Lissajous Orbit: L1 Libration Halo Orbit, Ax=\sim 175,000\text{ km}, Ay = 660,000\text{ km}, Az\sim 120,000\text{ km}, Class I

Spacecraft: Mass=480\text{ Kg}, Spin stabilized,

Notable: First Ever Libration Orbiter, First Ever Comet Encounter

Farquhar et al [1985] Trajectories and Orbital Maneuvers for the ISEE-3/ICE, Comet Mission, JAS 33, No. 3
Mission: Investigate Solar-Terrestrial Relationships, Solar Wind, Magnetosphere
Launch: November, 1994, Multiple Lunar Gravity Assist
Lissajous Orbit: Originally an L1 Lissajous Constrained Orbit, Ax~10,000km, Ay~350,000km, Az~250,000km, Class I
Spacecraft: Mass=1254kg, Spin Stabilized,
Notable: First Ever Multiple Gravity Assist Towards L1
**Mission:**

Produce an Accurate Full-sky Map of the Cosmic Microwave Background Temperature Fluctuations (Anisotropy)

**Launch:**
Summer 2001, Gravity Assist Transfer

**Lissajous Orbit:**
L2 Lissajous Constrained Orbit Ay~ 264,000km, Ax~ tbd, Ay~ 264,000km,
Class II

**Spacecraft:**
Mass=818kg, Three Axis Stabilized,
First Gravity Assisted Constrained L2 Lissajous Orbit

**Notable:**
### JWST

<table>
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<tr>
<th>System</th>
<th>Universe</th>
<th>Mission</th>
<th>Parameters</th>
<th>Orbit Files</th>
<th>Views</th>
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<td>2011/04/17</td>
<td>06:50:24.937841</td>
<td>471.20580</td>
<td>days</td>
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<td></td>
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</tbody>
</table>

**Lunar Orbit**

**Solar-Rotating Coordinates**

**Ecliptic Plane Projection**

**Mission:** JWST is part of Origins Program. Designed to be the successor to the Hubble Space Telescope. JWST observations in the infrared part of the spectrum.

**Launch:** JWST~2012, Direct Transfer

**Lissajous Orbit:** L2 large lissajous, Ay~294,000km, Ax~800,000km, Az~131,000km, Class I or II

**Spacecraft:** Mass~6000kg, Three Axis Stabilized, ‘Star’ Pointing

**Notable:** Observations in the infrared part of the spectrum. Important that the telescope be kept at low temperatures, ~30K. Large solar shade/solar sail
Using Chaos to Design Orbits
Current Capabilities and Developments

To design vision missions, we need unprecedented capabilities:

- High Fidelity Perturbation Theory Modeling
- Intuitive Numerical Targeting Methods
- Access to Environmental Models and Algorithms
- Commercial and NASA Mission Design Programs
- Inclusion of Dynamical System, Optimization, Control Flow
A General Design Process

**In Low Fidelity Software**
- Use Chaos mathematical expressions for preliminary orbit design, e.g. Circular Restricted Three-Body (CRTB) problem.
- Generate orbit families via differential correction and continuation.
- Analyze the properties of these orbits and to meet mission requirements.
- Obtain orbit architectures.
- Apply two-step differential correction scheme to selected orbits.
- Add multiple revolutions for baseline mission duration.

**In Higher Fidelity Software:**
- Differentially correct in full ephemeris model.
- Constrain orbit to desired goals, apply chaos to obtain $\Delta v$.
- Acquire $\Delta v$ and fuel budget for station-keeping by perturbing initial target states in unstable directions and adding $\Delta v$ errors.
- Analyze mission requirements and constraints (e.g. Sun angle limits and Facility access)
Chaos - System Application

**Numerical Systems**

- Limited Set of Initial Conditions
- Perturbation Theory
- Single Trajectory
- Intuitive DC Process

**Chaos Systems**

- Qualitative Assessments
- Global Solutions
- Time Saver / Trust Results
- Robust
- Helps in choosing numerical methods
  (e.g., Hamiltonian => Symplectic Integration Schemes?)
Chaos and Invariant Manifolds

✓ Use of invariant manifolds are directly applicable to weak stability boundary and libration trajectory design

✓ Together with differential corrections, the use of invariant manifolds provides an efficient method to obtain transfers and control

✓ Invariant manifolds results can be used as a initial conditions for NASA mission design software
Chaos System Approach Transfer

Design a Large Libration Orbit’s Transfer Trajectory - Projections of All Invariant Manifolds for Time Interval
Chaos System Approach Transfer

Design a Small Libration Orbit’s Transfer Trajectory Projections of All Invariant Manifolds for Time Interval
Design Quasi-Periodic Orbits → 2-D Torus

Natural Formations:
Lunar Orbit Design
Lunar Orbit Design

- Uses Traditional Approach
  ✓ Hohmann / Minimum Energy Transfer
  ✓ Targeting Goals of Lunar Orbit, Moon Position, B-plane, and Orbit Conditions Chosen on Requirements
  ✓ Numerical Differential Correction Process that Varies Initial Parking Orbit and Injection Velocity

- Successful and Easily Applied to Parametric And Monte Carlo Analysis

- Would Chaos Improve The Design?

Nominal Cis-lunar Trajectory
Solar Rotating Coordinates
Earth – Moon L\textsubscript{1} & L\textsubscript{2} Halo Orbit Families
L₁ & L₂ Vertical Orbit Families

Moon
$L_2$ Butterfly Orbit Family
Applications of Chaos in the Earth-Moon Region

- A natural transfer between co-linear orbits is defined by the Eigenstructure of the co-linear STM and the dynamics in question.

- The co-linear unstable and stable modes are in the familiar directions and indicate that direct transfers between libration points are straightforward.

- Use the unstable mode of $L_1/L_2$ to perform a $\Delta V$ (possibly near or at zero magnitude) to transfer to the stable mode of $L_2/L_1$ once the trajectory has moved to the other side of the secondary mass.

Earth-Moon Co-linear Departures from $L_1$ and $L_2$
Placing a facility at the South Pole of the Moon poses questions concerning the orbital architecture of the communicating satellites. Constant communication can easily be achieved with Earth-Moon libration point orbits. We analyze different architectures for nearly rectilinear halo orbits, vertical orbits, and other three-body variations for lunar coverage of the South Pole. Using invariant manifold theory, we also analyze the transfer and station-keeping costs for these orbits. Libration point orbits may be a cheaper alternative to pole-sitters or even two-body, highly eccentric orbits.
Applications of Chaos in the Earth-Moon Region

- Dynamical system provides the structure
- Numerical DC used for targeting process
- Need to improve methods
- Example: Finding intersections of Sun-Earth and Earth-Moon Manifolds for Transfer Trajectories
Comparison of Direct and Weak Stability Boundary Transfer to a 100km Circular Lunar Orbit

<table>
<thead>
<tr>
<th>Direct Transfer</th>
<th>Weak Stability Transfer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Launch Vehicle Injection (km/s)</td>
<td>3.13</td>
</tr>
<tr>
<td>Launch C3 (km²/s²)</td>
<td>-2.11</td>
</tr>
<tr>
<td>Equivalent launch mass for C3</td>
<td>1595</td>
</tr>
<tr>
<td>Total Delta-V to attain mission orbit (m/s)</td>
<td>821</td>
</tr>
<tr>
<td>Trip Time (days)</td>
<td>4.5</td>
</tr>
<tr>
<td>Max distance from Earth (million km)</td>
<td>0.367</td>
</tr>
</tbody>
</table>

$\Delta V$ Improvement of 19.12%

Assumptions:
- Polar lunar mission orbit at 100km altitude
- Launch Vehicle of Delta-II
- Final mass computed via rocket equation with starting launch mass, Isp=220, thrust = 22N, $\Delta V$ as above
New Orbits and Future Challenges

Upcoming missions also bring new challenges that individually may easily be met, but in combination they become problematic. These may include:

- Lunar Orbits for Relay Spacecraft
- Biased Orbits when using large sun shades
- Frequent Spacecraft Perturbations (momentum unloads)
- Constrained communications
- Shadow restrictions
- Very small libration orbit amplitudes (<10000km)
- Limited thruster directions
- Transfers Between Libration Orbits and the Moon
- Earth-Moon libration orbits
- Continuous control to reference trajectories
- Quasi-stationary orbits
- Human exploration
- Servicing of resources in libration orbits
NASA Exploration Goals Can Be Achieved With Chaos Dynamics.

Thank you for your attention