Rapid Assessment of Aircraft Structural Topologies for Multidisciplinary Optimization and Weight Estimation

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Algorithms for rapid generation of moderate-fidelity structural finite element models of air vehicle structures to allow more accurate weight estimation earlier in the vehicle design process have been developed. Application of these algorithms should help to rapidly assess many structural layouts before the start of the preliminary design phase and eliminate weight penalties imposed when actual structure weights exceed those estimated during conceptual design. By defining the structural topology in a fully parametric manner, the structure can be mapped to arbitrary vehicle configurations being considered during conceptual design optimization. Recent enhancements to this approach include the porting of the algorithms to a platform-independent software language Python, and modifications to specifically consider morphing aircraft-type configurations. Two sample cases which illustrate these recent developments are presented.

I. Introduction

THE application of multidisciplinary optimization to aero-structural design of flight vehicle structures has been a topic of considerable attention in recent years. Computational grid generation has advanced considerably and has been used to allow parametric aerodynamic design of vehicle shapes to be considered in this optimization process. However, the structural layout design has not been typically treated with the same level of detail. In most cases, the structural layout is fixed and only a structural sizing optimization is performed. In the few instances where the layout is considered as a variable itself, the model chosen is usually overly simplistic, not representative of actual vehicle designs, and suspect from an accuracy perspective. Aircraft manufacturers have very accurate structural models of existing vehicles, but these models take literally months to generate and are not suitable for rapid consideration of candidate designs. Thus, there is a need for a methodology to create structural models of at least intermediate level accuracy while incorporating the ability to generate these models automatically based on a parametric description of the structural layout. This will facilitate the application of these models to rapid assessment of candidate external vehicle shapes.

As a first step towards developing this automatic model generation capability, an abstraction of the aircraft structural elements and their layout was constructed so that models could be created quickly for a given structural layout regardless of changes in external shape. This abstraction was based on a thorough study of structural design trends in modern aircraft of various types[1]. Based on this abstraction, some initial algorithms to allow mapping of parametric structural topologies to arbitrary wing shapes were developed. These algorithms allow a structural designer to specify a layout in parametric terms, specifying locations in percentages of span and chord rather than actual dimensions. This specification is independent of the aircraft geometry so that upon making a change to the external vehicle shape, finite element models are automatically generated to allow sizing optimization to be

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performed. In the present effort, this process was developed further and implemented in a non-graphical manner to allow automation and integration with existing design tools. Ultimately, this capability will generate more accurate weight estimates for new aircraft concepts, including such innovation as morphing structures, and facilitate improved aero-structural optimization earlier in the design process. The framework for this capability was developed and demonstrated in the previous paper by the authors[2]. In this paper, continued development will be discussed and application to an innovative configuration similar to that proposed for future morphing aircraft will be presented.

II. Integrated Conceptual and Preliminary Design Processes

The ability to generate structural layout and associated finite element models automatically will facilitate the incorporation of multidisciplinary optimization into the conceptual and preliminary design phases. Fig. 1 shows a schematic representation of the conventional conceptual and preliminary design phases. Through tools like FLOPS[3], conceptual design has come to include an optimization of aircraft vehicle configuration. However, structural design is generally not addressed to any significant extent in conceptual design. Vehicle weights are estimated as a function of the configuration parameters using historical or generic analysis-based data. The actual structural design is typically not selected until the preliminary design phase. If the aircraft being designed is not radically different from those on which the weights database is constructed, the designed weights will be fairly close to the estimates obtained during the conceptual design. However, if the "new" configuration is substantially different or original, as is the case for several of the proposed morphing aircraft designs, a significant weight uncertainty can be introduced. By this time in the design process, making notable changes to the aircraft configuration can be extremely difficult as these changes will affect many systems, not just structures. Not only could this weight uncertainty be avoided by bringing structural design into the earlier stages of design, but multidisciplinary optimization could be enabled as well allowing the designer to strike the best balance between the vehicle configuration and the vehicle weight. In addition, this multidisciplinary optimization would be a critical component in the assessment of the effectiveness of various morphing strategies where the weight and efficiency of shape change mechanisms is crucial to the success of the vehicle design.

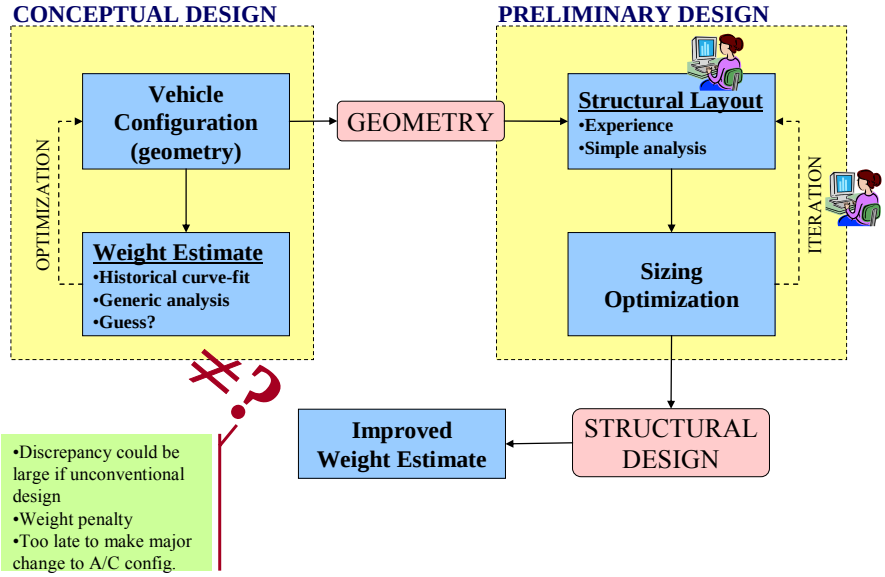


Figure 1. Schematic of convention conceptual and preliminary design processes.

In Fig. 2, an integrated approach to conceptual and preliminary design is illustrated. By automatically generating structural models for an arbitrary vehicle configuration, optimization of the structure can be incorporated into the conceptual/preliminary design phase. This will result in more accurate weight estimates for candidate vehicles and allow for multidisciplinary optimization to be conducted much earlier in the design process than in current practice. This paper describes the continuation of an effort to move towards development of automatic structural layout and finite element model-generation algorithms suitable for designing a wide variety of aircraft families, including revolutionary concepts such as morphing aircraft.

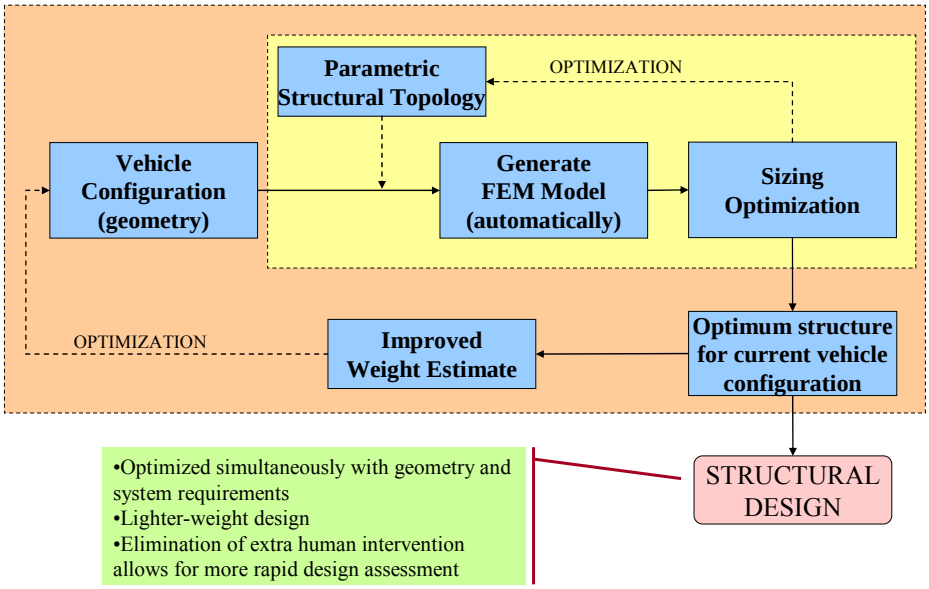


Figure 2. Proposed integration of conceptual and preliminary design with multi-level optimization

A key requirement in the development of these algorithms is the construction of a "demarcation line" between the geometrical specification of the wing exterior and the parametric specification of the topology of internal structural elements. The designer should be able to specify a structural topology with no reference to any shape variables (e.g., inboard spar at 20% chord), including any discontinuities in the shape. The algorithms which map the parametric topology to the geometry to produce a specific design can be used to generate a new design in response to an arbitrary change in the wing exterior shape with no further human interaction. Thus, the geometry itself becomes a "black box" which is tapped into by the mapping algorithm as needed to determine the specific location and orientation of each structural element. Once the mapping is complete, the three-dimensional structure can be converted into a computational domain (e.g. finite element mesh) and a multidisciplinary sizing optimization can be performed. Finally, an optimization loop can be wrapped around this entire process to optimize the wing shape while updating the structural layout at each step to achieve improved weight estimates and aeroelastic responses if required. This process is also shown in Fig. 2.

In Fig. 2, two embedded optimization loops were shown schematically. Although the structural optimization loop is shown as being nested within the overall vehicle system (geometry) optimization loop, these processes are more likely to be simultaneous in actual practice to enhance efficiency and perhaps avoid numerical difficulties. Though these optimizations are likely to be simultaneous, each requires some development to enable their application to the proposed vehicle design process. The present effort focuses on the structural optimization loop as shown in Fig. 3. The key to the success of this effort is the use of a fully parametric structural topology specification scheme which can easily be mapped onto an arbitrary vehicle geometry and structural models generated quickly and automatically.

III. Automatic Topology Generation Framework

The following section outlines the basic framework for the automatic model generation capability described above. Much of this framework was implemented and demonstrated in the previous paper by the authors[2]. For further details, the reader is referred to that publication. The basic concepts are repeated here for the sake of completeness. The initial implementation for this approach was through a simple GUI-based tool, OptWing, which was developed using Microsoft Visual Basic® as a testbed in which to implement and evaluate candidate algorithms for mapping parametric structural elements to arbitrary wing configurations. Since the ultimate objective of this effort is to allow the model generation framework to be implemented within an overall system optimization model on a variety of hardware platforms, recent effort has been focused on a "black-box" version of the software which is not limited to Windows™-based platforms. This will discussed in a subsequent section.

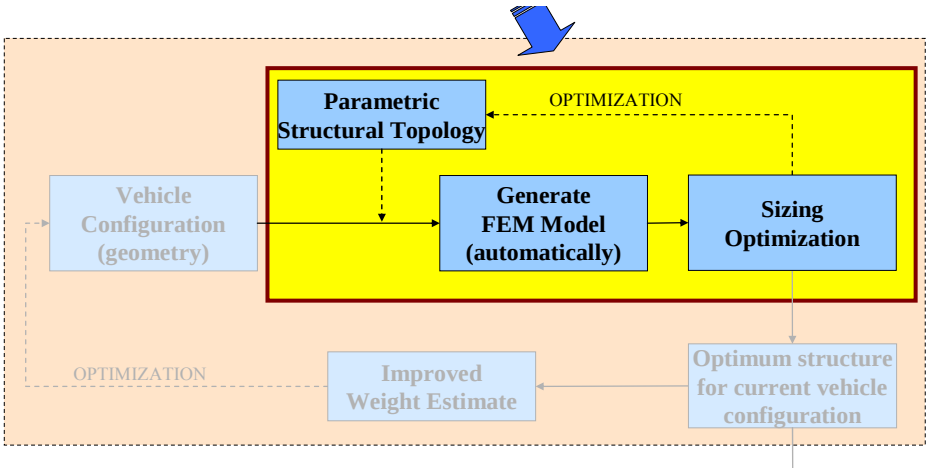


Figure 3. Structural optimization sub-loop

A. Wing Geometry

Currently, a fairly simple geometry model is assumed. Wing boundaries (i.e. leading and trailing edges) are considered to be straight within a wing segment. Each segment may have a constant sweep angle, twist angle, and dihedral angle, taper and an independent span length. A wing may be built up from an arbitrary number of these straight segments. Currently, each wing is assumed to have a constant airfoil shape (though obviously the chord may differ at different spanwise locations).

It is important to emphasize that the structural mapping algorithms and finite element mesh generation must be accomplished without any knowledge of the type of geometry model used. The geometry model must present itself as a "black-box" to the mapping algorithms and simply supply physical (x, y, z) coordinates from input parametric information.

B. Wing Parametric Coordinate System

The simplest possible parametric coordinate system is used for the purpose of specifying the locations of structural components. This coordinate system is shown in Figure 4. The parametric coordinates are:

- s spanwise coordinate (0 at root, 1 at tip)
- c chordwise coordinate (0 at local leading edge (LE), 1 at local trailing edge (TE))
- t thickness coordinate (+1 at local airfoil top surface, 1 at local airfoil bottom surface)

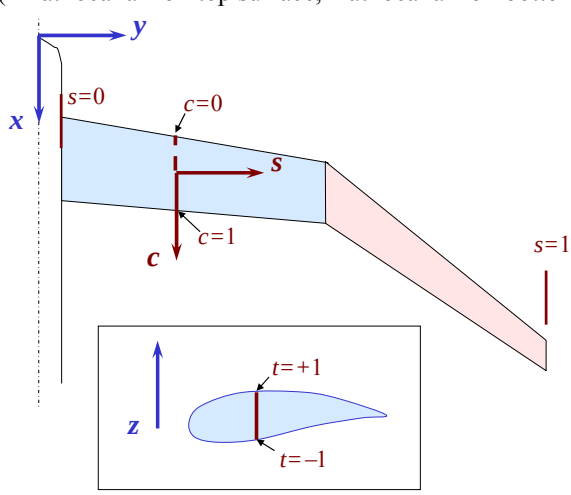


Figure 4. Parametric coordinate system used in OptWing

All location references for structural components must be linked to this parametric coordinate system or specified relative to other structural components. Thus, the locations and orientations must be stated in one of the following manners:

- Parametric coordinate locations (s, c, t)
- Locations relative to other structural elements (e.g. intersects at $s = 0.3$ with nearest spar towards LE)
- Orientations relative to durable parametric geometric references (e.g. parallel or perpendicular to a specific % chord line)
- Orientations relative to other structural elements (e.g. perpendicular to LE spar)

C. Wing Structure

The structural components that make up a wing are assumed to be:

- Spars (webs and caps)
- Ribs
- Stringers
- Skin

The options for defining each of these structure elements are presented in this section. These are not intended to be exhaustive, merely representative of the typical vehicle structural layouts observed by Sensmeier & Samareh[1].

1. Spar Definitions

Each spar in the wing is defined individually, and is assumed to be continuous from the top airfoil surface to the bottom airfoil surface. The presence of spar caps is implicitly assumed. The current options for spar definition include:

- *Spar Inboard Location* – The starting location of the spar is specified in terms of the parametric coordinates (s, c).
- *Spar Outboard Location* – The spanwise parametric coordinate (s) at the point where the spar ends
- *Spar Orientation* – Several options are available to describe the spar orientation
    - *Constant % of chord* – Spar stays at the specified chordwise parametric coordinate (c) over its entire length
    - *Specified % of chord* – Spar follows a straight line path from its specified origin to its ending (s, c) coordinates
    - *Joining nearest neighboring spar* – Spar can be specified to end by joining the nearest neighbor towards either the LE or TE at the specified ending coordinate (s)
    - *Remain // to % chord* – Spar remains parallel to a given % chord line over its entire length

2. Rib Group Definitions

Since most aircraft wings contain a large number of fairly evenly-spaced ribs, OptWing allows the user to specify groups of ribs. The current options for rib definition include:

- *Span Range* – The group of ribs can run the entire span of the wing or any subset, specified by the spanwise parametric coordinates s_{start}, s_{end}.
- *Span Reference line* – This line defines the starting and ending points of a group of ribs. The first and last ribs within a group intersect the reference line at s_{start} and s_{end} and are evenly divided between these two points.
- *Rib Extent* – For most aircraft, the structural loading of the wing is primarily borne by the main wingbox (the structure between the LE and TE spars). This option allows the user to specify that the finite element model only represents the main wingbox. The user has the option, though, to specify the extent of a rib group to include the region from the LE spar to the LE of the wing and/or the main wingbox and/or the region from the TE spar to the TE of the wing.
- *Rib Orientation* – Currently, there are four options for the orientation of a group of ribs:
    - *Parallel to fuselage* – All ribs in the group remain parallel to the fuselage centerline.
    - *Perpendicular to LE spar* – All ribs in the group are perpendicular to the LE spar at the point where that particular rib intersects the LE spar
    - *Perpendicular to TE spar* – All ribs in the group are perpendicular to the TE spar at the point where that particular rib intersects the TE spar
    - *Perpendicular to % chord line* – All ribs in the group are perpendicular to a specified local % chord line.
- *Rib Spacing* – The number of ribs within a group is specified by one of two methods:
    - The total number of rib divisions (so number of ribs equals number of divisions plus one)
    - An approximate fixed spacing (the number of ribs is an integer which most closely matches the specified spacing). This is the only part of the parametric structural definition that involves a specific physical dimension. This capability was included to reflect the observation from the aircraft structural design trend study[1] that most transport aircraft, regardless of size or wingspan, have the same rib spacing (approximately 24 inches).

3. Stringer Definitions

The Stringer definition is virtually identical to the Spar definition except for a flag specifying whether the stringer is on the top or bottom (or both) surface(s) of the airfoil.

4. Skin Definitions

The skin regions are fully defined by the presence and location of spars, ribs, and stringers, so there is nothing for the user to define. The algorithm for mapping and meshing the skin regions will be described in a later section. However, each skin "panel" could be associated with a flag to determine whether or not it is to be included in the finite element model. This would allow for non-structural skin regions to be eliminated from the model.

D. Wing Model

The crux of the present effort is the development of mapping algorithms which take the fully parametric structural topology definition and map it to the current state of the wing geometry. Once this mapping is accomplished, a finite element mesh is created based on this mapping. From that mesh, different load cases may be applied and the commands to perform a structural analysis and/or sizing optimization may be generated. In the ultimate system, this would be invisible within the overall vehicle configuration and structural optimization process. This process is currently manual within OptWing, as the user may make changes manually to the wing geometry and create the mapped model only for the current geometry.

The user has several options for output generated by Optwing:

- *Mesh only* – consists of only the nodes and elements.
- *Analysis deck* – consists of the nodes and elements, as well as the appropriate control cards to perform either a static (+ or – g) wing loading, linear buckling analysis, or modal analysis.
- *Sizing optimization deck* – consists of the mesh, any load cases which the user specifies to include in the optimization, and the design variable, constraints, and fitness function (volume, or weight). For the optimization, the user can specify any subset of the structural components to be specified by design variables and/or constraints.

Currently, OptWing will produce input decks for both ANSYS® and MSC/NASTRAN®. The types of elements used for each component and analysis code is shown in Table 1.

Table 1. Element types used to generate finite element model

Element Type	ANSYS®	MSC/NASTRAN®
3D Beam	BEAM4	CBAR
3D Shell	SHELL63	CQUAD4

IV. Recent Enhancements

In this section, enhancements and modifications made since the publication of the previous paper are described. An incremental development process has been used to add various capabilities to OptWing. The significant portion of the recent effort has focused on completing the initial framework and porting it to the "black-box" software version.

A. OptWing Python

As described in the previous paper, the initial testbed version of OptWing was developed as a GUI-based software tool limited to one type of operating system. Since the ultimate goal of this tool is to incorporate it into a larger system optimization framework which could be run on a variety of hardware platforms, it was necessary to begin implementing the tested algorithms into a batch, "black-box" version of the model generation software. The batch version needed to meet the following requirements:

- Be fully modular to allow easy modification
- The structural topology should be design-variable driven, not driven by user input
- Must run on a variety of operating systems and platforms
- Must access the wing geometry using a standard interface

To meet these requirements, it was decided to implement the batch version using the Python programming language, a multi-platform object-oriented open source language. To date, the basic model generation framework has been successfully ported from its original platform to the Python language. The program is not yet truly a black-box, but that will be part of continuing enhancements.

B. Skin Meshing

At the time of writing of the previous paper, only mapping algorithms for the spars and ribs had been successfully implemented in the testbed. A significant part of the recent enhancements have focused on the incorporation of skin meshing. The spar/rib mapping algorithms already created a set of nodes which, in effect, form the boundaries of the various skin segments. Thus, a constrained Delaunay triangular meshing algorithm[4] was selected as the easiest and most versatile option for meshing the skin regions.

1. 2-Dimensional Mapping of 3-Dimensional Wing

In order to implement this algorithm most efficiently, though, we decided to map the wing geometry and initial spar/rib surface nodes from 3-dimensional locations around the wing to a 2-dimensional mapping. This mapping is illustrated in Fig. 5, where the final 2-dimensional mapping represents the x' - y' coordinate system. This amounts to a virtual "unwrapping" of the wing such that the wing becomes a collection of trapezoids, whose sides represent the trailing edge of the wing and whose top and bottom represent the wing root, interfaces between wing regions, and wing tip. An example of a portion of this "unwrapped" wing showing the surface nodes of the spars and ribs is shown in Fig. 6. The points represented by the line at $x' = 0$ are along the leading edge of the wing, while the left and right edges of the diagram represent the trailing edge.

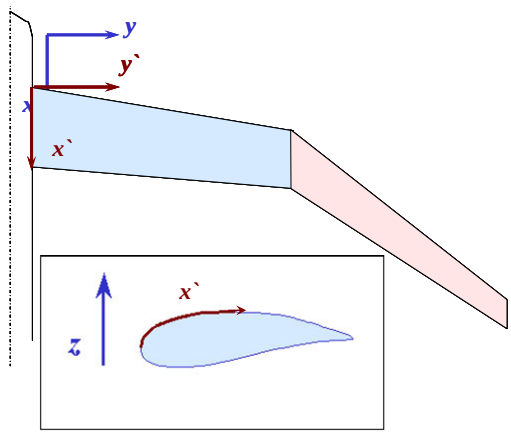


Figure 5. Parametric curvilinear coordinate system

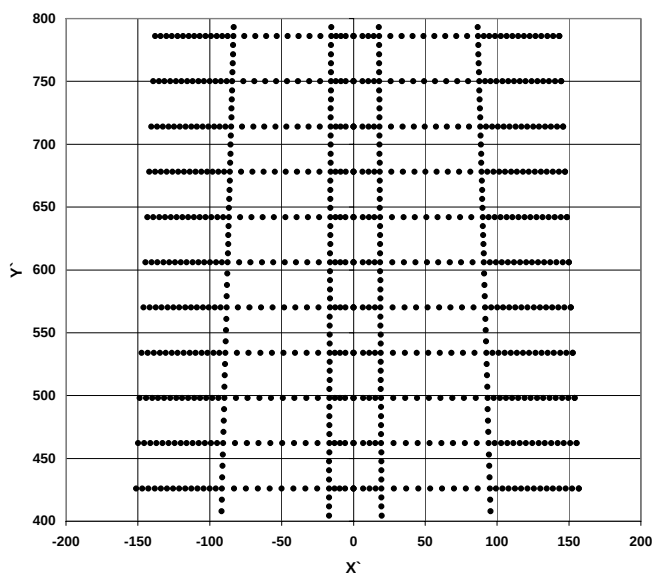


Figure 6. Typical spar/rib node locations for "unwrapped" wing

2. Node Insertion

The "unwrapped" wing described above is then subdivided into skin regions. These regions are bound by ribs, spars, and the leading and/or trailing edge of the wing. (Meshing of the regions forward of the leading edge spar and/or the region aft of the trailing edge spar can be turned off by the user if these regions are not expected to

contribute structurally to the wing.) Nodes are then randomly inserted into each region, with the following restrictions:

- The closest spacing allowed between points is equal to the average distance between the points in the already existing mesh (from the ribs and spars)
- Nodes are inserted until the spacing restriction can no longer be met or there are as many nodes in the interior of the area as there are along all of its edges

This method allows the skin mesh to automatically approximate the resolution of the rest of the model. Fig. 7 shows the same portion of the sample wing after insertion of interior nodes.

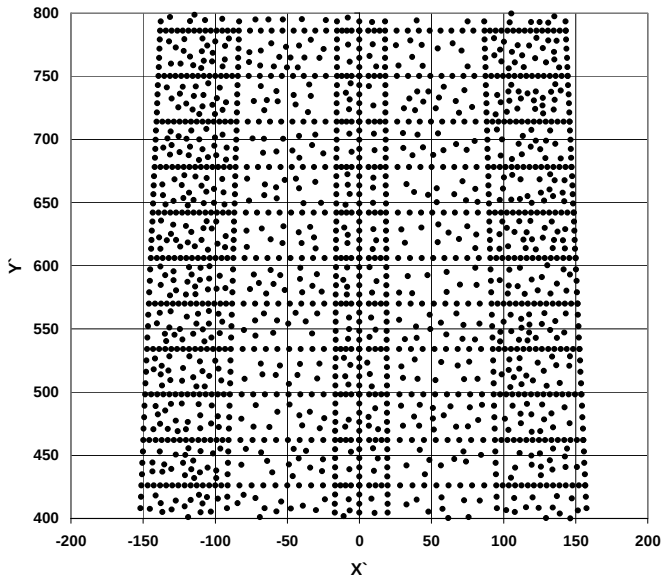


Figure 7. Sample mesh after random interior node insertion.

3. Delaunay Triangulation

After the points have been inserted into each region, the Delaunay triangulation of the entire wing surface is completed. This creates triangular shell elements in the skin regions. Fig. 8 shows the resulting mesh within the same region of the sample wing.

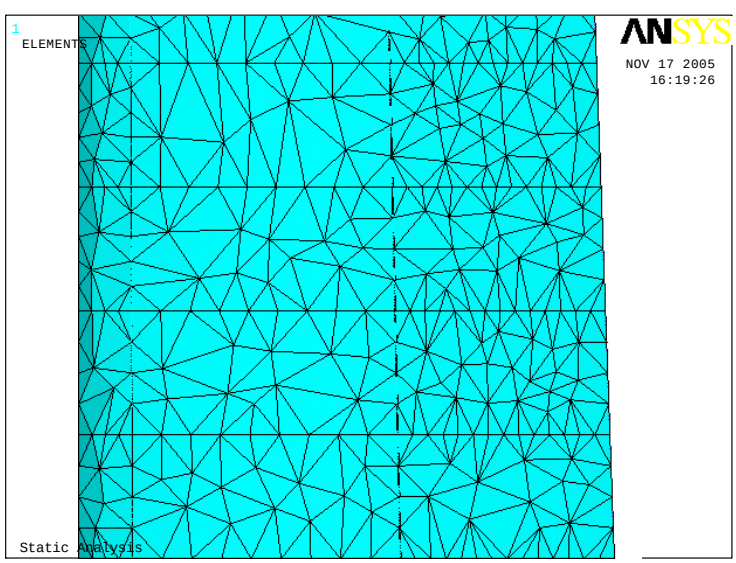


Figure 8. Sample Delaunay triangulation mesh.

C. Enhancements to Facilitate Modeling of Morphing Aircraft

Since one of the first key applications of the current approach is anticipated to be for the design of morphing aircraft, a portion of the effort has been focused on ensuring that OptWing can handle these types of structural configurations. Two of the most promising morphing mechanisms presently under consideration are scissor-type wings (which involve wings whose spar/rib lattices can "scissor" during flight to change the overall wing sweep) and folding wings (which involve wings whose segments can have large positive and/or negative dihedral angles). After assessing OptWing's capabilities, it was determined that wings with the former of these mechanisms can already be modeled simply by correctly defining the spar angles. The configuration of the scissor-type wings can be easily changed by modifying these spar angles.

However, modeling folding-type wings required some modifications to the existing algorithms. Initially, this involved correcting an error in the original geometry model for wing segments with dihedral and dealing with the transition area between regions with largely differing dihedrals. Since the original algorithms were written with only small amounts of dihedral in mind, it was necessary to modify the way the airfoil sections are implemented. Instead of the airfoil sections being represented in a vertical plane in the wing, they need to be represented in a plane perpendicular to the local spanwise direction of the wing. This modification prevents the wing from becoming too small in the thickness direction within regions of high dihedral. An example of the application of OptWing to a folding-wing structure will be shown in the following section.

V. Sample Applications

In the previous paper, two sample applications were presented to demonstrate how the current approach could be used to rapidly assess candidate vehicle configurations. Since the basic premise of this approach has already been demonstrated, the examples shown in this section will focus on the enhancements made to the algorithms (specifically skin meshing) and the folding-wing type morphing aircraft. Thus, the latest version of OptWing Python has been applied to two different types of aircraft wings:

- NASA's HALE (High-Altitude Long Endurance) aircraft
- A generic folding-type wing

It is important to point out here that the results presented in this section are meant only to demonstrate the utility of the current approach in enhancing the conceptual/preliminary design phases. Since the current state of the tool is not complete, actual analysis/optimization results are not quite realistic or representative of the actual aircraft.

A. HALE (High-Altitude Long Endurance) Aircraft

The newest version of a High-Altitude Long Endurance (HALE) aircraft being studied by NASA was used as an example in the previous paper. A current representation of the geometry of the aircraft is shown in Fig. 9. Since the internal structure for this vehicle has not been fully defined, a two-spar arrangement was assumed. The key difference in the present effort was the incorporation of skin elements into the model. The resulting model is shown in Fig. 10.

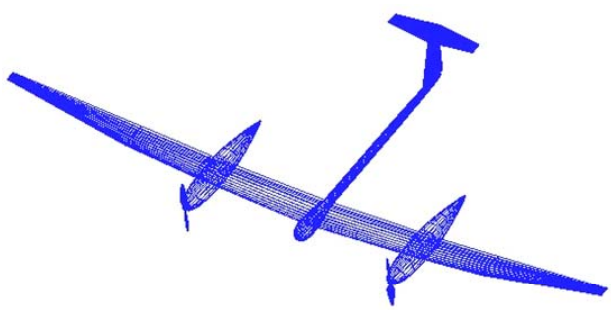


Figure 9. Geometry of proposed HALE vehicle.

The optimization cases for the HALE wing were run in ANSYS. For the baseline case, Fig. 11a shows the history of the wing volume (i.e. weight) during the optimization process. The history of the thickness design variable values is shown in Fig. 11b (THK01 = thickness of spar at 20% chord, THK02 = thickness of spar at 60% chord, THK03 = Rib thickness, THK04 = Skin thickness).

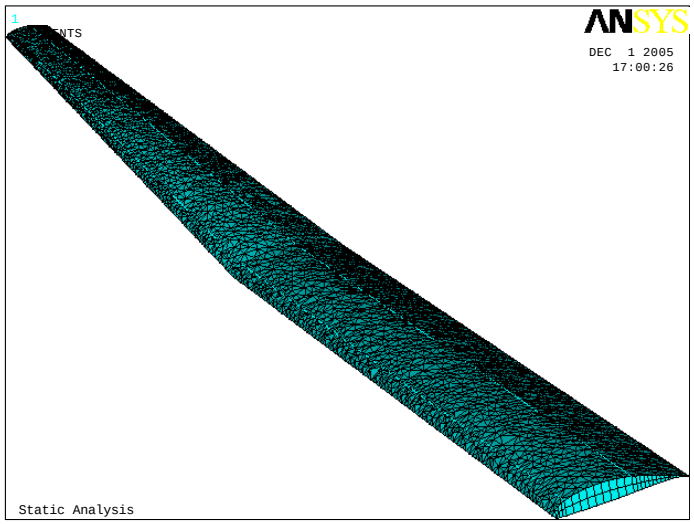
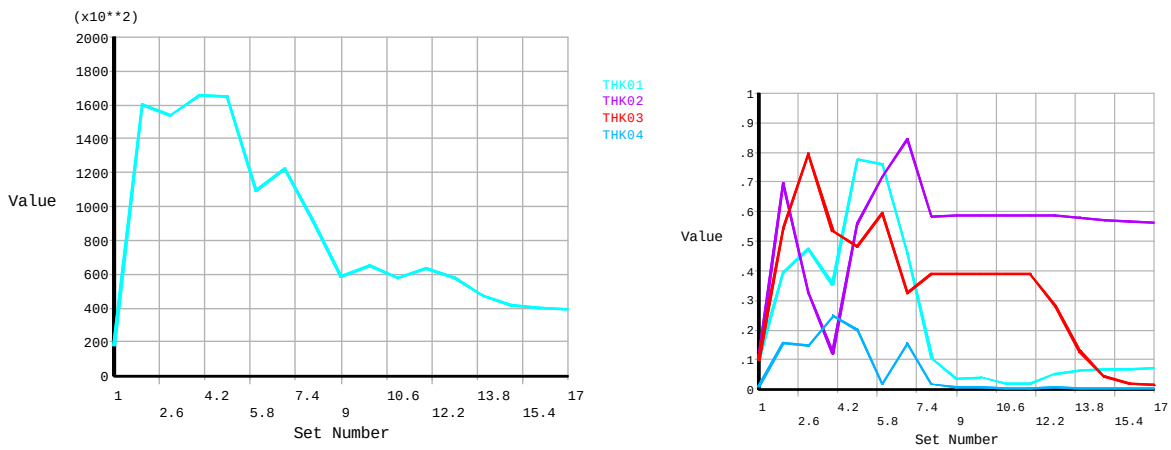


Figure 10. Finite element model for HALE wing.

A trade study was also performed for the HALE wing. In this case, though, the location of the leading edge spar was varied. The LE spar was placed at 10%, 20%, and 30% of chord. The wing volume (weight) was optimized for all three of these cases, and the results are shown in Fig. 12.



a) Wing volume (weight) history during optimization. b) Thickness design history during optimization.

Figure 11. Optimization results for HALE sample problem with skin mesh included.

B. Folding-Type Wing

Since actual dimensions of current morphing-type aircraft are considered company sensitive information, a generic-type folding wing was created just to demonstrate the capability of OptWing to successfully model such a wing structure. The resulting model is shown in Fig. 13. The optimization case for the folding-type wing was also run in ANSYS. Fig. 14a shows the history of the wing volume (i.e. weight) during optimization. The history of the thickness design values is shown in Fig. 14b.

VI. Continuing Development

An incremental development process continues to be used to add various capabilities to OptWing. Several enhancements currently underway or anticipated in the immediate future are:

• The native Delaunay triangulation that has been implemented into OptWing Python has proven to be fairly robust. However, the algorithm is not as fast as would be desired. Thus, OptWing is being modified to give the user the option to run our native code or to call Jonathan Shewchuck's open-source TRIANGLE code[5], which is written in C language and runs much more quickly.

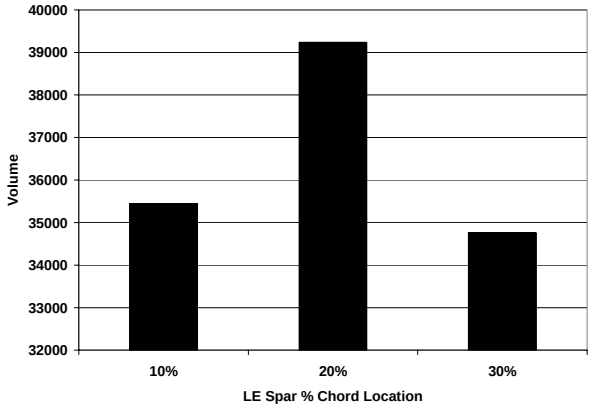
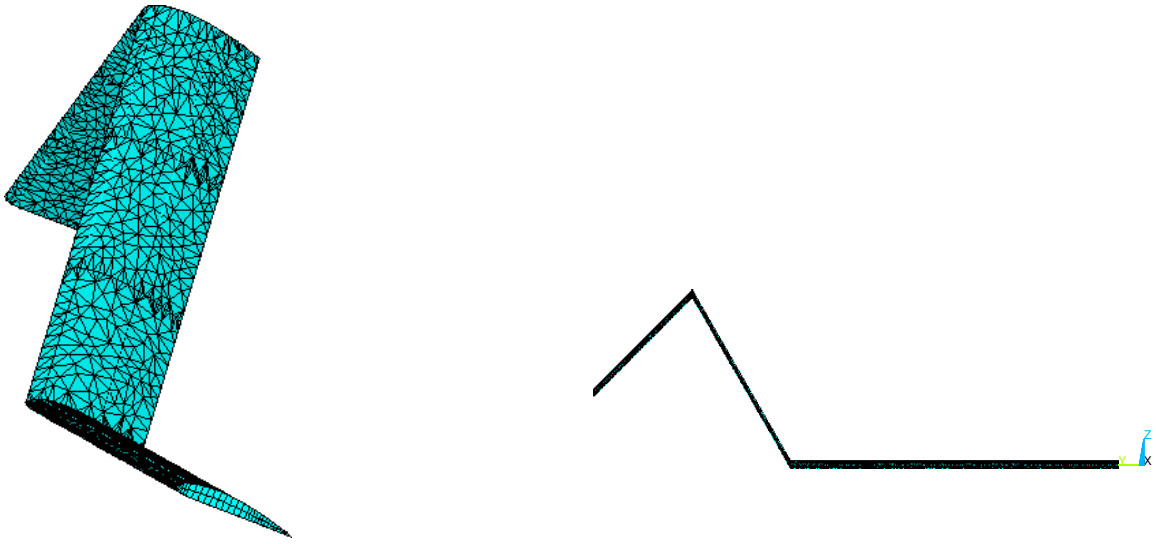
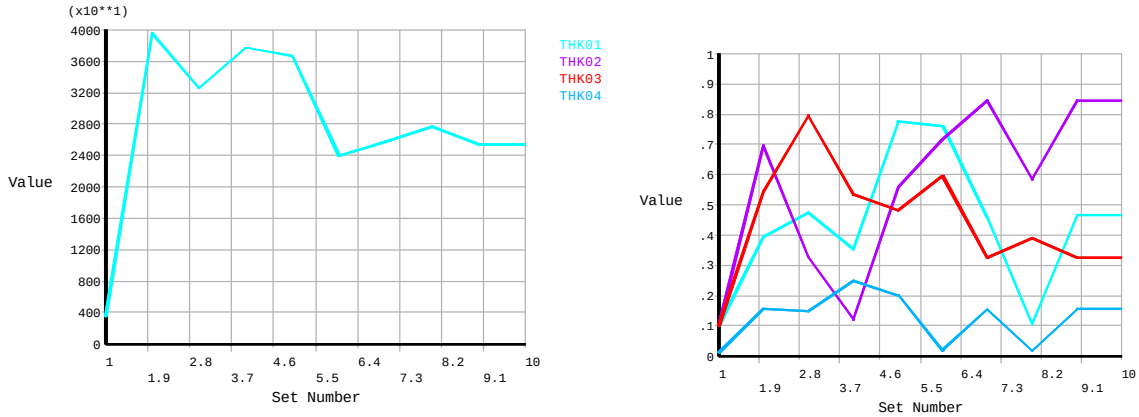


Figure 12. Results from HALE trade study on LE spar location.



a) Folding-type wing model **b) Front view of folding-type wing model**

Figure 13. Configuration of sample folding-type wing.



a) Wing volume (weight) history during optimization. **b) Thickness design history during optimization.**

Figure 14. Optimization results for HALE sample problem with skin mesh included.

The black-box version of OptWing will be modified

to be actually design variable-driven. Currently, the code requires that the wing geometry and structure be defined manually. Since this code will ultimately be implemented within a larger system optimization framework, it needs to operate in a fully automatic manner. A variety of design variable structures will be studied to best accomplish this, and the most promising will be implemented into the code.

• Improved aero-structural loads determination will be studied and implemented. This could involve the use of the aerodynamic capabilities inherent in ANSYS® or NASTRAN®, or could involve the implementation of unified theories such as those presented by Drela[6] or Meirovitch and Tuzcu[7].

• Improve the capability to handle morphing-type structures. The unified theories mentioned above would be particularly useful here, as they have been developed specifically for flexible structures which are representative of morphing-type wings.

VII. Conclusions

The use of rapid model generation has been proposed as a means to improve the conventional approaches to conceptual and preliminary design of flight vehicles. By mapping a parametric description of the structural topology layout to a give vehicle geometry, a finite element model of moderate fidelity can be generated. This model can then be used within an optimization algorithm to determine the optimum structural dimensions. This allows for a more accurate weight estimate than one obtained from, say, curve-fits of historical data.

An initial software tool, OptWing, which accomplishes this mapping and rapid model generation has been developed and demonstrated for several different cases. The original GUI-driven demonstrator tool has been ported to a platform-independent, black-box style program which can then be incorporated within the overall vehicle configuration optimization process to simultaneously optimize the aircraft geometry and structure.

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