UWB Tracking Algorithms – AOA and TDOA

Abstract

Ultra-Wideband (UWB) tracking prototype systems are currently under development at NASA Johnson Space Center for various applications on space exploration. For long range applications, a two-cluster Angle of Arrival (AOA) tracking method is employed for implementation of the tracking system; for close-in applications, a Time Difference of Arrival (TDOA) positioning methodology is exploited. Both AOA and TDOA are chosen to utilize the achievable fine time resolution of UWB signals.

This talk presents a brief introduction to AOA and TDOA methodologies. The theoretical analysis of these two algorithms reveal the affecting parameters’ impact on the tracking resolution. For the AOA algorithm, simulations show that a tracking resolution less than 0.5% of the range can be achieved with the current achievable time resolution of UWB signals. For the TDOA algorithm used in close-in applications, simulations show that the (sub-inch) high tracking resolution is achieved with a chosen tracking baseline configuration. The analytical and simulated results provide insightful guidance for the UWB tracking system design.
UWB Tracking Algorithms
AOA and TDOA

Jianjun (David) Ni*, Melinda Refford
Dickey Arndt, Phong Ngo, Chau Phan, Julia Gross, John Dusl

UWB Systems Group / EV4
Avionic Systems Division / EV
NASA Johnson Space Center

*This research was performed while the PI held a National Research Council Research Associateship Award at NASA Johnson Space Center.
Outline

• UWB Tracking Algorithm Introduction
• AOA Tracking Methodology
• AOA Resolution Analysis and Simulations
• TDOA Tracking Methodology
• TDOA Resolution Analysis and Simulations
• Conclusion and Future Work
Motivation
(UWB Fine Time Resolution to Precise Tracking)

• 1 ns → 1 foot, 3 ps → 1 mm (ranging, linear)
• Tracking (positioning in 2D/3D) resolution (nonlinear) ?
• Long Range application: AOA (Angle of Arrival)
• Proximity Application: TDOA (Time Difference of Arrival)
Angle of Arrival (AOA)

\[
\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}
\]

\[
T \left[ \frac{d \cos \theta_1 \sin \theta_2}{\sin(180^\circ - (\theta_1 + \theta_2))}, \frac{d \sin \theta_1 \sin \theta_2}{\sin(180^\circ - (\theta_1 + \theta_2))} \right]
\]
Two Cluster Design

Long Range Assumption: $r_1, r_2 \gg a$

\[ c_{12} \approx a \cos \theta_1 \]
\[ \theta_1 = \arccos \left( \frac{c_{12}}{a} \right) \]

\[ c_{43} \approx a \cos \theta_2 \]
\[ \theta_2 = \arccos \left( \frac{c_{43}}{a} \right) \]
Default Setting:

- Cluster Size $a=15$ meters
- Baseline Size $d=50$ meters
- Tracking Range $r=610$ meters (2000 feet)
- Tracking Angle $\theta=0\sim180$ degree
Default Setting:

- Cluster Size $a = 15$ meters
- Baseline Size $d = 50$ meters
- Tracking Range $r = 610$ meters (2000 feet)
- Tracking Angle $\theta = 30\sim150$ degree
- TDOA Noise Level $=10$ picoseconds
Two-Cluster-AOA-Tracking (noisy TDOA)

- Antenna Position
- Actual Position
- Tracked Position

X (m) Y (m) Antenna Position Actual Position Tracked Position

[Graph showing tracking results with various markers and lines indicating actual and tracked positions]
Resolution vs. Affecting Parameters

\[
MSE = f(a, d, r, \sigma)
\]

- \(a\) – cluster size (distance between two antennas)
- \(d\) – baseline size (distance between two receivers)
- \(r\) – tracking range (distance from target to origin, angle)
- \(\sigma\) – TDOA noise level (standard derivation of TDOA)
The distance between two antennas in one cluster (m)

Average Error Norm (m)

Resolution vs. Cluster Size

Perfect TDOA

Noisy TDOA

The distance between two antennas in one cluster (m)
Resolution vs. Baseline Size

The distance between two clusters (m)

Average Error Norm (m)

Perfect TDOA

Noisy TDOA

The distance between two clusters (m)
Resolution vs. Range

The Tracking Range (m)

Average Error Norm (m)

- Perfect TDOA
- Noisy TDOA
Resolution vs. TDOA Noise

- **Average Error Norm (m)**
- **Std of TDOA Noise (ns)**

Graph showing the relationship between TDOA noise and error norm.
Analysis shows that AOA algorithm can achieve fine tracking resolution using TDOA estimates with low noise level.

Analysis provides guidance for system design to improve the tracking resolution:

- To increase the cluster size
- To decrease the noise level of TDOA estimates (hardware/DSP techniques)
Time of Arrival (TOA)
Drawbacks of TOA

- Ranging: requires duplex transmission and incurs overhead (Asynchronization)
- Synchronization: hard to achieve the synchronization precision between the transmitter and receiver; 1 microsecond synchronization error can easily translate into 300 meters of range error
Time Difference of Arrival (TDOA)

Hyperbola: $b^2 x^2 - a^2 y^2 = a^2 b^2$
Time Difference of Arrival (TDOA)
Advantages of TDOA

- No synchronization between Tx and Rx
- Simplex (one-way) data estimation
- Cross-correlation works well to obtain TDOA
TDOA Equations (2D)

\[ \begin{align*} 
D_{12} &= \sqrt{(x_1 - x)^2 + (y_1 - y)^2} - \sqrt{(x_2 - x)^2 + (y_2 - y)^2} = \tau_{12} c \\
D_{13} &= \sqrt{(x_1 - x)^2 + (y_1 - y)^2} - \sqrt{(x_3 - x)^2 + (y_3 - y)^2} = \tau_{13} c \\
D_{23} &= \sqrt{(x_2 - x)^2 + (y_2 - y)^2} - \sqrt{(x_3 - x)^2 + (y_3 - y)^2} = \tau_{23} c 
\end{align*} \]
TDOA Algorithm

- Taylor Series Expansion Least Squares Iterative Algorithm
  (Initialization problem and convergence problem)

- Two-Stage Weighted Least Squares Algorithm
  (one more receiver)
Resolution Analysis (setting)
Resolution Analysis (MSE)

\[
\text{MSE} = \frac{c^2 \sigma^2 r_0^2 \sum_{i=1}^{3} \left( a_i^2 + b_i^2 \right)}{r^2 \left( \sum_{i=1}^{3} a_i^2 \sum_{i=1}^{3} b_i^2 - \left( \sum_{i=1}^{3} a_i b_i \right)^2 \right)}
\]

\[
a_i = \cos \phi_i + \frac{r_i - r_0}{r} \cos \theta
\]

\[
b_i = \sin \phi_i + \frac{r_i - r_0}{r} \sin \theta
\]

\[
r_i = \sqrt{r_0^2 + r^2 - 2r_0 r \cos(\theta - \phi_i)}
\]
Orbit Tracking

- Tracking Resolution vs. TDOA noise
- Tracking Resolution vs. Receiver Configuration
- Tracking Resolution vs. Dynamic Reference
## Error Analysis

<table>
<thead>
<tr>
<th>Standard Deviation of TDOA (ns)</th>
<th>Maximum Error (m)</th>
<th>Average Error (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>60.0461</td>
<td>4.9656</td>
</tr>
<tr>
<td>0.001</td>
<td>7.5611</td>
<td>0.5468</td>
</tr>
<tr>
<td>0.0001</td>
<td>0.7218</td>
<td>0.0544</td>
</tr>
</tbody>
</table>

- The tracking error is linear to the standard deviation of TDOA data
TDOA Summary

- TDOA algorithm can achieve fine tracking resolution with UWB fine time resolution
- Tracking resolution is proportional to TDOA noise level
- Receiver antenna configuration matters
- Dynamic reference can improve the tracking resolution
Future Work

• AOA
  To study the scalable baseline configuration to increase the tracking coverage

• TDOA
  To improve the tracking resolution using enhanced algorithm (Totally Least Square Algorithm)
  To study the optimal receiving antenna configuration

• AOA and TDOA
  Extend the tracking dimension (from 2D to 3D)