Carrier Modulation Via Waveform Probability Density Function

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Abstract

Beyond the classic modes of carrier modulation by varying amplitude (AM), phase (PM), or frequency (FM), we extend the modulation domain of an analog carrier signal to include a class of general modulations which are distinguished by their probability density function histogram. Separate waveform states are easily created by varying the pdf of the transmitted waveform. Individual waveform states are assignable as proxies for digital ONEs or ZEROs. At the receiver, these states are easily detected by accumulating sampled waveform statistics and performing periodic pattern matching, correlation, or statistical filtering. No fundamental physical laws are broken in the detection process. We show how a typical modulation scheme would work in the digital domain and suggest how to build an analog version. We propose that clever variations of the modulating waveform (and thus the histogram) can provide simple steganographic encoding.

Introduction

For many years the definitions of ac-carrier modulation types have remained constant (refs. 1 and 3), namely forms of amplitude modulation (AM) and angle modulation (PM) and subtypes and variations such as FM and QPSK. We recently have explored waveform types in which the information is impressed onto the carrier by varying the probability density function (pdf) of the wave (histogram), which can be done such that the long-term amplitude- or phase-envelope are of little concern. That is, in a noisy channel where the signal may suffer long-term or short-term variations of amplitude or phase or frequency, it is still possible via statistical analysis of the waveform to recognize patterns in the pdf which imply reception of digital ONE or ZERO. We view pdf-modulation as representing a new modulation domain, which although it might not be described as having a high bit-rate efficiency, it does have the advantage of permitting steganographic masking of information content.

Modulation Requirements

In AM and PM and their various subtypes, the resulting modulated waveforms generally appear as ac- or rf-carriers that resemble quasi-sine waves. A well-behaved modulated carrier signal must be stationary and ergodic [2]. We note here that the modulating waveform, to which information has been added, does not have to be subsequently transmitted over ac-coupled or wireless transmission channels. For example, modulation can be impressed onto a dc carrier, i.e., a baseband carrier. Our discussion does not rule out subsequent modulation of the baseband signal onto a secondary ac-coupled or rf carrier, so long as at the receiving end of the communications channel, the original baseband signal can be detected and extracted from the secondary ac-carrier without gross distortion. Comments on noise distortion of a non-noisy desired signal will follow later.

Modulation Model

We introduce here a basis for the new model, pdf-modulation. We begin by rejecting the premise that modulation of amplitude or angle is all that can be done. We generalize to say that if any controlled distortion of a carrier’s waveform shape can be recognized by a receiver at a later time or at a different location, then we meet all the needs for modulation of information onto a carrier. The probability density function of a waveform distribution function $F(x)$ can be defined as (ref. 2):

$$f(x) = \frac{dF(x)}{dx} \geq 0$$

We presume the ability to distort the carrier signal by implementing (1). Therefore in principle we have gone beyond amplitude or angle modulation as defined long ago (ref. 1).

In our model, the information to be modulated onto the carrier conceptually employs a Digital-to-Analog Converter (DAC) to realize a pre-defined controlled distortion to the quasi-sinewave carrier signal as in figure 2. Our DAC operates at a sample rate much higher than the information data rate, so that the final modulated carrier signal is comprised of a smoothed but modulated carrier signal comprised of a large number of small step-voltage changes in the DAC output, to become one complete cycle of the modulating signal. The exact nature of the smoothing technique is not important for this model and will not be further discussed.
In our model it is sometimes useful to have from 1-to-n whole half-cycles of carrier signal (“modulation packet”) to complete the modulation of a single “unit of information”. Thus the zero-crossings of the modulated signal might also be detected in order to keep track of the progress. However, implementation of dynamic LIFO (Last-In-First-Out) techniques and dynamic histogramming can bypass the concern about having to detect waveform zero crossings.

In the simplest version of the modulator, the DAC generates the modulating signal by being stepped through a large “source table” of numbers representing all the samples necessary to synthesize the packet of carrier signal necessary to carry one unit (e.g., bit) of information. To encode binary digital data, there would be two or more source tables, with each table used to generate one modulation packet for each unique signal state. Although our examples will discuss binary data transmission, but there is no requirement to limit the number of information states to only two if a multilevel digital system is being defined.

Secondly, our model assumes the existence of an Analog-to-Digital Converter (ADC) which transfers a continuous stream of samples of the received signal to a device which generates a data table representing a dynamic histogram of the modulation packets. Histograms (ref. 2), and the statistics for pdf-modulation decoding, are thus extracted from the data.

In our model, the resulting successive histograms of the received modulated signal are processed by some algorithmic “black-box,” such as by computer-driven statistical analyses, pattern-matching, correlation techniques, or by means such as neural networks, to produce a list of “units of information” that should match the original list that was encoded by the DAC and source-tables in the transmitter. We assume that there may be a “training time” involved in the algorithm for decoding the units of information. In the end, the algorithm must distinguish by some means when the probability density function of the extracted modulation has changed from one information state to another, as part of the novelty of our method.

Additional Subtleties

We have purposely not specified nor claimed that there exist only one set of tables used to perform optimum (in the mathematical sense) modulation of the carrier waveforms based on the information to be modulated. We have merely stated that the model requires creating modulating waveforms that have identifiable mathematical statistics after reception and demodulation, where the statistics are determined strictly by use of waveform probability density function (pdf) and dynamic histogramming. Setting aside the problem (outside the versions of this model) where zero-crossings in the received carrier signal might be useful in a design implementation, our model includes the use of any means, analog or digital, for recognition of differences in the modulation statistics based on the received probability density functions.

Example Forms of pdf-Modulation

Figures 1 to 4, show two different sets of pdf-modulation waveforms, which we will also subsequently show have differing advantages and disadvantages. We do not claim either of these two sets to be the only possible forms, nor optimized forms. They merely serve as trial examples for some instructive simulations. In figures 3 and 4, a rather obvious distinction in waveform pdf is created by use of a sawtooth waveform and a square-wave waveform for representing the two states of a binary-based state system, state A and state B. In figures 1 and 2 the two binary states are represented by quasi-sinewave waveform having two symmetric but variable levels where kinks occur. If the kinks are all at the same relative sampling level, one state is implied. If one half of the kinks are raised in level, the other binary state is implied. Note that the shape of the distortions, and hence pdf, of the waveforms is important, not the peak-to-peak values.

Figure 1.—Modified sinewave modulation with steps even, for state A.

Figure 2.—Modified sinewave modulation with steps offset, for state B.

Figure 3.—Triangle modulation for state A.
The reader should at this point discern that a large number of possible \( \text{pdf-modulation} \) carrier waveforms exist even though we show only two example sets.

**Signal to Noise Ratio Issues**

Because communications channels typically add undesired noise to the signal (ref. 4), we test sensitivity of the above pdf-modulation processes to errors by simulating statistical noise in the received signal. We have tested our example modulation types by creating spreadsheet simulations, in which random quasi-white noise is added (with the RAND function), and by using the statistical skewness function from statistics on a distribution, we have empirically derived some indications of process signal-to-noise (S/N) ratio robustness. ((2) is one representative definition of skewness (ref. 7)).

\[
\text{Skewness} = \frac{n}{n-1} \sum \left( \frac{x_j - \bar{x}}{\sigma} \right)^3
\]

Table I shows a comparison of noise sensitivity of the quasi-sinewave small scale distortion process example of figure 5 when compared to the large-scale process of figure 6. However, merely listing some values tends to be inconclusive. Therefore we assume a binary “slicing level” exists between the skewness of state A and the skewness of state B, such that if the ratio of levels calculated over reference slicing level is greater than unity, then state A “wins” and if it is less than unity, then state B “wins”. Since we already know which waveform represents state B and which waveform represents state A, a false “win” is actually an error condition where the noise level has overridden the signal level and therefore S/N is too low. Note that the following data is based on many random trials, while looking for the minimum values. Anyone performing similar simulations will see slightly different data depending on the number of “runs” attempted.

<table>
<thead>
<tr>
<th>Detection windows (see Set I)</th>
<th>Small-scale process</th>
<th>Detection windows (see Set II)</th>
<th>Large-scale process</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noise seed for “rand” function</td>
<td>Typical state A skewness</td>
<td>Typical state B skewness</td>
<td>Noise seed for “rand” function</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.43</td>
<td>1.09</td>
<td>50</td>
</tr>
<tr>
<td>1</td>
<td>-0.55</td>
<td>1.52</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>0.056</td>
<td>3.22</td>
<td>75</td>
</tr>
<tr>
<td>2</td>
<td>0.54</td>
<td>1.43</td>
<td>75</td>
</tr>
<tr>
<td>3</td>
<td>0.77</td>
<td>2.15</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>0.54</td>
<td>0.88</td>
<td>100</td>
</tr>
<tr>
<td>4</td>
<td>-0.98</td>
<td>0.29</td>
<td>125</td>
</tr>
<tr>
<td>4</td>
<td>0.55</td>
<td>0.35</td>
<td>125</td>
</tr>
</tbody>
</table>

Viewing the ratios in table II, we see that set I sensitivity to noise is obviously worse than in Set II. In Set I it was possible to measure \( \text{SKEWNESS} \) clear across the entire half-waveform in spite of noise level. In Set I, the small-scale steps needed to be resolved by measuring only the \( \text{SKEWNESS} \) of the histograms in the areas where the steps were presumed to exist. This fact, coupled with the results in
表 II. 显示了大尺度示例比小尺度示例远不那么敏感于噪声。但我们不声称这一结果证明大尺度过程是数学上最优的。

<table>
<thead>
<tr>
<th>Detection windows (see Set I)</th>
<th>Small-Scale</th>
<th>Detection windows (see Set II)</th>
<th>Large-scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noise seed for “rand” function</td>
<td>Lowest skewness ratio after many trials</td>
<td>Noise seed for “rand” function</td>
<td>Lowest skewness ratio after many trials</td>
</tr>
<tr>
<td>0</td>
<td>NA</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1.04</td>
<td>TBD</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>0.17</td>
<td>TBD</td>
<td>75, 1.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>78, &gt;1.0, TBD</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>82, &lt;1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>88, 0.97</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>125</td>
</tr>
</tbody>
</table>

定I：测量 SKEWNESS@ array (104:119) 在第一周期对于STATE A
测量 SKEWNESS @ array (104:119) 在第二周期对于STATE B
定II：测量 SKEWNESS @ array (0:255) 在锯齿波对于STATE A
测量 SKEWNESS @ array (0:255) 在正方形波对于STATE B
TBD：未确定（是一个统计值，最好通过大量实验确定）。

模拟检测模型

作为我们模型的另一种变化，我们提出了一种概念上的模拟信号解码和直方图统计分析方法。如图7所示，我们首先提出使用窗口比较器与集成输出（注5）来捕捉每个波形水平的窗口。如图8所示，我们提出使用一个带比较器的树，其中输出是动态和连续评估的模拟水平，使用一个通用的工业级求和放大器/比较器电路（注6）。除了这些工业标准，窗口比较器/积分器，或者求和放大器/比较器，使用任何新奇技术。但无论任何模拟电路都是具体连接到一个模拟直方图检测的pdf-调制，结合模型是这种新奇的。

图7.—代表性的模拟窗口比较器/积分器。

图8.—代表性的模拟求和放大器/比较器。

嵌入式通信问题

我们已经讨论了鲁棒性和系统噪声问题，当使用 pdf-调制时。但模型的含义是，系统必须有某种手段来适应信号衰减和其他信号变化，这会导致直方图的长期变化。因此我们假设接收侧“训练”会持续，直到接收过程持续。也就是说，每个直方图更新整个接收相关过程的状态。因此，接收信号的平均统计信号是可变的，而且以一种方式，使得接收器可以通过连续训练来适应信号的变化。我们因此提出，除了零偏移、线性变化、从正弦波到三角波等，可以故意用于复杂编解码器的形状，只要干扰不会混淆接收器的训练。

（或者在发送者，一个过程会做出一个显著的波形统计变化，然后允许一个固定时间让接收器重新校正，然后再发送真实信息）。请注意，我们不包括使用扩展频谱技术在我们的模型。

在一个电信电路中，如果通道可以“被窃听”，故意使用新奇信号模式会使得信号看起来像是噪声，从而故意想混淆某人。因此，是假设的。这里未证明，但没有证据表明，pdf-调制允许噪声调制来传输信息，因为其性质可能与真实信道噪声相似。虽然受制于设计一个过程来提取噪声调制的物理噪声影响，成功传输和检测信息的效果会使得隐蔽监控更难。
Conclusions

We have shown via simulation of pdf-modulation that the model successfully transfers information through a communications channel provided that the signal-to-noise ratio is manageable. We have shown that successful operation of a pdf-modulation process in the presence of noise requires an increasingly higher signal-to-noise ratio when there is a lower degree of distortion or “cleverness” of the modulation waveform. Simulated signals with noise have shown effects that are entirely expected by classical communications theory (ref. 4), but with effects that are difficult to define in closed-form equations. The model does not violate any rules covering signal-to-noise ratio versus channel bit rate. Pdf-modulation, being fundamentally unlike the “older” existing industry-practice forms such as AM and PM, merits inclusion in the modulation domain as pdf- or histogram-modulation. We have proposed both analog and digital models for implementing the process.

References

7. HELP for SKEW worksheet function, Microsoft Excel 2002 (Microsoft Corporation), which is defined differently than the standard SKEWNESS in other sources.
Beyond the classic modes of carrier modulation by varying amplitude (AM), phase (PM), or frequency (FM), we extend the modulation domain of an analog carrier signal to include a class of general modulations which are distinguished by their probability density function histogram. Separate waveform states are easily created by varying the pdf of the transmitted waveform. Individual waveform states are assignable as proxies for digital one's or zero's. At the receiver, these states are easily detected by accumulating sampled waveform statistics and performing periodic pattern matching, correlation, or statistical filtering. No fundamental physical laws are broken in the detection process. We show how a typical modulation scheme would work in the digital domain and suggest how to build an analog version. We propose that clever variations of the modulating waveform (and thus the histogram) can provide simple steganographic encoding.