Measuring rainfall is one of the most difficult challenges in meteorology because of its extreme spatial and temporal variability. The Tropical Rainfall Measuring Mission (TRMM), a satellite program, was designed to systematically measure tropical rainfall from space. Tipping Bucket (TB) rain gauges were deployed to validate the satellite-inferred rainfall estimates and generate rain products and their quality evaluation. The TB gauge used in this study is a simple mechanical device that directly measures rainfall in increments of 0.01 inch, or a tip, at a discrete, point location on the earth's surface. The TB gauge is equipped with an onboard data logger that records the time of each tip to the nearest second.

This paper describes the generation of one-minute rain rates from TB gauge measurements. The cubic spline, constructed of third-order polynomials, is used to interpolate discrete TB tip records into one-minute rain rates during a rain event. A simulated TB gauge from a Joss-Waldvogel (JW) disdrometer is employed to evaluate effects of time scales and rain event definitions on errors of the rain rate estimation. The comparison between rain rates measured from the JW disdrometer and those estimated from the simulated TB gauge shows good overall agreement; however, the TB gauge
suffers sampling problems, resulting in errors in the rain rate estimation. These errors are very sensitive to the time scale of rain rates. One-minute rain rates suffer substantial errors, especially at low rain rates. When one-minute rain rates are averaged to 4-7 minute or longer time scales, the errors dramatically reduce.

The rain event duration is very sensitive to the event definition but the event rain total is rather insensitive, provided that the events with less than 1 mm rain totals are excluded. Estimated lower rain rates are sensitive to the event definition whereas the higher rates are not. The median relative absolute errors are about 22% and 32% for 1-minute TB rain rates higher and lower than 3 mm h\(^{-1}\), respectively. These errors decrease to 5% and 14% when TB rain rates are used at 7-minute scale.
Estimating Rain Rates
from Tipping-Bucket Rain Gauge Measurements

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ABSTRACT

This paper describes the cubic spline based operational system for the generation of the TRMM one-minute rain rate product 2A-56 from Tipping Bucket (TB) gauge measurements. Methodological issues associated with applying the cubic spline to the TB gauge rain rate estimation are closely examined. A simulated TB gauge from a Joss-Waldvogel (JW) disdrometer is employed to evaluate effects of time scales and rain event definitions on errors of the rain rate estimation. The comparison between rain rates measured from the JW disdrometer and those estimated from the simulated TB gauge shows good overall agreement; however, the TB gauge suffers sampling problems, resulting in errors in the rain rate estimation. These errors are very sensitive to the time scale of rain rates. One-minute rain rates suffer substantial errors, especially at low rain rates. When one-minute rain rates are averaged to 4-7 minute or longer time scales, the errors dramatically reduce.

The rain event duration is very sensitive to the event definition but the event rain total is rather insensitive, provided that the events with less than 1 mm rain totals are excluded. Estimated lower rain rates are sensitive to the event definition whereas the higher rates are not. The median relative absolute errors are about 22% and 32% for 1-minute TB rain rates higher and lower than 3 mm h$^{-1}$, respectively. These errors decrease to 5% and 14% when TB rain rates are used at 7-minute scale. The radar reflectivity–rainrate (Ze–R) distributions drawn from large amount of 7-minute TB rain rates and radar reflectivity data are mostly insensitive to the event definition.
1. Introduction

The Tropical Rainfall Measuring Mission (TRMM) is a satellite program designed to systematically measure tropical rainfall (Simpson et al. 1996; Kummerow et al. 1998). Measuring rainfall from space is a real challenge due to the extreme spatial and temporal variability of tropical rainfall and the imperfect measuring technologies. There are also limitations in our understanding of precipitation processes that affect how accurately surface rainfall can be measured remotely from space. The TRMM Ground Validation (GV) program, based in the TRMM Satellite Validation Office (TSVO) at the NASA Goddard Space Flight, was established to validate the satellite-inferred rainfall estimates. Many rain products and their quality evaluations have been produced by TSVO. Rain gauges have played a crucial role in this validation effort (Wolff et al. 2005).

TSVO has deployed tipping-bucket (TB) rain gauge networks over several tropical sites. TSVO’s operational system interpolates one-minute rain rates with a cubic spline algorithm, using data collected from the TB gauge networks. The cubic spline effectively transforms a time series of discrete TB tip records into a quasi-continuous time series of one-minute rain rates. The rain product generated with this algorithm is known as the TRMM Standard Product 2A-56. The 7-minute averages of 2A-56 rain rates are used in the Window Probability Matching Method (WPMM) to derive radar reflectivity–rainrate (Ze–R) relations (Rosenfeld et al. 1994; Amitai 2000), and then rainfall estimations. The 2A-56 is also widely used for the calibration of the ground-based radar estimates, the validation of monthly satellite rainfall estimates, as well as being disseminated as data files to the scientific community in general (Habib and Krajewski 2002; Datta et al. 2003; Tokay et al. 2003a; Fisher 2004; Wolff et al. 2005).
The uncertainty of rain rate estimates varies when considered over the full spectrum of rain rates. It is least accurate at the low end of the rain rate spectrum where consecutive rain records are separated by longer intervals of time much greater than one-minute. By averaging the one-minute rain rates over longer time intervals, sampling related errors could be effectively reduced. The cubic spline algorithm also requires a somewhat arbitrary rain event definition based on the time gap between consecutive tips in the time series. TSVO defines the end of an event when this time gap exceeds 15 minutes, but other definitions are possible. The rain rates generated from the cubic spline are affected with this definition.

There are only a few studies about the TB gauge rain rate estimation. Williams and Erdman (1988) estimated rain rates by simply dividing the TB bucket volume by the time between tips. Sadler and Busscher (1989) employed a cubic spline to fit the accumulated rainfall time series, and then differentiated the spline to generate the rain rates. Habib, Krajewski and Kruger (2001) used an optical rain gauge to simulate the TB gauge and linearly estimated rain rates from the simulated TB data, and then discussed the sampling errors of TB gauge measurements.

This study is to describe the cubic spline based operational system used by TSVO for the generation of the TRMM one-minute rain rate product 2A-56 from rain gauge measurements. The sampling related errors of TB rain rate estimation are also discussed. A description about rainfall measurements from the gauge and disdrometer along with their comparisons is given in Section 2. Section 3 provides the methodology of the rain rate estimation. Section 4 discusses details on employing the cubic spline in the rain rate estimation. Section 5 describes the simulation of TB gauge measurements using a disdrometer. In Section 6, sampling related errors of TB rain rate estimation are evaluated.
for different time scales and different rain event definitions. A summary of conclusions is offered in Section 7.

2. Rainfall measurements

TRMM GV program selected Kwajalein Atoll in the Republic of the Marshall Islands as one of four direct data sites for the purpose of climatological validation of TRMM rain estimates. Kwajalein Atoll in the Central Pacific Ocean is located on the northern edge of the Intertropical Convergence Zone (ITCZ). There are 8 TB gauge sites on 8 coral islets of Kwajalein Atoll, including 6 gauges on the islet of Roi Namur and 2 gauges on each of other 7 islets. There are also 2 Joss Waldvogel (JW) disdrometers on the islets of Kwajalein and Roi Namur. Two TB gauges and a JW disdrometer were closely installed on Roi Namur on 08 April 2005. The distances among two TB gauges and the JW disdrometer are about 1 foot. In this study, only the reliable data from one TB gauge and the JW disdrometer on Roi Namur were used in the analysis.

a. TB gauge measurements

The TB rain gauge is a simple mechanical device that directly measures rainfall in increments of 0.01 inch, or a tip, at a discrete, point location on the earth’s surface. The rain gauge at Kwajalein is equipped with an onboard data logger that records the time of each tip to the nearest second. Non-rainy periods are determined by the absence of rain records. The TB gauge suffers sampling problems as well as systematic errors, mechanical and electrical failures.
b. JW disdrometer measurements

An impact type JW disdrometer measures the raindrop size distribution at the ground. It records raindrop numbers within each of 20 dropsize categories from 0.3 to 5.5 mm diameter during a time interval of one-minute (Joss and Waldvogel 1967). When a raindrop hits the 50-cm² conical styrofoam surface of the JW disdrometer, an electrical impulse of the impacting drop is converted to the diameter of the raindrop. Environmental noise and man-made noise can decrease the JW disdrometer’s detection efficiency for small drops. In processing data, raindrop spectra with fewer than 20 drops per minute were removed because they were usually triggered by non-meteorological factors such as debris and insects. Raindrop spectra with rain rates less than 0.2 mm h⁻¹ were also removed because they were prone to large sampling errors (Hangen and Yuter 2003).

c. Comparisons between gauge and disdrometer measurements

Comparisons among the two co-located TB gauges and the JW disdrometer on Roi Namur show one gauge constantly failed to record rain events, whereas other gauge agreed with the disdrometer. Therefore, only the good gauge and the disdrometer were used in this study. The JW disdrometer stopped recording between 3 June and 1 July 2005 due to a computer software problem. It also failed between 25 and 30 August 2005 for unknown reasons. The gauge failed to record rain events from 23 November to 03 December 2005, then it did not seem to work properly for the rest of 2005 by comparing with the disdrometer. The data used in this study consisted of times when the disdrometer and gauge were both working.
Figure 1 shows the daily rainfall and accumulated rainfall computed from the gauge and disdrometer from 09 April to 22 November 2005, excluding the periods from 3 June to 1 July, and from 25 to 30 August 2005. The excluded periods are denoted as heavy lines on the abscissa in Fig. 1. The gauge recorded 735.58 mm and disdrometer recorded 901.19 mm for the entire available reliable data period. The gauge accumulated rain total was 81.62% of the disdrometer. The correction coefficient of the two measurements was 0.996. The overall agreement between the gauge and disdrometer was good although the gauge measurements were usually lower than the disdrometer.

Several researchers have shown that the rain differences between a gauge and disdrometer were mostly from 10% to 20% (McFarquhar and List 1993; Sheppard and Joe 1994; Tokay et al. 2003 b; Hagen and Yuter 2003). These differences might result from the rainfall variability, instrument precisions or environmental effects. According to the JW disdrometer manufacturer, the difference of event rain totals measured by the rain gauge and co-located JW disdrometer should be within 15% for a rain event with total rainfall of 5 to 10 mm or more (Tokay et al. 2005).

A rain event is usually defined based on the time gap between two consecutive records from a TB gauge or JW disdrometer. Here any two records in the time series separated by 15 minutes or more is interpreted as the end of one event and the beginning of the next one. The rain event definition will be further discussed in Section 6b in conjunction with the errors of the rain rate estimation. Now if the above event definition were used for the original JW disdrometer records, the number of determined JW disdrometer rain events and their event durations would be very different from those of the TB gauge. The difference is due to the mechanical sampling of rainfall by each device. Whereas the TB
observes rainfall in 0.01-inch increments, the JW disdrometer is an impact device that responds instantly to drops impinging on its measuring surface. The rain event as defined using the disdrometer should be more accurate. But the purpose of this study is to estimate rain rates from TB gauge measurements. Moreover, disdrometers are usually not available for most gauge networks. In order to determine the approximately same rain events for both devices, it is necessary to simulate the JW disdrometer as a TB gauge. The simulation will be discussed in Section 5. Table 1 shows the event rain totals, start times and durations measured from the TB gauge and event rain totals from the simulated gauge based on the JW disdrometer for all rain events with rain totals at least 5 mm. The disdrometer was 7.26% higher than the gauge for the rain total of all these 30 events. For most events, the difference was less than 15%. When the rain events with gauge rain totals at least 10 mm were compared, only one event on 8 October had the difference greater than 15%. The agreement was better for heavier rain events. A total of 382.78 mm for all rain events in Table 1 (at least 5 mm) was 52.04% of the rain total observed by the gauge, whereas 412.74 mm in Table 1 was 45.80% of the rain total observed by the disdrometer. Besides the small-scale variability of rainfall and systematic errors of both devices, this may imply that the gauge has a tendency to under-catch light rainfall, which could be caused by the evaporation of a partial rain tip in the bucket, clogging of the funnel that drains into the bucket, wind, turbulence and so on due to its design (Groisman and Legates 1994; Nespor and Sevruk 1999).

Figure 2 is the scatter plot along with the regression line for these 30 rain events listed in Table 1. The correlation coefficient between the gauge and disdrometer event rain totals was 0.999 and the mean absolute error was 1.07 mm. The standard deviation of the
event rain total differences was 0.91 mm. It is immediately apparent that the gauge and disdrometer agreed reasonably well. Therefore, we would use the disdrometer as a reference to study the sampling related errors of the rain rate estimation from the gauge measurements in Section 6.

3. Methodologies for rain rate estimation

The numbers of raw TB gauge tips in a rain event are summed into accumulated tip numbers at each tip minute. Then these accumulated tip numbers are converted into the accumulated rainfall in millimeter by multiplying the TB bucket size of 0.254 mm. The accumulated rainfall is the monotonically increasing function of the tip minute, which should be more suited to interpolation methods than the raw TB rain tips. Several methods can be considered for this kind of interpolations, such as linear, quadratic, and cubic methods.

The linear method is the simplest way to estimate the rain rate, which can be described as

\[ R_{i+1} - R_i = \frac{\Delta V}{(x_{i+1} - x_i)} \]  

where \( R_{i+1} - R_i \) is rain rate during two consecutive TB tip minutes \( x_i \) and \( x_{i+1} \) for \( i=1,2,\ldots,n-1 \) (\( n \) is the total number of tip minutes), \( \Delta V \) is the gauge bucket size which is 0.254 mm here. The linear interpolation always results in constant rain rates between any two consecutive tip minutes. This may not be acceptable for coarse sampling intervals with high rain gradients. A typical problem associated with the linear interpolation is that interpolated rain rates are never zero even during rain intermittences. For example, for one-
tip rainfall in 10-minute interval with rain peaks and intermittences, the linear estimates of rain rates are 1.524 mm h\(^{-1}\) for the entire 10-minute interval.

The quadratic method produces a saw tooth rate curve, which is apparently not well suited for the rain interpolation (Sadler and Busscher, 1989). Only cubic and higher order methods produce a smooth and continuous rate curve. Because of its easy implementation as discussed below and its prior satisfactory experience, the cubic spline method is often chosen to interpolate rain rates (Sadler and Busscher 1989; Tokay et al. 2003a).

The cubic spline is a spline constructed of third-order polynomials used to interpolate the data. The basic idea behind the cubic spline is from an engineer’s drafting tool, a flexible rod, often called spline, which is used to help draw smooth curves connecting widely spaced points. The mathematical cubic spline accomplishes the same result for numerical data points (Press et al. 1992; Bartels et al. 1998). A series of unique cubic polynomials \( f(x) \) are constructed to fit the curves between each of the data points. The fitting curves must be continuous and appear smooth.

Given a tabulated dataset \((x_i, y_i)\) \((i=1,2,...,n)\), where \(x_i\) is the same as in (1) and \(y_i\) is rain accumulation at the tip minute \(x_i\). A cubic spline essentially is a piecewise function in the form of

\[
f(x) = f_i(x) \text{ if } x_i \leq x < x_{i+1} \text{ for } i=1,2,...,n-1
\]

where \(f_i(x)\) is a cubic polynomial at each interval \([x_i, x_{i+1})\), defined as

\[
f_i(x) = a_i (x - x_i)^3 + b_i (x - x_i)^2 + c_i (x - x_i) + d_i
\]

By definition, the cubic spline satisfies following conditions:

1) \(f(x)\) interpolates all data points,

\[f(x_i) = y_i \text{ for } i=1,2,...,n;\]
2) \( f(x) \) is continuous across the entire interval \([x_1, x_n]\),

\[ f_i(x_i) = f_{i-1}(x_i) \quad \text{for} \quad i = 2, 3, \ldots, n; \]

3) \( f'(x) \) is continuous across the entire interval \([x_1, x_n]\),

\[ f'_i(x_i) = f'_{i-1}(x_i) \quad \text{for} \quad i = 2, 3, \ldots, n; \]

4) \( f''(x) \) is continuous across the entire interval \([x_1, x_n]\),

\[ f''_i(x_i) = f''_{i-1}(x_i) \quad \text{for} \quad i = 2, 3, \ldots, n; \]

Based on 1) - 4), coefficients of Equation (2) can be derived as

\[
\begin{align*}
   a_i &= \frac{f''_{i+1}(x_{i+1}) - f''_i(x_i)}{6(x_{i+1} - x_i)} \\
   b_i &= \frac{f''_i(x_i)}{2} \\
   c_i &= \frac{y_{i+1} - y_i}{x_{i+1} - x_i} - \frac{f''_{i+1}(x_{i+1}) + 2f''_i(x_i)}{6}(x_{i+1} - x_i) \\
   d_i &= y_i
\end{align*}
\]

for \( i = 1, 2, \ldots, n-1 \).

A set of Equations (3) can be rearranged as following symmetric tridiagonal system

\[
\begin{align*}
   \frac{x_i - x_{i-1}}{6} f''_{i-1}(x_{i-1}) + \frac{x_{i+1} - x_i}{3} f''_i(x_i) + \frac{x_{i+1} - x_i}{6} f''_{i+1}(x_{i+1}) &= \frac{y_{i+1} - y_i}{x_{i+1} - x_i} - \frac{y_i - y_{i-1}}{x_i - x_{i-1}} \\
   \quad \text{for} \quad i = 2, 3, \ldots, n-1.
\end{align*}
\]

This gives \( n - 2 \) linear equations and \( n \) unknowns \( f''_i(x_i) \) for \( i = 1, 2, \ldots, n \).

Therefore, (4) is under-determined. Two other conditions are required in order to get a unique solution for (4). Most commonly, second derivatives are set to zeros at the both endpoints \( x_1 \) and \( x_n \).
\[ f_i''(x_1) = f_n''(x_n) = 0 \quad (5) \]

A set of Equations (4) and two boundary conditions (5) lead to a simple linear tridiagonal system that can be easily solved by tridiagonal algorithm (Press et al. 1992). Thus coefficients of the cubic polynomial (2) can be determined using (3). The cubic spline is now unique. This produces a so-called “natural” cubic spline.

The cubic spline is popular because of its easy implementation and seamless fitting curve. But notice that the cubic spline is only piecewise continuous, which means it may not be the best choice for data sensitive to the smoothness of the third or higher derivatives. It is a reasonable assumption that the time series of accumulated rainfall during a rain event is piecewise continuous, so the cubic spline can be used for the rain rate estimation on the rain event basis.

4. Estimation of rain rates using cubic spline

The cubic polynomial as in Equation (2) can be analytically constructed for the accumulated rainfall at each tip interval in a rain event. For the longest and largest rain event from 1838 UTC on 11 July to 0052 UTC on 12 July 2005, 347 tips were recorded by the TB gauge during 374 minutes. More than one tip was recorded at some minutes whereas no tip was recorded at other minutes. There were 176 minutes recorded rain tips at unequally tipped minutes of 1838, 1842, ..., 2356, 0000, 0003, ..., 0045, 0052 UTC (Table 2). Thus there were 175 tip minute intervals and 175 cubic polynomials. In Equation (2), set \( x_i = 0 \) as the first tip minute at 1838 UTC, then \( x_i = 4, 8, ..., 367 \) for \( i = 2, 3, ..., 175 \) which are the 2\(^{nd}\), 3\(^{rd}\), ..., 175\(^{th}\) tip minutes. Table 2 lists coefficients a, b and c of the first and last 5 cubic
polynomials constructed for this event. Coefficient $d$ is not listed in Table 2 because $d$ is equal to the accumulated rainfall at a given tip minute according to the last equation of (3).

Equation (2) gives the accumulated rainfall as a function of time, which can be evaluated at each integer minute during a tip interval. The rainfall during each minute can be obtained by subtracting the accumulated rainfall at the previous minute from that at the current minute. This one-minute rainfall can be easily converted into one-minute rain rate in mm h$^{-1}$ by multiplying 60. For the first tip interval in Table 2, we have parameters of Equation (2) for the first cubic polynomial

$$i=1$$

$$x_1=0$$

$$a_1=0.000143$$

$$b_1=0$$

$$c_1=0.06579$$

$$d_1=0.254$$

Equation (2) for the first tip interval can be written as

$$f_1(x)=-0.000143x^3+0.06579x+0.254$$

where $x=0,1,2,3$ at 1838, 1839, 1840, 1841 UTC.

Similarly, for the second tip interval,

$$f_2(x)=0.000716(x-4)^3-0.001718(x-4)^2+0.058919(x-4)+0.508$$

where $x=4,5,6,7$ at 1842, 1843, 1844, 1845 UTC.

Notice that $f_1(4)=f_2(4)=0.508$ when $x=4$ at 1842 UTC because $f(x)$ is continuous across the entire $x$ interval from 0 to 374, or from 1838 UTC on 11 July to 0052 UTC on 12 July 2005, according to the cubic spline definition in Section 3. The one-minute rain rate at 1839 UTC
can be estimated as \([f_1(1) - f_1(0)] \times 60 = 3.94\) (mm h\(^{-1}\)). One-minute rain rates at any integer minutes can be similarly estimated.

The cubic spline is employed within a rain event. The inputted dataset \((x_i, y_i)\) are the TB tip time in minute and the accumulated rainfall at the tip time in the event. A typical problem associated with TB gauge measurements and the any rain rate interpolations is the lack of information about the start and end times of a rain event. The first or last tip time in a rain event recorded by a TB gauge does not indicate the real start or end time of the event. A rain event always starts before the first tip time and ends after the last tip time. A rain event often terminates before the bucket tips, leaving it partially filled. This partial tip of rainfall amount is recorded in association with the first tip of the next event. Similarly, a partial tip after the last tip of a rain event is left into the next event. The exact amount of the partial tip is impossible to be estimated. Here a fair assumption for the partial tip is one-half tip. The maximum possible error of an event rain total caused by this assumption would be one tip. This is trivial for most rain events. The start time of a rain event could be estimated by backward extrapolating the first cubic polynomial to the time when accumulated rainfall reaches one-half tip. This time is estimated as the start time. Similarly, the end time of a rain event could be estimated by forward extrapolating the last cubic polynomial to one-half tip beyond the last tip. This arbitrary assumption is not expected to result in good extrapolations for all rain events, but it preserves the event rain totals. For the event of 11-12 July 2005, the estimated start time is 1837 UTC on 11 July that is one minute ahead of the first tip. The estimated end time is 0055 UTC on 12 July that is 3 minutes after the last tip.

Figure 3 shows the time series of the rain rates measured from the disdrometer (solid line), and estimated from the gauge using cubic spline (dot line) and linear (dash line)
methods for the event of 11-12 July 2005. The TB gauge recorded rain total of 88.14 mm during 374 minutes. The disdrometer recorded slightly more rainfall. The accumulated rainfall from the TB gauge cubic spline and linear estimates were nearly identical. Both showed the similar shapes as that of the disdrometer. Generally speaking, rain rates estimated from both cubic spline and linear methods agreed well with the rain rates measured from the disdrometer. The gauge lagged the disdrometer a couple of minutes. This might be caused by the different sampling mechanisms of two devices besides the rain small-scale variability. The cubic spline revealed finer features than the linear interpolation, especially for a light rain period such as 1849-1857, 1914-1928 or 2243-2253 UTC on 11 July where the linear interpolation only evenly distributed the rain amount to each minute of the entire period.

The objective of generating the TRMM Standard Product 2A-56 is to provide meaningful rain rate information, and so a dilemma arises regarding the rain rate estimation for one-minute events. Since the 2A-56 is also used to ascertain rainfall accumulation over various time periods, it is of nearly equal importance that the measured rainfall accumulation be preserved in the estimated data product 2A-56.

There are situations where the cubic spline cannot be applied such as in the case where a rain event only consists of one minute. In this case, neither a first or second derivative in the cubic spline algorithm can be determined from the TB rain information. The one-minute event rainfall is usually one tip recorded at the minute by the TB gauge. Actually this amount of rainfall could be accumulated during a heavy rain period of several seconds or a light rain period of several hours with rain intermittences if observed from a disdrometer or radar. Through discussions with TRMM field campaign participants and
other TRMM scientists, rain rates are arbitrarily estimated by evenly distributing the one-minute event rainfall over a five-minute interval begins at 3 minutes prior to the tip minute. For example, for a one-minute event with only a single tip, the rain rate is inferred to be 3.048 mm h\(^{-1}\) at each of the five minutes. This estimation for a one-minute event is necessitated more by the need for accurate rainfall totals than the need for the rain rate information.

Another special case occurs for a two-minute rain event. In this case, the natural cubic spline becomes a linear line because second derivatives are set to zero at the both minutes. A linear extrapolation is actually applied to estimate rain rates as well as the start and end times of the event.

The above determination of the start and end times for a rain event is rather arbitrary. Different rain events have very different temporal features, especially at the start and end phases of events. No single technique can be suitable to all events. The TB gauge provides no temporal information before the first tip and after the last tip. The inaccurate estimation of the event start and end times could result in inaccurate estimation of rain rates, especially low rain rates.

Another problem in the rain rate estimation using the cubic spline is that the interpolated rain rates could occasionally be negative. This occurs when large rain gradients exist at low rain rates. An example is around 1926-1930 UTC on 11 July 2005 in Fig. 3. When this problem happens, the interpolated negative rates are set to zero. This results in a bias between estimated and observed event rain totals. Therefore, it is necessary to adjust all estimated rain rates in the event so that the bias after the adjustment is zero. The bias before the adjustment is usually less than 5%. When the bias is greater than 50%, the linear
method, instead of the cubic spline, is employed for the event. There were 3 events with biases larger than 50% among all 374 events defined by the 15-minute gap for the TB gauge from 09 April to 22 November 2005. They were all short events with large rain gradients adjacent to rain intermittences.

5. Simulation of TB gauge measurements using a JW disdrometer

Since the JW disdrometer has its own inherent detection problems as discussed in Section 2, we removed the records with less than 20 drops per minute and rain rate less than 0.2 mm h\(^{-1}\) as noise. The quality-controlled JW rain rate is considered as a reference for the evaluation of the TB gauge accuracy. We accumulate the one-minute rainfall measured from the JW disdrometer to integer multiples of the TB bucket size of 0.254 mm, or the number of tips, then record the tip minute. The fractional remainder of the tip at the tip time is carried forward and used to accumulate into next integer tip. By doing so, the rain total is preserved. The cubic spline is then applied to rain events defined from the simulated TB gauge to get the one-minute simulated TB gauge rain rates.

Figure 4 shows the time series of rain rates measured from the JW disdrometer (solid line) and estimated by the cubic spline from the simulated TB gauge (dot line) for the rain event of 11-12 July 2005. The upper part of Fig. 4 shows the tip times of the simulated TB gauge (plus sign) and real TB gauge (dot bar). The overall agreement among the JW disdrometer, TB gauge and simulated TB gauge was reasonably good. The accumulated rainfall of the JW disdrometer ideally matched that of the simulated TB gauge. We should not expect a perfect match between the tip times of the TB gauge and simulated gauge...
because the TB gauge and JW disdrometer suffer a lot of environmental and operational factors that affect their measurements. The simulated gauge slightly lagged the disdrometer because of the TB gauge sampling mechanism. The simulated TB gauge has the same sampling problem as the real TB gauge. They record the tip time only when rain accumulates to the full bucket size, which means it cannot be able to respond immediately to the changes in rain rates. The simulated TB gauge, as well as the TB gauge, cannot tell the start and end times of the rain event. The comparison between the JW rain rates and the simulated TB tips shows that rain started before the first simulated tip at 1825 UTC on July 11 and did not stop immediately after the last simulated tip at 0055 UTC on July 12. The simulated gauge cannot reproduce all detailed features observed by the JW disdrometer, especially for low rain rates and high gradients of rain rates. However, the example in Fig. 4 does show that the simulated gauge is able to reproduce the general feature of a rain event and suffer the same sampling problem as a real TB gauge. Therefore, we can study the sampling related errors of the TB rain rate estimation by comparing the simulated TB gauge with the JW disdrometer for different time scales and different rain event definitions. By using the simulated TB gauge rather than the real TB gauge, systematic errors, mechanical and electrical failures are excluded in the comparison.

6. Errors of TB rain rate estimation

TSVO produces TRMM Standard Product 2A-56, a time series of one-minute rain rates, using the cubic spline algorithm discussed in Sections 3 and 4. The 2A-56 plays a key
role in the generation of radar rain products and their quality evaluations. It is important to understand errors of the rain rate estimation.

Many factors could affect the TB gauge measurements and rain rate estimation (Humphrey et al. 1997; Nespor and Sevruk 1999; Habib et al. 2001; Wolff et al. 2005). As a precision instrument, the TB gauge suffers the systematic problem due to inadequate calibration before and after its deployment. The calibration should be performed in a sheltered site or a laboratory. This is often impossible at remote gauge sites. Mechanical and electrical failures occasionally happen mainly due to the harsh environment where the gauge is usually deployed. The gauge may fail to tip or the onboard logger may fail to correctly record tips during a rain event.

The TB gauge sampling mechanism is also a significant source of rain rate estimation errors. The TB gauge cannot provide actual start and end times of a rain event. The extrapolation method provided in Section 4 can be used to estimate the start and end times. But its result cannot be expected to be good for all rain events with different structures and types. As an accumulating instrument, the TB gauge always delays to response the rain rate changes, especially in a light rain period. The TB gauge is often unable to provide accurate information about the rain rate temporal distributions due to sparse tip intervals and high gradients of rain intensities.

The deployment of co-located gauges was suggested by Krajewski et al. (1998), Tokay et al. (2003b) and Wolff et al. (2005). The careful quality control of the co-located gauge data can help to detect most systematic, mechanical or electronic problems. The cubic spline method is developed to estimate one-minute rain rates from the rain tips recorded by the TB gauge. The estimated rain rates suffer the TB gauge sampling related
errors. The errors of the rain rate estimation need to be investigated because these estimates are instrumental in the calibration of the GV radar and the validation of the satellite rainfall estimates (Rosenfeld et al. 1994; Fisher 2004; Wolff et al. 2005; Fisher in press).

a. Effect of time scales on errors of rain rate estimation

In order to estimate the effect of time scales on errors of rain rates, similar to Habib, Krajewski and Kruger (2001), we use the rain rate measured from the JW disdrometer as a reference and define the TB gauge error as follows:

\[ E_{TB} = R_{TB} - R_{JW} \quad (R_{TB} > 0 \text{ or } R_{JW} > 0) \]  

Here \( R_{TB} \) is the estimated rain rate from the simulated TB gauge at a given time scale. \( R_{JW} \) is the measured rain rate from the JW disdrometer at the same time scale. \( E_{TB} \) is the error of \( R_{TB} \) estimate at the time scale. All one-minute rain rates from all rain events, including one-minute rain events, are used here. Both one-minute rain rates from the simulated TB gauge and the JW disdrometer are averaged to 2-, 4-, 7-, 10-, 15-, 30- and 60-minute scales. The TB gauge error is evaluated individually for different time scales. Formula (6) is performed only when either the simulated TB rain rate or JW disdrometer rate at the certain time scale is greater than zero. By setting this condition, non-rain periods are excluded from the comparison.

The error scatter plots are shown in Fig. 5 where \( E_{TB} \) is the ordinate and the rain rate is the abscissa. The sample size of \( E_{TB} \), standard deviation (STD) of \( E_{TB} \), and mean absolute error (MAE) between \( R_{TB} \) and \( R_{JW} \) decreased but the correlation coefficient (Corr) between \( R_{TB} \) and \( R_{JW} \) increased with the increasing time scales as shown in the inserted texts of Figs. 5a-5h. The scatter of the error \( E_{TB} \) decreased as the time scale increased from 1 minute to 60
minutes. At the 4- and 7-minute time scales, the errors (Figs. 5c and 5d) dramatically reduced in comparison with 1- and 2-minute scales (Figs. 5a and 5b). At the 10- and 15-minute time scales, the errors further reduced (Figures 5e and f). If the time scale increased to longer than 30 minutes, the errors were very close to zero (Figs. 5g and 5h) and therefore could be negligible. It is interesting to notice that errors in Fig. 5 were bounded to a linear line \( E_{TB} = R_{TB} \) when \( R_{JW} \) was zero or \( E_{TB} \) was at its maximum value \( R_{TB} \).

Figure 5 shows larger TB gauge errors located at low rain rates. This is caused by the poor performance of the TB gauge during light rain periods. The TB gauge often takes a longer time to accumulate a tip when rain rates are low. It cannot provide detailed information to determine the start and end times of a rain event. Light rain often occurs during initial and final phases of the event, and between rain peaks. During a light rain period in a rain event, a JW disdrometer might record a lot of small rain rates while a TB gauge might record only one tip at the time when the bucket gets full. For this type of light rain, the cubic spline algorithm might interpolate this one tip to several relatively higher rates around the tip time and many zero-rates at other times in the entire light rain period. This might cause the light rain rate overestimation around the tip time and underestimation at the other times.

From Fig. 5a, we can clearly see the binning of about 15.24 mm h\(^{-1}\) for the simulated rain rates. This is due to the 1-minute sampling resolution of the simulated TB gauge with the bucket size of 0.254 mm. The TB gauge always records the number of tips or rain amount as multiples of the bucket size at a tip minute. The binning disappears with increasing time scales. It is as expected that the error defined in (6) increases with the bucket
size. Habib, Krajewski and Kruger (2001) showed that a bucket size larger than 0.254 mm would increase uncertainties of the TB gauge measurements.

From above analysis, we can conclude that substantial errors associated with the TB gauge sampling mechanism do exist in low rates at 1- or 2-minute scale. These errors could be alleviated when the time scale of the rain estimates increases. The estimated TB rain rates would be more reliable if used at the time scales of 4-7 minutes or longer. This is one of reasons that TSVO uses the 7-minute average of rain rates in WPMM Ze-R development (Rosenfeld et al. 1994; Wolff et al. 2005).

b. Effect of event definitions on errors of rain rate estimation

The cubic spline method has been applied to the estimation of rain rates on a rain event basis. The definition of a rain event is often based on the time gap between two consecutive TB gauge rain tips. A new event is defined when the gap is longer than a certain criterion. A shorter gap could result in more defined rain events by separating a long light event, such as a drizzle, to several shorter events. In the other hand, a longer gap could result in fewer defined rain events by combining several shorter events into a longer event, and introduce more rain intermittences into the event. The rain intermittence in an event often causes inaccurate estimation of rain rates. Sadler and Busscher (1989) set a 10-minute gap whereas Cosgrove and Garstang (1995), Habib and Krajewski (2002), Tokay et al. (2003a, b) set a 30-minute gap as the criteria. Tokay et al. (2003b) also set 15-minute and 60-minute gaps to study the sensitivity of the rain event statistics to the definition using observed disdrometer rain rates, and found that no significant change in rain intensity was evident while the rain duration was sensitive to the definition.
Here we separate the rain events when no TB tip is recorded in a 10-, 15-, 20-, 30- or 60- minute interval. The cubic spline algorithm is used to estimate rain rates for all these differently defined events for the simulated TB gauge. Figure 6 shows rain rates measured from the JW disdrometer and estimated by the cubic spline from the simulated TB gauge with different rain event definitions for the same period from 0406 to 0835 UTC on 16 Oct 2005. This would be one rain event if defined from the JW disdrometer using the 60-minute definition. It would be 7, 6, 4, 2 or 1 event if defined from the simulated TB gauge using 10-, 15-, 20-, 30- or 60-minute definition, respectively. These events are listed in Table 3. There was a one isolated tip event at 0651 UTC in the 10- and 15- minute definitions. This one tip event was combined into a longer event in 20-, 30- and 60-minute definitions. The number of rain intermittences in a rain event would increase when the event was defined from 10- to 60-minute gap (Fig. 6). The determination of a rain event was very sensitive to the event definition (Table 3), but the estimated rain rate was not so sensitive (Fig. 6). Almost all peak rain rates were reasonably estimated by the cubic spline with different event definitions although a few peak rates were either somewhat overestimated or underestimated. A close look at the details of Fig. 6 shows that the cubic spline with longer gap definitions interpolated more low rain rates while they were in fact zeros during rain intermittences.

Figure 6 along with Table 3 is just a case study for comparisons of rain rates and rain events among different event definitions. Figure 7 shows the boxplots for event durations, event rain totals and estimated rain rates for all differently defined rain events with event totals at least 1 mm during the entire data period. Event durations, event rain totals and rain rates are highly positively skewed. The median and inter-quartile range (IQR) are robust and resistant to
outliers, therefore considered as the better center and dispersion characteristics. The IQR is simply the difference between the upper (75th) and lower (25th) quartiles, and plotted as the black box in Fig. 7. The white bar inside the box is the median. The upper whisker is truncated and the maximum is listed at the upper border of each boxplot. The numbers of events were 156, 154, 151, 143 and 129 in the boxplots for the 10-, 15-, 20-, 30- and 60- minute definitions respectively. A rain event could be as long as 572 minutes with event rain total of 96 mm (Figs. 7a and 7b) or as short as several seconds with event rain total of 0.254 mm. Because only the events with rain total greater than 1 mm were plotted, the shortest event duration and minimum event rain total shown in Figs. 7a and 7b were 1.4 minutes and 1 mm, respectively. As expected, the event duration increased as a rain event was defined by a longer time gap. However, the event rain total was rather insensitive to the event definition because the event defined by the longer gap was often associated with rain intermittences and the events with less than 1 mm rainfall were excluded in the analysis. While a rain event was defined from the 10- to 60-minute time gap, the number of estimated 1-minute rates increased from 3448 to 6931 and the IQR decreased rapidly whereas the median rate decreased slightly and the maximum rate was constant (Fig. 7c). The similar situation happened for 7-minute rates (Fig. 7d) but less rapidly. Since lower rain rates suffered larger errors as shown in Fig. 5, we separated 1-minute rain rates into two groups, higher and lower than 3 mm h⁻¹. Their boxplots were shown in Figs. 7e and 7f. The higher rates (Fig. 7e) were not sensitive to the event definition but it was not the case for the lower rates (Fig. 7f). A greater number of lower rates were estimated when a rain event was defined by a longer time gap. The sensitivity of rain rates to the rain event definition was different from Tokay et al.'s research (2003b). In their case, rain rates were observed directly from JW disdrometers whereas for the case of above analysis, rain rates were estimated
by the cubic spline algorithm from TB gauge tips. The determination of a rain event from TB tips is more sensitive than that from the disdrometer rain rates because of their different sampling mechanisms as discussed earlier.

In order to quantitatively evaluate the error level of estimated TB rain rates, we define the relative absolute error (RAETB) as

$$\text{RAE}_{TB} = \frac{|R_{TB} - R_{JW}|}{R_{JW}} \quad (R_{JW} > 0)$$  \hspace{1cm} (7)

Here $R_{TB}$ is the one-minute rain rate estimated by the cubic spline from the simulated TB gauge or the rain rate averaged from the one-minute rates over a 7-minute time scale. $R_{JW}$ is the one-minute rain rate measured from the JW disdrometer or the rain rate averaged over the same time scale. All rain rates from all rain events with event rain totals at least 1 mm are used in (7). Formula (7) is individually performed for two TB rate groups (higher or lower than 3 mm h$^{-1}$) with different event definitions (i.e. 10-, 15-, 20-, 30- or 60-minute definition). Table 4 shows the median RAETB in percentage for 1-minute and 7-minute (in parenthesis) TB rain rates. The median RAETB for higher rates was about 10% less than that for the lower rates. The light rain rate might be either overestimated or underestimated as discussed in Section 6a. The underestimation was 100% from (7) if the estimated TB rain rate was zero for any recorded non-zero JW rain rate.

From Table 4, the median RAETB was about 22% and 32% for 1-minute TB rain rates higher and lower than 3 mm h$^{-1}$, respectively, for the 15-minute event definition. When the TB rain rates were averaged to 7-minute scale, the median RAETB decreased to about 5% and 14% for higher and lower rates respectively. When a rain event was defined by a longer time gap, the median RAETB increased, especially for lower rain rates. A rain event defined by a longer tip gap often contains more rain intermittences. The cubic spline
algorithm might interpolate these rain intermittences as light rain. The rain intermittence is a major difficulty in interpolating rain rates from TB gauge measurements using the cubic spline, linear or any other methods. In this regard, we prefer to choose a shorter definition, for example, 15-minute, in order to avoid more rain intermittences in an event.

TSVO also investigated the sensitivity of the WPMM Ze-R distributions to rain event definitions (http://trmmfc.gsfc.nasa.gov/trmm_gv/gv_products/level_2/event_definition/rain_event_compare.html). The 7-minute rain rates based on 15- and 60-minute definitions were used in monthly WPMM Ze-R development for Kwajalein, Republic of the Marshall Islands and Melbourne, Florida. By combining large amounts of rain rates from all TB gauges for each site, the effect of the rain event definition was diminished. The resulting Ze-R distributions were nearly identical for both definitions.

7. Conclusions

This study probes the cubic spline based operational system for the generation of the TRMM one-minute rain rate product 2A-56 from TB gauge measurements. The sampling related errors of the 2A-56 TB rain rates are investigated. These errors are very sensitive to the time scale of rain rates. One-minute rain rates suffer substantial errors, especially at low rain rates. When one-minute rain rates are averaged to 4 -7 minute scales, the errors dramatically reduce. At the 10- and 15-minute time scales, the errors further reduce. If the time scale increases to longer than 30 minutes, the errors become negligible.

The rain event duration is very sensitive to the event definition that is somewhat arbitrary, but the event rain total is rather insensitive to the event definition, provided that
the events with less than 1mm rain totals are excluded (see Fig. 7b). Estimated lower rain rates are sensitive to the event definition whereas the higher rates are not. The median relative absolute errors are about 22% and 32% for 1-minute TB rain rates higher and lower than 3 mm h\(^{-1}\), respectively. These errors decrease to 5% and 14% when TB rain rates are used at 7-minute scale. The WPMM Ze-R distributions drawn from large amount of 7-minute TB rain rates and radar reflectivity data are mostly insensitive to the event definition.

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REFERENCES


Figure Captions

Figure 1. Comparison of gauge and disdrometer rainfall measurements from 09 April to 22 November 2005, excluding the periods from 3 June to 1 July and from 25 August to 30 August. The daily rainfall (left ordinate) and accumulated rainfall (right ordinate) from the gauge (dot line) and disdrometer (solid line) are shown. Two heavy lines on the abscissa denote two excluded periods.

Figure 2. Scatter plot for event rain totals from the gauge and disdrometer. The solid line is a regression line. The diagonal dash line indicates 1:1 correspondence. Also shown are the regression equation between the gauge and disdrometer event rain totals, their correlation coefficient and mean absolute error, as well as the standard deviation of the event rain total differences and the number of events.

Figure 3. Time series of one-minute rain rates measured from the JW disdrometer (solid line), and estimated from the TB gauge using the cubic spline (dot line) and linear (dash line) methods for the event of 11-12 July 2005. Also shown is the accumulated rainfall from the measured JW rates and estimated TB rates using both methods. The left and right ordinates are for rain rates and accumulated rainfall, respectively.

Figure 4. Performance of the simulated TB gauge. The solid and dot lines are one-minute rain rates measured from the JW disdrometer and estimated from the simulated TB gauge using the cubic spline algorithm, respectively. In the upper part of the figure, the plus sign
and vertical dot bar indicate the tip times of the simulated TB gauge and real TB gauge, respectively. Also shown is the accumulated rainfall from the measured JW rates and simulated TB rates. The left and right ordinates denote rain rates and accumulated rainfall, respectively.

Figure 5. Error scatter plots for 1-, 2-, 4-, 7-, 10-, 15-, 30- and 60-minute simulated rain rates. The abscissa is the simulated rain rate at a certain time scale. The ordinate is the TB gauge error ($E_{TB}$). The sample size and standard deviation (STD) of $E_{TB}$ as well as the correlation coefficient (Corr) and mean absolute error (MAE) between the rain rates simulated from the TB gauge and measured from the JW disdrometer are shown in the inserted texts.

Figure 6. Time series of one-minute rain rates measured from the JW disdrometer (solid line) and estimated from the simulated TB gauge using the cubic spline algorithm with 15-, 30- and 60-minute event definitions for the period from 0406 to 0835 UTC on 16 Oct 2005. The solid, dot, dash, dash dot lines denote the disdrometer rain rates and gauge rain rates with 15-, 30- and 60-minute event definitions, respectively.

Figure 7. (a) Boxplots for event durations for rain events with event totals at least 1 mm. These events are determined by 10-, 15-, 20-, 30- and 60-minute definitions. The black box is the inter-quartile range. The white bar inside the box is the median. The upper whisker is truncated and maximum is listed at the upper border of each boxplot. (b) Same as (a) except for event rain totals. (c) Same as (a) except for all estimated 1-minute rain rates. (d) Same as
(c) except for 7-minute rain rates. (e) Same as (c) except for rain rates greater than 3 mm h\textsuperscript{-1}.

(f) Same as (c) except for rain rates at most 3 mm h\textsuperscript{-1}.
Figure 1. Comparison of gauge and disdrometer rainfall measurements from 09 April to 22 November 2005, excluding the periods from 3 June to 1 July and from 25 August to 30 August. The daily rainfall (left ordinate) and accumulated rainfall (right ordinate) from the gauge (dot line) and disdrometer (solid line) are shown. Two heavy lines on the abscissa denote two excluded periods.
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Table 1. Event rain total comparison between the TB gauge and JW disdrometer.

<table>
<thead>
<tr>
<th>Date (2005)</th>
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<th>Duration (min)</th>
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Table 2. Coefficients of cubic polynomials for the event of 11-12 July 2005.

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<tr>
<td>...</td>
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<td>...</td>
<td>...</td>
</tr>
<tr>
<td>0015-0023</td>
<td>86.868-87.122</td>
<td>0.000315</td>
<td>-0.004983</td>
<td>0.051429</td>
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<tr>
<td>0023-0029</td>
<td>87.122-87.376</td>
<td>-0.000151</td>
<td>0.002586</td>
<td>0.032255</td>
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<tr>
<td>0029-0035</td>
<td>87.376-87.630</td>
<td>-0.000107</td>
<td>-0.000134</td>
<td>0.046970</td>
</tr>
<tr>
<td>0035-0045</td>
<td>87.630-87.884</td>
<td>0.000120</td>
<td>-0.002051</td>
<td>0.033862</td>
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<tr>
<td>0045-0052</td>
<td>87.884-88.138</td>
<td>-0.000074</td>
<td>0.001564</td>
<td>0.028988</td>
</tr>
</tbody>
</table>
Table 3. Simulated TB rain events of 16 Oct 2005 with 10-, 15-, 20-, 30- and 60-minute definitions

<table>
<thead>
<tr>
<th>Event definition (min)</th>
<th>Event (UTC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0406-0410, 0429-0438, 0525-0555, 0607-0633, 0651-0651, 0712-0801, 0827-0835</td>
</tr>
<tr>
<td>15</td>
<td>0406-0410, 0429-0438, 0525-0633, 0651-0651, 0712-0801, 0827-0835</td>
</tr>
<tr>
<td>20</td>
<td>0406-0438, 0525-0651, 0712-0801, 0827-0835</td>
</tr>
<tr>
<td>30</td>
<td>0406-0438, 0525-0835</td>
</tr>
<tr>
<td>60</td>
<td>0406-0835</td>
</tr>
</tbody>
</table>
Table 4 Median relative absolute errors (RAE_{TB}) in percentage for one-minute TB rain rates and 7-minute TB rain rates (in parenthesis). RAE_{TB} is individually calculated for two TB rate groups (higher or lower than 3 mm h^{-1}) for differently defined events (10-, 15-, 20-, 30- or 60-minute definition).

<table>
<thead>
<tr>
<th></th>
<th>10-min</th>
<th>15-min</th>
<th>20-min</th>
<th>30-min</th>
<th>60-min</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt;3 mm h^{-1}</td>
<td>22.00 (5.20)</td>
<td>22.12 (5.07)</td>
<td>22.62 (5.33)</td>
<td>22.89 (6.28)</td>
<td>24.68 (8.53)</td>
</tr>
<tr>
<td>≤3 mm h^{-1}</td>
<td>32.10 (12.87)</td>
<td>31.87 (13.87)</td>
<td>32.11 (15.62)</td>
<td>33.33 (17.67)</td>
<td>37.69 (27.03)</td>
</tr>
<tr>
<td>All</td>
<td>23.90 (6.31)</td>
<td>24.58 (7.33)</td>
<td>25.29 (8.10)</td>
<td>26.67 (9.43)</td>
<td>30.24 (14.27)</td>
</tr>
</tbody>
</table>