Numerical Strip-Yield Calculation of CTOD

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Outline

- CTOD background

- Using Boundary Elements to calculate crack face displacements
  - Theory
  - Practical procedure
  - Example cases

- Summary and future plans
CTOD background: plastic zone sizes

- **Irwin (1958)**
  - LEFM gives $\sigma \sim 1/\sqrt{r}$; however: real materials yield
  - Crack behaves as if it were longer: $a_{\text{eff}} = a + \rho$
  - Plastic zone size estimated from stress redistribution

- **Dugdale (1960)**
  - Yielding confined to narrow strip ahead of crack (the “strip yield” model)
  - Stresses at “effective” crack tip ($a + \rho$) are finite
    - Yield zone loading neutralizes stress singularity due to remote loading
    - Plastic zone size estimated from setting $K(a + \rho) = 0$
Knowledge of $\rho$ enabled derivation of explicit CTOD expression
- Complex-variable analysis used (no full elastic-plastic analysis)
- Elastic-plastic behavior modeled by superposition of 2 elastic solutions

Wells (1963)
- CTOD is proportional to overall tensile strain, even after general yielding
  ⇒ CTOD became widely accepted as a useful fracture criterion when effects of the crack tip plastic zone are important
CTOD background: some calculation methods

- Dugdale’s model
  - Based on thin infinite plate, plane stress, remote tension
  - Extensions to other infinite geometries limited to a few particular cases
  - Arbitrary finite geometries require tailor-made elastic solutions

- Weight function, green’s function, collocation methods
  - Developed for particular finite geometries
  - Potentially heavy computational burden (e.g. reference solutions)

- Finite elements
  - General-purpose, but also severe computational toll
  - Where behind the crack tip to measure CTOD?
    - 1st node, 2nd node, 45° intercept, or some prescribed distance
  - Re-meshing burden for analyses of multiple loads or cracks
Using Boundary Elements to calculate crack face displacements: theory

- Direct application of conventional BEMs to fracture problems leads to mathematically degenerate formulation
  - Cause: geometric proximity of crack surfaces
  - Information about crack face tractions is lost
  - Can circumvent by developing additional integral equation for crack face tractions

- One approach:
  - Derive crack face traction equation from displacement eq^n via
    - Strain-displacement relations, Hooke’s law, limiting process
  - Resulting equation contains hypersingular kernel
    - Requires special interpretation; challenging to evaluate numerically
Better approach (Prof. Mear et al, Univ of Texas):

- Hypersingularity avoided by eliminating the offending terms in the displacement equation **before** deriving traction equation:
  - Appropriate choice of stress function for the stress kernel
  - Integration by parts to obtain a “modified” displacement equation

- Crack face traction equation is then derived as before
  (strain-displacement relations, Hooke’s Law, limiting process)

- Other practical benefits: small mesh size and fast solution times

- This is the basis of NASGRO’s BE component
Using Boundary Elements to calculate crack face displacements: theory (cont’d)

- Gradients of relative crack face displacements $\Delta D$
  - Described by dislocation density function $A$
  - $A$ is approximated by functions containing the requisite singularity
  - $A_j$ are nodal quantities in the vector of unknowns solved by NASBEM

- Technique implemented in NASBEM to integrate $A$
  - $\Delta D$ is sum of contributions from each crack element between the tip and the point of interest

\[
A(\zeta) = \frac{i\mu}{\pi(\kappa + 1)} \frac{\partial[\Delta D(\zeta)]}{\partial s}
\]

\[
A(t) = A[\zeta(t)] = \frac{1}{2\sqrt{\alpha_j}} \left( \frac{1-t}{\sqrt{\rho_j + t}} A_j + \frac{1+t}{\sqrt{\rho_j + t}} A_{j+1} \right)
\]

\[
\Delta D(\zeta) = \frac{\pi(\kappa + 1)}{i\mu} \int_0^\zeta A[\zeta(s)] ds
\]
NASGRO is an analysis software suite with four distinct modules:

- Fracture mechanics and fatigue crack growth analysis (NASFLA series)
- Fracture and fatigue crack growth material property database; fitting of experimental data (NASMAT)
- 2D boundary element stress analysis and stress intensity factor calculation (NASBEM)
- Fatigue crack formation (initiation) analysis (NASFORM)
NASGRO history

- **1980s:**
  - NASA/FLAGRO development initiated to provide fracture control analysis for manned space programs
  - NASA Fracture Control Methodology Panel formed to standardize methods and monitor NASA/FLAGRO development

- **1990s:**
  - NASA Interagency Working Group (NASA, DoD, FAA, ESA) formed to provide guidance for NASA/FLAGRO development
  - Additional NASA, FAA, USAF support for aging aircraft

- **2000s:**
  - NASA and Southwest Research Institute® sign Space Act Agreement for joint NASGRO development
  - NASGRO Industrial Consortium formed by SwRI; members include government agencies and industrial representatives
Example of typical NASBEM use: Orbiter feedline flowliner

- Fatigue cracks in flowliner (LH$_2$ supply to SSME)
  - 1’ Ø, 8-12’ L
  - Bellows within gimballing joints
  - Flowliners inside bellows to smooth flow

- Concern:
  - Engine failure due to debris
  - Loss of mission or vehicle

- NASBEM used to get $K$ vs $a$
Using NASBEM to calculate CTOD: procedure

- Use NASBEM to construct model
  - “Mathematical” crack consisting of
    - Physical crack $a$
    - Cohesive load zone $\rho$
  - Applied loading
  - Cohesive yield loading

- Following Dugdale’s idea
  - Plastic zone is sized so that $K$ due to cohesive loading cancels $K$ due to applied loading:
    \[ K_{\sigma y} = -K_{\sigma} \]
For a given yield stress $\sigma_Y$, achieve $K(a+\rho) = K_{\sigma_Y} + K_\sigma = 0$

- by setting the plastic zone size $\rho$ and iterating on the remote stress $\sigma$
  - advantage: no need to remesh while iterating
- or by setting the remote stress $\sigma$ and iterating on $\rho$
  - advantage: CTOD obtained for specific values of $\sigma$

CTOD value is given by crack face displacement at tip of physical crack $a$
Using NASBEM to calculate CTOD: results

- **Mesh**
  - Quadratic boundary elements, linear crack elements

- **Smaller mesh size than other BEM formulations**
  - Typical error <3% with 20 elements or less per boundary or crack
  - Crack face loading discontinuity requires a finer mesh
  - Fast results (example cases run in 2-3 seconds)

- **Configurations studied**
  - Center crack in finite and infinite sheets
  - Edge crack in finite sheet
  - Cracks from holes in infinite sheets
  - Periodic cracks in infinite sheet
  - 3-hole tension specimen
CTOD verification case: reproducing Dugdale’s result

- First verification case
  - Reproducing Dugdale’s model really should work!

- Dugdale model consists of
  - Center crack in infinite plate
  - Remote uniform tension
  - Cohesive yield stress on crack faces near crack tips
CTOD verification case: reproducing Dugdale’s result, cont’d

- Results virtually identical over wide range of $\sigma/\sigma_Y$
  - 4 significant digits
  - non-uniform error due to manually iterating to $K=0$

<table>
<thead>
<tr>
<th>$\sigma / \sigma_Y$</th>
<th>CTOD/$(\sigma_Y a/E)$ BEM</th>
<th>CTOD/$(\sigma_Y a/E)$ Dugdale</th>
<th>Error (%)</th>
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CTOD verification case: edge crack

- Verification case: C(T) specimen with W=3, a=1
  - Plastic zone size and CTOD were calculated ($\rho$ is shown here)
  - Excellent agreement with collocation results by Newman & Mall, and also Terada (both 1983)
CTOD verification case: 1 crack from a hole, infinite plate

- Verification case: 1 crack from a hole in an infinite plate under remote uniaxial loading
  - Plastic zone size calculated
  - Excellent correlation to analytical results by Rich (complex-variable analysis with conformal mapping, 1968)
Verification case: plastic zone size studied for various values of $a/R$

- Difference between NASBEM and Rich < 2.5%
- Larger $\rho$ in small cracks due to higher stress concentration at hole
- Solution approaches Dugdale solution for large $a/R$
CTOD verification case: periodic cracks in an infinite sheet

- Practical considerations:
  - How to model an infinite number of cracks?
  - 7 cracks seems a good approximation -- idea taken from literature on modelling large arrays of fuselage fasteners
- BE compared to Tada (Westergaard stress function, 1974)
CTOD verification case: periodic cracks in an infinite sheet, cont’d

- Excellent correlation for plastic zone size vs applied stress
CTOD verification case: periodic cracks in an infinite sheet, cont’d

- Excellent correlation for CTOD v plastic zone size
CTOD calculations: center crack in finite-width sheet

- BE compared to test data (Forman, 1966)
  - Tests on 0.020” AM350CRT steel sheet for toughness variation with specimen size
  - Plastic zone sizes were measured photographically
  - NASBEM compares well with test data
**CTOA calculations**

- **Using NASBEM as a fracture predictor:**
  - Crack tip opening angle (CTOA) has been noted by many to be a useful fracture criterion.
  - CTOA is calculated at ~0.04” (1 mm) behind the crack tip.
    - Comparisons to analytical results on previous pages were for CTOD at crack tip (“δ₅”) – not a practical location for real measurements.
  - CTOA = 2 * tan⁻¹(CTOD/2x), where x is distance behind crack tip.

- **Comparisons for**
  - M(T) specimen: Al 7075-T6, Al 2024-T81
  - 3-hole tension specimen: Al 7075
CTOA calculations:
center crack in finite-width sheet

- BE compared to test data (Forman, 1966)
  - M(T) specimens, 0.060” sheet
  - Al 7075-T6, 2024-T81

- Idea was to see if calculated CTOD or CTOA was reasonably constant over crack size \( a \)
  - looks good for 7075
  - less so for 2024
CTOA calculations:
3-hole tension specimen

- 3-hole tension (THT) specimen simulates $K$ for a cracked stiffened panel
  - $K$ curve taken from ASTM STP 896 (1985)
CTOA calculations: 3-hole tension specimen

- CTOA calculated for failure loads taken from ASTM round-robin on experimental and predictive fracture analysis methods (ASTM STP 896)
  - K and calculated CTOA show that THT is a complex configuration
  - Work is in progress
Many existing methods to calculate CTOD can be costly and complicated, or apply only to particular configurations.

A new numerical method for calculating CTOD was investigated:

- NASGRO’s Boundary Element module NASBEM was adapted to calculate displacements at any point on the crack.
- Demonstrated for a number of crack configurations:
  - finite and infinite domains
  - center and edge cracks
  - complex cases with several cracks and holes
- Great accuracy at minimal computational cost
Future

- Still a work in progress:
  - CTOA investigated ... more work needs to be done
  - Is $K_c$ corrected for Dugdale plastic zone size a better fracture criterion than $K_c(a)$ alone, or $K_c$ corrected for Irwin plastic zone size?
  - Multi-site damage issues to be investigated
  - Strain hardening -- easy to model

- CTOD capability is currently still a research tool
  - Turn capability into production-level tool
  - Implement automation of CTOD calculations
    - Manual meshing and convergence to $K=0$ for multiple cracks is tedious!