An Initial Non-Equilibrium Porous-Media Model for CFD Simulation of Stirling Regenerators

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The objective of this paper is to define empirical parameters (or closure models) for an initial thermal non-equilibrium porous-media model for use in Computational Fluid Dynamics (CFD) codes for simulation of Stirling regenerators. The two CFD codes currently being used at Glenn Research Center (GRC) for Stirling engine modeling are Fluent and CFD-ACE. The porous-media models available in each of these codes are equilibrium models, which assume that the solid matrix and the fluid are in thermal equilibrium at each spatial location within the porous medium. This is believed to be a poor assumption for the oscillating-flow environment within Stirling regenerators; Stirling 1-D regenerator models, used in Stirling design, use non-equilibrium regenerator models and suggest regenerator matrix and gas average temperatures can differ by several degrees at a given axial location and time during the cycle. A NASA regenerator research grant has been providing experimental and computational results to support definition of various empirical coefficients needed in defining a non-equilibrium, macroscopic, porous-media model (i.e., to define “closure” relations). The grant effort is being led by Cleveland State University, with subcontractor assistance from the University of Minnesota, Gedeon Associates, and Sunpower, Inc. Friction-factor and heat-transfer correlations based on data taken with the NASA/Sunpower oscillating-flow test rig also provide experimentally based correlations that are useful in defining parameters for the porous-media model; these correlations are documented in Gedeon Associates’ Sage Stirling-Code Manuals. These sources of experimentally based information were used to define the following terms and parameters needed in the non-equilibrium porous-media model: hydrodynamic dispersion, permeability, inertial coefficient, fluid effective thermal conductivity (including thermal dispersion and estimate of tortuosity effects), and fluid-solid heat transfer coefficient. Solid effective thermal conductivity (including the effect of tortuosity) was also estimated. Determination of the porous-media model parameters was based on planned use in a CFD model of Infiniti’s Stirling Technology Demonstration Convertor (TDC), which uses a random-fiber regenerator matrix. The non-equilibrium porous-media model presented is considered to be an initial, or “draft,” model for possible incorporation in commercial CFD codes, with the expectation that the empirical parameters will likely need to be updated once resulting Stirling CFD model regenerator and engine results have been analyzed. The emphasis of the paper is on use of available data to define empirical parameters (and closure models) needed in a thermal non-equilibrium porous-media model for Stirling regenerator simulation. Such a model has not yet been implemented by the authors or their associates. However, it is anticipated that a thermal non-equilibrium model such as that presented here, when incorporated in the CFD codes, will improve our ability to accurately model Stirling regenerators with CFD relative to current thermal-equilibrium porous-media models.

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Illustrations of Stirling engine, 2-D simulation results, regen.

Evidence that assumption of gas/solid thermal equilibrium in regenerator not valid

For reference, thermal non-equilibrium porous-media conservation equations

Porous-media model quantities in these equations needing definition or "closure"

Summary of available information for defining these quantities

Concluding remarks
Schematic of Infinia’s TDC 55 We Stirling Engine

Pistons have Clearance Seals => No Contact with Cylinders

Uses Helium “Working” Gas
GRC Temperature Contours: Generated with Dyson’s Fluent 2-D Axisymmetric CFD Model of Infinia’s TDC Stirling Engine (Using Thermal-Equilibrium Porous-Media Model for Regenerator Simulation)
Currently Used Regenerator Random Fiber Material

Bekaert
316 Stainless Steel
Nominal 12 Micron Round Fibers
Mean Eff. D.=13.4 microns
& Fibers Are Not Round
90 % Porosity

---from David Gedeon memo. reporting on DOE Regenerator Research Contract Work, Photo By GRC.
Evidence that Gas/Solid Thermal Equilibrium Assumption is not Valid

- UMN Tests with Engine Values of Dimensionless Variables show Significant Temperature Differences between Gas and Solid - To be shown
- Sage 1-D Model of Infinia’s TDC 55 We Engine shows significant Gas/Solid Temperature Differences in Regenerator - To be shown
- Enthalpy Flux Losses through the Regenerator from the Hot to the Cold calculated by Sage are significant (to be shown) and are expected to be sensitive to thermal equilibrium/non-equilibrium assumption

- Enthalpy Flux at Point along 1-D Flow Axis is:

\[ \int \dot{m} \ c_p \ T_{gas} \ dt \]
Regenerator Research Test Section at UMN

**FIGURE 9.** The Schematic of the UMN Experimental Facility and the Test Section.

1----oscillatory flow generator, 2----piston, 3----flow distributor, 4----cooler, 5----plenum,
6----regenerator, 7----screen matrix, 8----electrical heating coil, 9----isolation duct,
10----hot-wire, 11----thermocouple, 12----cooling water in, 13----cooling water out

Frequency = 0.4 Hz (24 RPM), Stroke = 356 mm, Piston Diameter = 356 mm
Regen. Porosity = 90%, Dimensionless Parameters Similar to an Infinia Engine
UMN Wire Screen Regenerator
(Considered to Yield an Approximation of Random Fiber)

• 90 % Porosity
• Stainless Steel 304 Welded Screens
• 200 Layers of 6.3 mm x 6.3 mm Mesh
• Wire Diameter = 0.81 mm
• Each Screen Rotated 45 Deg. Rel. to Next

• Representative Stirling Engine Chosen to define Test Section Dimensionless Parameters (Max. Reynolds # & Valensi #)
• Hot-Wire Anemometry Measurements

Glenn Research Center
at Lewis Field
UMN Test Data: Temperature Difference between Gas/Solid in Regenerator

Measurements made with Hot Wire in Axial Center at 80th Layer of Screen

Crankangle (degrees)
CSU CFD “Micro-” and “Macro-” Calculations & UMN Test Data
(Micro-Calculations made using REV of actual geometry)
(Macro-Calculations made with CFD-ACE Thermal Equilibrium Porous Media Model)
Regenerator Gas and Solid Temperatures at Hot-End, Mid-Point and Cold-End as Predicted by 1-D Sage Code—for Infinia’s TDC 55 We Engine (Using 1-D Thermal Non-Equilibrium Regenerator Model)

TDC Operating Conditions & Predictions

- Operating Conditions (Not Design) were
  - Hot-End Temperature = 823 K (550 C)
  - Cold-End Temperature = 363 K (90 C)
  - Frequency of Operation = 82.5 Hz
  - Helium Mean Pressure = 2.59 MPa (376 psi)
- Performance Predictions were
  - Engine Electrical Power = 55.9 We
  - Engine Efficiency = 19.4%
  - Heat Into Engine = 288 W
  - Regenerator Enthalpy Flux Loss = 22.4 W

Regen. Enthalpy Flux Loss \( \equiv \int m c_p T_{gas} \, dt \)
Non-Equilibrium Porous-Media Conservation Equations

**Continuity:**
\[
\frac{\partial \langle \rho \rangle^f}{\partial t} + \frac{1}{\beta} \nabla \cdot \left[ \langle \rho \rangle^f \langle u \rangle \right] = 0
\]

**Momentum:**
\[
\frac{1}{\beta} \frac{\partial \left( \langle \rho \rangle^f \langle u \rangle \right)}{\partial t} + \frac{1}{\beta^2} \nabla \cdot \left[ \langle \rho \rangle^f \langle u \rangle \langle u \rangle \right] = -\nabla \langle p \rangle^f + \nabla \cdot \left( \frac{\langle \rho \rangle^f}{\beta} \nabla \langle u \rangle - \langle \rho \rangle^f \frac{C_f}{\sqrt{K}} \langle u \rangle \right) - \frac{\langle \mu \rangle^f}{K} \langle u \rangle - \langle \rho \rangle^f \frac{C_f}{\sqrt{K}} \langle u \rangle \langle u \rangle
\]

**Fluid Energy:**
\[
\frac{\partial \left( \langle \rho \rangle^f \langle h \rangle^f \right)}{\partial t} + \frac{1}{\beta} \nabla \cdot \left[ \langle \rho \rangle^f \langle u \rangle \langle h \rangle^f \right] = \nabla \cdot \left[ \frac{k_{fe}}{\langle u \rangle} \nabla \langle T \rangle^f \right] + \left( \frac{\mu}{K} + \langle \rho \rangle^f \frac{C_f}{\sqrt{K}} \langle u \rangle \right) \langle u \rangle \cdot \langle u \rangle + \frac{dA_sf}{dV_f} (\langle T \rangle^s - \langle T \rangle^f) + \frac{d\langle p \rangle^f}{dt}
\]

**Solid Energy:**
\[
\frac{\partial \left( \rho_s C_s \langle T \rangle^s \right)}{\partial t} = \nabla \cdot \left[ \frac{k_{se}}{\langle u \rangle} \nabla \langle T \rangle^s \right] - h_{sf} \frac{dA_sf}{dV_s} (\langle T \rangle^s - \langle T \rangle^f)
\]
Porous-Media Quantities Needing Definition or “Closure”

Momentum Equation:
\[
\frac{1}{\beta} \frac{\partial}{\partial t} \left( \langle \rho \rangle^f \langle \mathbf{u} \rangle^f \right) + \frac{1}{\beta^2} \nabla \cdot \left[ \langle \rho \rangle^f \langle \mathbf{u} \rangle^f \langle \mathbf{u} \rangle^f \right] = -\nabla \langle p \rangle^f + \nabla \cdot \left( \frac{\langle v_{eff} \rangle^f}{\beta} \langle \rho \rangle^f \nabla \langle \mathbf{u} \rangle^f - \frac{\langle \rho \rangle^f}{\beta} \langle \mathbf{u} \mathbf{u} \rangle^f \right) - \frac{\langle \mu \rangle^f}{K} \langle \mathbf{u} \rangle^f - \frac{\langle \rho \rangle^f}{\sqrt{K}} \langle \mathbf{u} \rangle^f \frac{C_f}{K} \langle \mathbf{u} \rangle^f \langle \mathbf{u} \rangle^f
\]

Fluid Energy:
\[
\frac{\partial}{\partial t} \left( \langle \rho \rangle^f \langle h \rangle^f \right) + \frac{1}{\beta} \nabla \cdot \left[ \langle \rho \rangle^f \langle \mathbf{u} \rangle^f \langle h \rangle^f \right] = \nabla \cdot \left[ \frac{\bar{f}}{k_{fe}} \mathbf{u} \mathbf{u} \right] + \left( \frac{\mu}{K} + \langle \rho \rangle^f \frac{C_f}{\sqrt{K}} \right) \mathbf{u} \cdot \mathbf{u} + \frac{d}{dt} \frac{A_{sf}}{dV_f} \left( \langle T \rangle^s - \langle T \rangle^f \right) + \frac{d}{dt} \frac{\langle p \rangle^f}{dV_f}
\]

Solid Energy:
\[
\frac{\partial}{\partial t} \left( \rho_s C_s \langle T \rangle^s \right) = \nabla \cdot \left[ \frac{\bar{f}}{k_{se}} \mathbf{u} \mathbf{u} \right] - \frac{d}{dt} \frac{A_{sf}}{dV_s} \left( \langle T \rangle^s - \langle T \rangle^f \right)
\]
Determination of Permeability and Inertial Coefficient

(1) U. of Minn. Large-Scale Wire Screen Measurements

(2) Assume Quasi-Steady Flow, Equate Darcy-Forchheimer Steady 1-D Momentum Equation with Darcy Friction Factor Momentum Eq. & Use NASA/Sunpower Oscillating-Flow Rig Darcy Friction-Factor Data

\[
\frac{\nabla p}{L} = \frac{\mu}{K} u + \frac{C_f}{\sqrt{K}} \left\langle \rho \right\rangle^f u^2
\]

\[
\frac{\nabla p}{L} = \frac{f_D}{d_h} \frac{1}{2} \left\langle \rho \right\rangle^f u^2
\]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>UMN Large-Scale Screens ((d_w=8.1E-4 \text{ m}))</th>
<th>TDC Random Fiber</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>UMN Old, Experimental</td>
<td>UMN New, Experimental</td>
</tr>
<tr>
<td>K ((\text{m}^2))</td>
<td>1.07E-7</td>
<td>1.86E-7</td>
</tr>
<tr>
<td>K/d_w^2</td>
<td>0.163</td>
<td>0.283</td>
</tr>
<tr>
<td>C_f</td>
<td>0.049</td>
<td>0.052</td>
</tr>
</tbody>
</table>
Fluid & Solid Effective Thermal Conductivity Tensors

- Assume only diagonal elements of tensors are non-zero

- Then, in terms of 3-D cylindrical coordinates--

\[
\bar{k}_{fe} = \begin{bmatrix} k_{fe,rr} & 0 & 0 \\ 0 & k_{fe,\theta\theta} & 0 \\ 0 & 0 & k_{fe,xx} \end{bmatrix} = \begin{bmatrix} k_f + k_{f,tor,rr} + k_{dis,rr} & 0 & 0 \\ 0 & k_f + k_{f,tor,\theta\theta} + k_{dis,\theta\theta} & 0 \\ 0 & 0 & k_f + k_{f,tor,xx} + k_{dis,xx} \end{bmatrix} = \begin{bmatrix} k_f + k_{f,tor,rr} & 0 & 0 \\ 0 & k_f + k_{f,tor,\theta\theta} & 0 \\ 0 & 0 & k_f + k_{f,tor,xx} \end{bmatrix}
\]

\[
\bar{k}_{se} = \begin{bmatrix} k_{se,rr} & 0 & 0 \\ 0 & k_{se,\theta\theta} & 0 \\ 0 & 0 & k_{se,xx} \end{bmatrix} = \begin{bmatrix} f(k_s, k_{s,tor,rr}) & 0 & 0 \\ 0 & f(k_s, k_{s,tor,\theta\theta}) & 0 \\ 0 & 0 & f(k_s, k_{s,tor,xx}) \end{bmatrix}
\]
Experimental Values of Fluid Thermal Dispersion Conductivity

- In Table below, $k_{dis,yy}$, $k_{dis,xx}$, respectively, are the radial and axial thermal dispersion conductivities determined by various experimental measurements.

<table>
<thead>
<tr>
<th>Porous media</th>
<th>Estimated Thermal Dispersion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Welded Screen</td>
<td>$\varepsilon_{M, eddy} \frac{k_{dis,yy}}{\rho_f c_p} = 0.02d_h U$ or $\frac{k_{dis,yy}}{k_f} = 0.02 Pe$</td>
</tr>
<tr>
<td>Fibrous Media</td>
<td>$\frac{k_{dis,yy}}{k_f} = 0.0011 Pe$</td>
</tr>
<tr>
<td>Packed Spheres</td>
<td>$\frac{k_{dis,yy}}{k_f} = (0.03-0.05) Pe$ and $\frac{k_{dis,xx}}{k_f} = 0.073 Pe^{1.59}$</td>
</tr>
<tr>
<td>Woven Screen</td>
<td>$\frac{k_{dis,xx}}{k_f} = 0.50 Pe^{0.62} \beta^{-2.91}$ or $\frac{k_{dis,xx}}{k_f} \approx 0.06 Pe$ for $\beta = 0.9, Pe = 560$</td>
</tr>
</tbody>
</table>
Estimates of Fluid-Stagnant and Solid-Effective Thermal Conductivities
--for 90% porosity wire screen or random fiber matrix---#1

• Base calculations below on air, \( k_f = 0.026 \text{ W/m-K} \) and stainless steel, \( k_s = 13.4 \text{ W/m-K} \)

• For radial & azimuthal directions, assume approximately parallel solid & fluid flow paths & thus based on the parallel model:

\[
\begin{align*}
{k_f}_{stag} &= k_f \beta = (26 \times 10^{-3} \text{ W/mK}) (0.90) = 0.0234 \text{ W/mK} \\
{k_{se}} &= k_s (1-\beta) = (13.4 \text{ W/mK}) (0.1) = 1.34 \text{ W/mK}
\end{align*}
\]

• McFadden’s (UMN) calculations based on actual welded screen geometry suggest parallel model value of solid effective thermal conductivity should be multiplied by 0.625

\[
\therefore \quad k_{se} = k_s (1-\beta) 0.625 = 1.34 \text{ W/mK} \times 0.625 = 0.838
\]
Estimates of Fluid-Stagnant and Solid-Effective Thermal Conductivities
--for 90% porosity wire screen or random fiber matrix---#2

- For the axial direction use a modification of the series model
- A lumped effective solid + fluid effective thermal conductivity (not including thermal dispersion), based on the series model is --

\[
k_{eff,s+f} = \left( \frac{1}{\frac{\beta}{k_f} + \frac{1 - \beta}{k_s}} \right) = \left( \frac{1}{0.90 \times 10^{-3} + 13.4} \right) = 0.0289 \text{ W/mK}
\]

- Only slightly larger than molecular fluid cond. of 0.026 W/m-K & likely too small since series model implies wires not touching in axial direction
- 3-D CFD microscopic simulation of REV of the UMN welded screen by Rong of CSU suggests above series value should be multiplied by 2.157

\[
\therefore \ k_{eff,s+f} = \left( 0.0289 \text{ W/mK} \right) \left( 2.157 \right) = 0.0623 \text{ W/mK}
\]

- The above would be appropriate for an equilibrium model. Seeing no obvious way to separate values for a fluid & solid in the axial direction—propose using the same value for fluid-stagnant and solid-effective cond. in the axial direction, hoping that overall effect will be reasonable.
Heat Transfer Coefficients Between Fluid & Solid Matrix

- Good sources of heat transfer coefficient correlations for wire screen and random fiber matrices are Gedeon’s Sage manuals and NASA Technical Memorandums containing NASA/Sunpower oscillating-flow rig data.

- The following correlations are in terms of Nusselt No., Peclet Number (Reynolds No. x Prandtl No.), and porosity:
  - For wire screen:  \[ Nu = (1.0 + 0.99 \ Pe^{0.66}) \beta^{1.79} \]
  - For random fiber:  \[ Nu = (1.16 + 1.16 \ Pe^{0.66}) \beta^{2.61} \]
  - where--

\[
Nu = \frac{hd}{h}, \quad Pe = Re Pr = \frac{\rho u d}{\mu} \frac{c_p}{\mu} \frac{\mu}{k}
\]

Oscillating-Flow Rig
Hydrodynamic Dispersion Term (Possibly Negligible Effect for this Application—This Needs to be Checked)

- Hydrodynamic dispersion term--fluid momentum equation: \( \frac{1}{\beta} \langle \tilde{u}\tilde{u} \rangle \approx \tilde{u}\tilde{u} \)

\( \beta = \text{porosity}; \quad u = \text{average-channel fluid, or local, velocity inside matrix} \)

\( \tilde{u}\tilde{u} = ii \tilde{u}\tilde{u} + ij \tilde{u}\tilde{v} + ik \tilde{u}\tilde{w} + ji \tilde{v}\tilde{u} + jj \tilde{v}\tilde{v} + jk \tilde{v}\tilde{w} + ki \tilde{w}\tilde{u} + kj \tilde{w}\tilde{v} + kk \tilde{w}\tilde{w} \)

- Most important terms from the above tensor expression are for transport normal to the flow and are: \(|\tilde{u}\tilde{v}|\) or \(|\tilde{v}\tilde{u}|\)

- Niu of UMN—based on regenerator experiments--showed that:

\[
\frac{\langle \tilde{u}\tilde{v} \rangle}{\beta^2} = \frac{\langle \tilde{v}\tilde{u} \rangle}{\beta^2} \approx \frac{\langle u'v' \rangle}{\beta^2} = -\frac{1}{\beta^2} \varepsilon M \frac{\partial U}{\partial r} = -\frac{1}{\beta^2} 0.02 dh U \frac{\partial U}{\partial r} = -\frac{1}{\beta^2} \lambda dh U \frac{\partial U}{\partial r}
\]

- This information could be used to check the significance of the term—but has not been done, yet.
Concluding Remarks

• A thermal-non-equilibrium porous-media model is needed to replace the existing thermal equilibrium models in CFD codes for accurate simulation of Stirling regenerators.

• Transient, compressible-flow, conservation equations are summarized for reference in discussing porous-media model parameters needing definition for a macroscopic, thermal-non-equilibrium porous-media model.

• Available experimental information is discussed for definition of the hydrodynamic-dispersion term and the permeability & inertial coefficients in the momentum equation, and the heat transfer coefficient & thermal dispersion terms in the energy equations.

• Methods are outlined for estimating fluid-stagnant and solid-effective thermal conductivities.

• Adequate information is given for definition of an initial thermal non-equilibrium porous-media model for use in a CFD model of the regenerator of a TDC Stirling engine. Still need to implement.

• Application of this model in Stirling CFD codes may demonstrate that further refinement of these parameters, or of the model itself may be required.